HANDBOOK FOR
INTERMEDIATE FIRST YEAR

MATHEMATICS

BOARD OF INTERMEDIATE EDUCATION

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CHAPTER NAME

SETS

Exercise 1(a)

I) 1) Which of the following are sets? Justify your answer.

(i) The collection of all the months of a year beginning with the letter J.

(ii) The collection of ten most talented writers of India,

(iii) A team of eleven best-cricket batsmen of the word.

(iv)The collection of all boys in your class

(v)The collection of all natural numbers less than 100

(vi) A collection of novels written by the writer munshi Premchand

(vii) The collection of all even integers.

(viii) The collection of questions in this chapter.

(ix) A collection of most dangerous animals of the world.

Sol.: (i)The collection of all months of a year beginning with letter J are, January, June , July, the collection is well defined, so it is a set.

(ii)It is not a set, since one cannot decide a writer is most talented, because it depends on ones own perspective.

(iii)It is not a set, a batsmen may be best for one person and may not be for the other

(iv) It is a set, since the collections is well defined.

(v)It is a set, since the collection is well defined.

(vi) It is a set, since the collection is well defined.

(vii) It is a set, since the collection is well defined.

(viii) It is a set, since the collection is well defined.

(ix) It is not a set, An animal is most dangerous for one person and may not be for another, so the collection is not well defined.

2) Let $A=\{1,2,3,4,5,6\}$ **Insert the appropriate symbol** $\in$ **or** $\notin$ **in the blank spaces.**

(i)5….A (ii)8…A (iii)o…A (iv) 4...A (v) 2...A (vi)10…A

Sol.: - (i) $5 \in A$ (ii) $8 \notin A$ (iii) $o \notin A$ (iv) $4 \in A$ (v) $2 \notin A$ (vi) $10 \notin A$

3) Write the following sets in roster from:

(i) $A=\{x : x \text{ is an integer and } -3 \le x < 7\}$

(ii) $A=\{x : x \text{ is a natural number less than } 6\}$

(iii) $C=\{x : x \text{ is a two} - \text{digit natural number such that the sum of its digits is } 8\}$

(iv) $D=\{x : x \text{ is a prime number which is divisor of } 60\}$

(v) E = *The set of all letters in the word TRIGONOMETRY*

(vi) F = The set of all letters in the word BETTER

Sol.: (i) $A=\{-3,-2,-1,0,1,2,3,4,5,6\}$

(ii) $B=\{1,2,3,4,5\}$

(iii) $C=\{17,26,35,44,53,62,71,80\}$

(iv) $D=\{2,3,5\}$

(v) $E=\{T,R,I,G,O,N,M,E,Y\}$

(vi) $F=\{B,E,T,R\}$

4) Write the following sets in the set- builder from

(i) $\{3,6,9,12\}$ (ii) $\{2,4,8,16,32\}$ (iii) $\{5,25,125,625\}$

(iv) $\{2,4,6,....\}$ (v) $\{1,4,9,...100\}$

Sol.: (i) $\{x : x \text{ is natural number multiple of } 3 \text{ and } x<15\}$ (ii) $\{x : x=2^n, n\in N \text{ and } n\le 5\}$

(iii) $\{x : x=5^n, n\in N \text{ and } n\le 4\}$ (iv) $\{x : x \text{ is an even natural number}\}$

(v) $\{x : x=n^2, n\in N \text{ and } n\le 10\}$

5) List all the elements of the following sets:

(i) $A=\{x : x \text{ is an odd natural number}\}$

(ii) $B=\left\{x : x \text{ is an integer}, -\frac{1}{2}<x<\frac{9}{2}\right\}$

(iii) $C=\{x : x \text{ is an integer}, x^2\le 4\}$

(iv) $D=\{x : x \text{ is a letter in the word "LOYAL"}\}$

(v) $E=\{x : x \text{ is month of a year not having } 31 \text{ days}\}$

(vi) $F=\{x : x \text{ is a consonant in the English alphabet which prcedes } k\}$

Sol.:(i) $A=\{1,3,5,7....\}$

(ii) $B=\{0,1,2,3,4\}$

(iii) $C=\{-2,-1,0,1,2\}$

(iv) $D = \{L, O, Y, A\}$

(v) $E = \{February, April, June, September, November\}$

(vi) $F = \{b, c, d, f, g, h, j\}$

6) Match each of the set on the left in the roster from with the same set on the right described in set –builder from:

(i) $\{1, 2, 3, 6\}$ (Sol.: $\{x : x \text{ is a prime number and a divisor of } 6\}$

(ii) $\{2, 3\}$ (b) $\{x : x \text{ is an odd natural number less than } 10\}$

(iii) $\{M, A, T, H, E, I, C, S\}$ (c) $\{x : x \text{ is a natural number and divisor of } 6\}$

(iv) $\{1, 3, 5, 7, 9\}$ (d) $\{x : x \text{ is a letter of the word } MATHEMATICS\}$

Sol.: (i)c (ii)a (iii)d (iv)b

Exercise 1 (b)

I)1. Which of the following are examples of the null set.

(i)Set of odd natural numbers divisible by 2.

(ii)Set of even prime numbers

(iii)$\{x{:}x$ is a natural numbers, $x < 5$ *and* $x > 7\}$

(iv) $\{y{:}y$ is a point common to any two parallel lines $\}$

Sol.: (i)Set of odd natural numbers divisible by 2 is a null set

(ii)Set of even prime numbers = $\{2\}$ so it is not a null set

(iii) $\{x{:}x$ is a natural number, $x < 5$ and $x > 7\}$ is a null set

(iv) $\{y{:}y$ is a point common to any two parallel lines $\}$ is a null set

2) Which of the following sets are finite or infinite

(i)The set of months of a year

(ii) $\{1, 2, 3, ...\}$

(iii) $\{1, 2, 3, ...99, 100\}$

(iv)The set of positive integers greater than 100

(v) The set of prime numbers less than 99

Sol.: (i)The set contains 12 elements so it is a finite set

(ii)The set is an infinite set

(iii) The set contains 100 elements, so it is a finite set

(iv) The set is an infinite set

(v) Prime numbers less than 99 are finite number. So it is a finite set.

3) State whether each of the following set is finite or infinite.

(i)The set of lines which are parallel to the *x-axis*

(ii) The set of letters in the English alphabet.

(iii) The set of numbers which are multiple of 5

(iv) The set of animals living on the earth

(v) The set of circles passing through the origin (o,o)

Sol.: **(i)**There will be infinite number of lines parallel to *x-axis*. So the set is an infinite set

(ii) The set contains 26 elements, so it is a finite set

(iii) the set of numbers which are multiples of 5 is an infinite set

(iv) The set of animals living on the earth is an infinite set

(v) The set of circles passing through the origin (0,0) is an infinite set.

4) In the following, state whether A= B or not:

(i) A = {a, b, c, d} B= {d, c, b, a}

(ii) A= {4, 8, 12, 16} B = {8,4,16,18}

(iii)A= {2, 4, 6, 8, 10} B= (x:x is positive even integer and $x \le 10$ }

(iv) A= { x:x is a multiple of 10 }, B = {10,15,20,25,30,…..}

Sol.: (i) $\mathbf{A} = \mathbf{B}$, (ii) $\mathbf{A} \ne \mathbf{B}$

(iii) $\mathbf{A} = \{2,4,6,8,10\}$, $\mathbf{B} = \{2,4,6,8,10\}$. $\therefore \mathbf{A} = \mathbf{B}$

(iv) $\mathbf{A} = \{10,20,30....\}$ $\mathbf{B} = \{10,15,20,25,30,....\}$ $\therefore \mathbf{A} \ne \mathbf{B}$

5) Are the following pair of sets equal ? Give reasons.

(i) A= {2,3} B = { x:x is Sol.:of $x^2 + 5x + 6 = 0$ }

(ii) A= {x:x is a letter of the word FOLLOW}

B: {y:y is a letter in the word WOLF}

Sol.: **(i)**A = {2,3}, B={-2,-3} , $\therefore$ A $\ne$ B

(ii) A = {F,O,L,W} B = {W,O,LF} $\therefore$ A = B

6) From the sets given below, select equal sets.

A= {2,4,8,12} B= {1,2,3,4} C = {4,8,12,14} D ={ 3,1,4,2}

E= {-1,1} F = {0,a} G = { 1,-1} H ={ 0,1}

Sol.: B = D, E = G

Exercise 1 (c)

I.(1) Make correct statements by filling in the symbol $\subset$ or $\not\subset$ in the blank spaces.

(i) { 2,3,4,}….{1,2,3,4,5}

(ii) { a ,b,c}….{b,c,d}

(iii) {x:x is a student of class XI of your school }….. { x:x student of your school}

(iv) { x:x is a circle in the plane }..{ x:x is a circle in the same plane with radius 1 unit}

(v) { x:x is an equilateral triangle in a plane }..{ x:x is a rectangle in the plane }

(vi) {x:x is an equilateral triangle in a plane }.. { x:x is a triangle in the same plane}

(vii) { x:x is an even natural number }…..{x:x is a integer)

Sol.: (i) { 2,3,4,} $\subset${1,2,3,4,5}

(ii) { a ,b,c} $\not\subset$ {b,c,d}

(iii) {x:x is a student of class XI of your school } $\subset$ { x:x student of your school}

(iv) { x:x is a circle in the plane } $\not\subset$ { x:x is a circle in the same plane with radius 1 unit}

(v) { x:x is an equilateral triangle in a plane } $\not\subset$ { x:x is a rectangle in the plane }

(vi) {x:x is an equilateral triangle in a plane } $\subset$ { x:x is a triangle in the same plane}

(vii) { x:x is an even natural number } $\subset${x:x is a integer)

2) Examine whether the following statements are true or false.

(i){a,b} $\not\subset$ {b,c,a}

(ii) { a,e} $\subset$ {x:x is a vowel in the English alphabet }

(iii) { 1,2,3} $\subset${ 1,3,5}

(iv) {a} $\subset${a,b,c}

(v) {a} $\in$ { a,b,c}

(vi) { x:x is an even natural number less than 6 } $\subset$ {x:x is a natural number which divides 36}

Sol.: (i){ a,b } $\not\subset$ { b,c,a} is False .

(ii) { a,e} $\subset${ x” x is a vowel in the English alphabet } is True

(iii) { 1,2,3} $\subset${1,3,5} is False

(iv){ a } $\subset$ { a,b,c} is True

(v) {a} $\in$ { a,b,c} is False

(vi) { x:x is an even natural number less than 6} $\subset$ { x:x is a natural number with divides 36} is True

3) Let A = { 1,2,{3,4},5} which of the following statements are incorrect and why?

(i) { 3,4} $\subset$ A (ii) { 3,4} $\in$ A (iii) {{3,4}} $\subset$ A (iv) 1 $\in$ A

(v) 1 $\subset$ A (vi){ 1,2,5} $\subset$ A (vii) { 1,2,5} $\in$ A (viii){1,2,3} $\subset$ A

(ix) $\phi \in \mathbf{A}$ (x) $\phi \subset \mathbf{A}$ (xi) $\{\phi\} \subset \mathbf{A}$

Sol.: (i){3,4} $\subset$ A is incorrect because {3,4} is an element of A, but not a set.

(ii) {3,4} $\in$ A correct

(iii) {{3,4}} $\subset$ A correct

(iv) $1 \in \mathbf{A}$ correct

(v) $1 \subset \mathbf{A}$ is incorrect because 1 is an elements of A, but not a set

(vi) { 1,2,5} $\subset$ **A** correct

(vii) { 1,2,5} $\in$ **A** is incorrect because { 1,2,5} is a set but not an element

(viii) {1,2,3} $\subset$ **A** is incorrect because 3 is not an element of A.

(ix) $\phi \in \mathbf{A}$ is incorrect because ϕ is a set but not an element .

(x) $\phi \subset \mathbf{A}$ is correct

(xi) $\{\phi\} \subset \mathbf{A}$ is incorrect since ϕ is an empty set, but not an element

4) Write down all the subsets of the followings sets .

(i) {a} (ii) {a,b} (iii){1,2,3} (iv) ϕ

Sol.: **(i)**Subsets of {a} are ϕ, {a}, **(ii)**Subsets of {a,b} are ϕ, {a},{b},{a,b}

(iii) Subsets of {1,2,3} are ϕ, {1}, {2}, {3}, {1,2},{1,3},{2,3},{1,2,3}

(iv) Subsets of ϕ is ϕ

5) Write the following as intervals.

i) $\{x : x \in R, -4 < x \leq 6\}$ ii) $\{x : x \in R, -12 < x < -10\}$

iii) $\{x : x \in R, o \leq x < 7\}$ (iv) $\{x : x \in R, 3 \leq x \leq 4\}$

Sol.: (i) (-4,6] (ii) (-12,-10) (iii) [0,7) (iv) $[3,4]$

6)Write the following intervals in set- builder from.

(i) (-3,0) (ii) $[6,12]$ (iii) $(6,12]$ iv) $[-23,5)$

Sol.: (i) $\{x : x \in R, -3 < x < 0\}$, (ii) $\{x : x \in R, 6 \leq x \leq 12\}$

(iii) $\{x : x \in R, 6 < x \leq 12\}$, (iv) $\{x : x \in R, -23 \leq x < 5\}$

7) What universal set(s) would you propose for each of the following

(i) The set of right triangles (ii) The set of isosceles triangles.

Sol.: (i)Universal set for set of right angle triangles is the set of all triangles

(ii)Universal set for the set of isosceles triangles is the set of all triangles.

8) Given the sets A= {1,3,5} B = {2,4,6} and C = {0,2,4,6,8}

Which of the following may be considered as universal set (s) for all the three sets A, B and C

(i)$\{0,1,2,3,4,5,6\}$ (ii) ϕ (iii)$\{0,1,2,3,4,5,6,7,8,9,10\}$

(iv) $\{1,2,3,4,5,6,7,8\}$

Sol.: $\{0,1,2,3,4,5,6,7,8,9,10\}$ is the universal set for all the three sets A, B and C.

Exercise I (d)

I. 1. Let A = {a,b}, B = {a,b,c}. Is $A \subset B$? what is $A \cup B$?

Sol.: $\mathbf{A \subset B}$. $\mathbf{A \cup B}$ = { a,b,c} = **B**

2) If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Sol.: Since $\mathbf{A \subset B}$, $\mathbf{A \cup B}$ = B.

3) Which of the following pairs of sets are disjoint

(i)$\{1,2,3,4\}$ and $\{x$:x is a natural number and $4 \leq x \leq 6\}$

(ii) { a,e,i,o,u} and { c,d,e,f}

(iii)$\{x$:x is an even integer } and $\{x$:x is an odd integer$\}$.

Sol.: (i)Let A = {1,2,3,4}

And B = $\{x$:x is a natural number and $4 \leq x \leq 6\}$ = {4,5,6}.

$\mathbf{A} \cap B$ = {4}, $\therefore$ Given sets are not disjoint.

(ii)Let A = {a,e,i,o,u} B= {c,d,e,f}

$\mathbf{A} \cap B$ = {e}, $\therefore$ Given sets are not disjoint.

(iii) Let A = { x:x is an even integer }= {2,4,6,8,10,…}

B = {x;x is an odd integer} = {3,5,7,9,…}

$\mathbf{A \cap B} = \phi$, $\therefore$ Given sets are disjoint sets.

4) If $X = \{a,b,c,d\}$ and $Y = \{f,b,d,g\}$ find

(i)X-Y (ii)Y-X (iii) $\mathbf{X \cap Y}$

Sol.: (i) $\mathbf{X - Y} = \{a,c\}$, (ii) $\mathbf{Y - X} = \{f,g\}$, (iii) $\mathbf{X \cap Y} = \{b,d\}$

5) If R is the set of real numbers and Q is the set of rational numbers, then what is R-Q?

Sol.: R- Q = set of irrational numbers.

6) State whether each of the following statement is true or false justify your answer

(i){2,3,4,5} and {3,6} are disjoint sets, (ii){a,e,i,o,u} and {a,b,c,d} are disjoint sets

(iii) {2,6,10,14} and { 3,7,11,15} are disjoint sets,

(iv){2,6,10} and {3,7,11} are disjoint sets

Sol.: (i)Let A= {2,3,4,5} , B = {3,6,}

$\mathbf{A} \cap \mathbf{B} = \{3\}$, Given sets are not disjoint , so the statement is false.

(ii)Let A = {a,e,i,o,u,}, B ={a,b,c,d}

$\mathbf{A} \cap \mathbf{B} = \{a\}$, Given sets are not disjoint, so the statement is false.

(iii)Let A = {2,6,10,14}, B= {3,7,11,15}

$\mathbf{A} \cap \mathbf{B} = \phi$, Given sets are disjoint , so the statement is true.

(iv) Let A = {2,6,10} , B= {3,7,11}

$\mathbf{A} \cap \mathbf{B} = \phi$, Given sets are disjoint, so the statement is true

II 1) Find the union of each of the following pairs of sets

(i) X = {1,3,5} Y = {1,2,3}

(ii) A = { a,e,i,o,u} B = { a, b,c }

(iii) A = {x:x is a natural number and multiple of 3 },

B ={x:x is a natural number less than 6 }

(iv) A= {x:x is a natural number and $1 < x \le 6$ }

B= { x:x is a natural number and 6<x<10}

v) A = {1,2,3}, B= ϕ

Sol.: (i) $X \cup \mathbf{Y}$ = { 1,2,3,5}

(ii) $\mathbf{A} \cup \mathbf{B}$ = { a,b,c,e,i,o,u}

(iii) A = $\{3, 6, 9, 12, ...\}$, B = $\{1, 2, 3, 4, 5\}$

$\mathbf{A} \cup \mathbf{B}$ = { 1,2,3,4,5,6,9,12,….}

(iv) A= {1,2,3,4,5,6}, B = {7,8,9}

$\mathbf{A} \cup \mathbf{B}$ = { 2,3,4,5,6,7,8,9}

(v) $\mathbf{A} \cup \mathbf{B}$ = {1,2,3}

2). Find the intersection of each pair of set of question 1 above.

Sol.: (i) $X \cap Y = \{1,3\}$ (ii) $A \cap B = \{a\}$ (iii) $A \cap B = \{3\}$

(iv) $A \cap B = \emptyset$ (v) $A \cap B = \emptyset$

3) If A = {1,2,3,4}, B = {3,4,5,6}, C= {5,6,7,8} and D = {7,8,9,10} find

(i) $\mathbf{A \cup B}$ ii) $\mathbf{A \cup C}$ iii) $\mathbf{B \cup C}$ iv) $\mathbf{B \cup D}$ v) $\mathbf{A \cup B \cup C}$

vi) $\mathbf{A \cup B \cup D}$ vii) $\mathbf{B \cup C \cup D}$

Sol.: (i): $\mathbf{A \cup B} = \{1,2,3,4,5,6\}$, (ii) $\mathbf{A \cup C} = \{1,2,3,4,5,6,7,8\}$

(iii) $B \cup C = \{3,4,5,6,7,8\}$, (iv) $B \cup D = \{3,4,5,6,7,8,9,10\}$

(v) $\mathbf{A} \cup B \cup \mathbf{C} = \{1,2,3,4,5,6,7,8\}$, (vi) $\mathbf{A} \cup B \cup \mathbf{D} = \{1,2,3,4,5,6,7,8,9,10\}$

(vii) $\mathbf{B \cup C \cup D} = \{3,4,5,6,7,8,9,10\}$

4) If A= {3,5,7,9,11} , B = {7,9,11,13} , C = {11,13,15} and D = {15,17} find

(i) $\mathbf{A \cap B}$ ii) $\mathbf{B \cap C}$ iii) $\mathbf{A \cap C \cap D}$ iv) $\mathbf{A \cap C}$

v) $\mathbf{B \cap D}$ vi) $\mathbf{A \cap B \cup C}$ vii) $\mathbf{A \cap D}$ viii) $\mathbf{A \cap (B \cap D)}$

ix) $(\mathbf{A \cap B}) \cap (\mathbf{B \cup C})$ x) $(\mathbf{A \cup D}) \cap (\mathbf{B \cup C})$

Sol.:

(i) $\mathbf{A \cap B} = \{7,9,11\}$

(ii) $\mathbf{B \cap C} = \{11,13\}$,

(iii) $\mathbf{A \cap C \cap D} = \{\ \}$ or ϕ

(iv) $\mathbf{A \cap C} = \{11\}$,

(v) $\mathbf{B \cap D} = \{\ \}$ or ϕ

(vi) $\mathbf{B \cup C} = \{7,9,11,13,15\}$
$\mathbf{A \cap (B \cup C)} = \{7,9,11\}$

(vii) $\mathbf{A \cap D} = \{\ \}$ or ϕ

(viii) $\mathbf{B \cup D} = \{7,9,11,13,15,17\}$
$\mathbf{A \cap (B \cup D)} = \{7,9,11\}$

(ix) $\mathbf{A \cap B} = \{7,9,11\}$
$\mathbf{B \cup C} = \{7,9,11,13,15\}$
$(\mathbf{A \cap B}) \bigcap (\mathbf{B \cup C}) = \{7,9,11\}$

(x) $\mathbf{A \cup D} = \{3,5,7,9,11,15,17\}$
$\mathbf{B \cup C} = \{7,9,11,13,15\}$
$(\mathbf{A \cup D}) \cap (\mathbf{B \cup C}) = \{7,9,11\}$

III 1) If A ={ *x*:*x* is a natural number} , B = {*x*:*x* is an even natural number } ,

C = {*x*:*x* is an odd natural number } and D = {*x*:*x* is a prime number } find

i) $\mathbf{A \cap B}$ ii) $\mathbf{A \cap C}$ iii) $\mathbf{A \cap D}$ iv) $\mathbf{B \cap C}$ v) $\mathbf{C \cap D}$

Sol.: A = {1,2,3,4,5….} B= {2,4,6,8…}

C = {3,5,7,9,…} D = {2,3,5,7,11…}

(i) $\mathbf{A} \cap \mathbf{B} = \{2,4,6......\}$, (ii) $\mathbf{A} \cap \mathbf{C} = \{3,5,7,9......\}$

(iii) $\mathbf{A} \cap \mathbf{D} = \{2,3,5,7......\}$, (iv) $\mathbf{B} \cap \mathbf{C} = \phi$

(v) $\mathbf{B} \cap \mathbf{D} = \{2\}$, (vi) $\mathbf{C} \cap \mathbf{D} = \{3,5,7,11,......\}$

2) If A= { 3,6,9,12,15,18,21} , B= {4,8,12,16,20} C= {2,4,6,8,10,12,14,16}, D = {5,10,15,20}, find (i) A- B (ii)A-C (iii)A-D (iv) B-A (v) C-A (vi)D-A

(vii) B-C (viii) B-D (ix) C-B (x) D-B (xi) C-D (xii)D-C

Sol.: (i)A- B = $\{3,6,9,15,18,21\}$, (ii) A - C = $\{3,9,15,18,21\}$

(iii)A-D = $\{3,6,9,12,18,21\}$, (iv) B-A= $\{4,8,16,20\}$

(v) C-A = , $\{2,4,8,10,14,16\}$, (vi) D-A = $\{5,10,20\}$

(vii)B-C = $\{20\}$, (viii) B- D = $\{4,8,12,16\}$

(ix) C-B = $\{2,6,10,14\}$, (ix) C- B = $\{2,6,10,14\}$

(x) D- B = $\{5,10,15\}$, (xi) C- D = $\{2,4,6,8,12,14,16\}$

(xii) D-C = $\{5,15,20\}$

<u>Exercise I (e)</u>

I) 1. If U = $\{1,2,3,4,5,6,7,8,9\}$, **A=** $\{2,4,6,8,\}$ **and B=** $\{2,3,5,7\}$ **Verify that**

(i) $(\mathbf{A} \cup \mathbf{B})^1 = \mathbf{A}^1 \cap \mathbf{B}^1$ ii) $(\mathbf{A} \cap \mathbf{B})^1 = \mathbf{A}^1 \cup \mathbf{B}^1$

Sol.: (i) $\mathbf{A} \cup \mathbf{B} = \{2,3,4,5,6,7,8\}$

$(\mathbf{A} \cup \mathbf{B})^1 = \{1,9\}$

$\mathbf{A}^1 = \{1,3,5,7,9\}$ $\mathbf{B}^1 = \{1,4,6,8,9\}$

$\mathbf{A}^1 \cap \mathbf{B}^1 = \{1,9\}$

$\therefore (\mathbf{A} \cup \mathbf{B})^1 = \mathbf{A}^1 \cap \mathbf{B}^1$

(ii) $\mathbf{A} \cup \mathbf{B} = \{2\}$

$(\mathbf{A} \cap \mathbf{B})^1 = \{1,3,4,5,6,7,8,9\}$

$\mathbf{A}^1 = \{1,3,5,7,9\} \quad \mathbf{B}' = \{1,4,6,8,9\}$

$\mathbf{A}^1 \cup \mathbf{B}^1 = \{1,3,4,5,6,7,8,9\}$

$\therefore (\mathbf{A} \cap \mathbf{B})^1 = \mathbf{A}^1 \cup \mathbf{B}^1$

2) Let $\cup$ be the set of all triangles in a plane. If A is set of all triangles with at least one angle different from 60^0, what is $\mathbf{A}^1$?

Sol.: $\mathbf{A}^1$ is the set of all triangles with every angle equal to 60^0 = set of equilateral triangles

3) Fill in the blanks to make each of the following a true statement

(i) $\mathbf{A} \cup \mathbf{A}^1 =$________ , (ii) $\phi' \cap \mathbf{A} =$______ , (iii) $\mathbf{A} \cap \mathbf{A}^1 =$ ___________
(iv) $U^1 \cap \mathbf{A} =$ _______

Sol.: (i) $\mathbf{A} \cup \mathbf{A}^1 = \cup$ (Universal set)
(ii) $\phi' \cap \mathbf{A} = U \cap \mathbf{A} = \mathbf{A}$
(iii) $\mathbf{A} \cap \mathbf{A}' = \phi$
(iv) $U^1 \cap \mathbf{A} = \phi \cap \mathbf{A} = \phi$

II 1) If $U = \{a,b,c,d,e,f,g,h\}$ find the complements of the following sets

(i) $\mathbf{A} = \{a,b,c\}$ (ii) $\mathbf{B} = \{d,e,f,g\}$ (iii) $C = \{a,c,e,g\}$ D) $\{f,g,h,a\}$

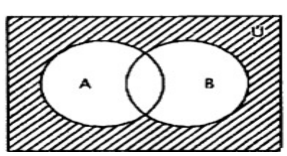
Sol.: (i) , $\mathbf{A}^1 = U - \mathbf{A} = \{d,e,f,g,h\}$ (ii) $\mathbf{B}' = U - \mathbf{B} = \{a,b,c,h\}$
(iii) $C^1 = U - C = \{b,d,f,h\}$ (iv) $\mathbf{D}' = U - \mathbf{D} = \{b,c,d,e\}$

2) Draw appropriate Venn diagram for each of the following

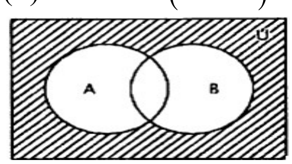
(i) $(A \cup B)^1$ (ii) $A^1 \cap B^1$ (iii) $(A \cap B)^1$ (i) $A^1 \cup B^1$

Sol:

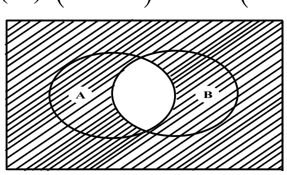
(i) $(\mathbf{A} \cup \mathbf{B})' = U - (\mathbf{A} \cap \mathbf{B})$,



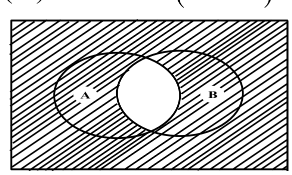
(ii) $\mathbf{A}' \cap \mathbf{B}' = (\mathbf{A} \cup \mathbf{B})' = U - (\mathbf{A} \cap \mathbf{B})$



(iii) $(\mathbf{A} \cap \mathbf{B})' = U - (\mathbf{A} \cap \mathbf{B})$



(iv) $\mathbf{A}' \cup \mathbf{B}' = (\mathbf{A} \cap \mathbf{B})' = U - (\mathbf{A} \cap \mathbf{B})$



III 1)Let $\cup = \{1,2,3,4,5,6,7,8,9\}$, $\mathbf{A} = \{1,2,3,4,\}$, $\mathbf{B} = \{2,4,6,8\}$ and $\mathbf{C} = \{3,4,5,6\}$ find

(i) $\mathbf{A}'$ (ii) $\mathbf{B}'$ iii) $(\mathbf{A} \cup \mathbf{C})'$ iv) $(\mathbf{A} \cup \mathbf{B})'$ v) $(\mathbf{A}')'$ vi) $(\mathbf{B} - \mathbf{C})'$

Sol.: (i) $\mathbf{A}' = U - \mathbf{A} = \{5,6,7,8,9\}$ (ii) $\mathbf{B}' = U - \mathbf{B} = \{1,3,5,7,9\}$
(iii) $\mathbf{A} \cup \mathbf{C} = \{1,2,3,4,5,6\} \quad \Rightarrow (\mathbf{A} \cup \mathbf{C})' = \{7,8,9\}$

(iv) $\mathbf{A} \cup \mathbf{B} = \{1,2,3,4,6,8\} \Rightarrow (\mathbf{A} \cup \mathbf{B})' = \{5,7,9\}$

(v) $(\mathbf{A}')' = U - \mathbf{A}' = \{1,2,3,4\} = \mathbf{A}$

(vi) $\mathbf{B} - \mathbf{C} = \{2,8\} \Rightarrow (\mathbf{B} - \mathbf{C})' = \{1,3,4,5,6,7,9\}$

2) Taking the set of natural numbers as the universal set, write down the complements of the following sets.

(i){x:x is an even natural number }
(ii){ x:x is an odd natural number }
(iii){ x:x is a positive multiple of 3 }
(iv){ x:x is a prime number}
(v) { x:x is a natural number divisible by 3 and 5 }
vi){ x:x is a perfect square}
Vii){ x:x is a prefect cube }
Viii) { x:x : $x+5=8$ }
ix) $\{x : 2x+5=9\}$
x) $\{x : x \geq 7\}$
(xi) $\{x : x \in \mathbf{N}\ and\ 2x+1 > 10\}$

Sol.: (i)Complement of given set = {x:x is an odd natural number}
(ii){ x:x is an odd natural number }' = { x:x is an even natural number}
(iii){ x:x is a positive multiple of 3 }' = {x:$x \in \mathbf{N}$, x is not a multiples of 3}
(iv){ x:x is a prime number}'={ x:$x \in \mathbf{N}$, is a positive composite number or $x=1$ }
v) {x : x is a natural number divisible by 3 and 5}'
= {x : x is not a natural number divisible by 3 and 5}
vi) {x : x is a perfect square}'={x : $x \in \mathbf{N}$ is not a perfect square}
vii) {x : x is a perfect cube}' = {x : $x \in \mathbf{N}$ in not a perfect cube}
viii) $\{x : x+5=8\}' = \{x : x \in N\ and\ x \neq 3\}$
ix) $\{x : 2x+5=9\}' = \{x : x \in N\ and\ x \neq 2\}$
x) $\{x : x \geq 7\}' = \{x : x \in N, x < 7\}$
xi) $\{x : x \in N\ and\ 2x+1 > 10\}' = \{x : x \in N\ and\ x \leq 9/2\}$.

Exercise I (f)

1) Decide, among the following sets, which are subsets of one and another

$\mathbf{A} = \{x : x \in R\ and\ x$ **satisfy** $x^2 - 8x + 12 = 0\}$

B { 2,4,6} , C = {2,4,6,8…..} D= {6}.

Sol.: A= $\{x : x \in R$ and x satisfy $x^2 - 8x + 12 = 0\} = \{2,6\}$

B = $\{2,4,6\}$

C = $\{2,4,6,8....\}$ D = $\{6\}$

Here D $\subset$ A , D $\subset$ B, D $\subset$ C
A $\subset$ B, A $\subset$ C, B $\subset$ C

2) In each of the following , determine whether the statement is true or false. If it is true, prove it. If it is false, give an example .

(i)If $x \in \mathbf{A}$ and $\mathbf{A} \in B$, then $x \in \mathbf{B}$
(ii) If $\mathbf{A} \subset \mathbf{B}$ and $\mathbf{B} \in \mathbf{C}$, then $\mathbf{A} \in C$.
(iii) If $\mathbf{A} \subset \mathbf{B}$ and $\mathbf{B} \subset \mathbf{C}$,then $\mathbf{A} \subset \mathbf{C}$
(iv) If $\mathbf{A} \not\subset \mathbf{B}$ and $\mathbf{B} \not\subset \mathbf{C}$ then $\mathbf{A} \not\subset \mathbf{C}$
v) If $x \in \mathbf{A}$ and $\mathbf{A} \not\subset \mathbf{B}$, then $x \in \mathbf{B}$
vi) If $\mathbf{A} \subset \mathbf{B}$ and $x \notin \mathbf{B}$, then $x \notin \mathbf{A}$.

Sol.: i) It is false. Ex: Let A ={a}, B={{a},b,c} but $a \notin B$.

ii)It is false. Ex: Let A= {a}, B = {a,b,c}, C = {{a,b,c},d} but {a} $\notin C$.

iii) It is true.

Let $a \in \mathbf{A}$ then If $\mathbf{A} \subset \mathbf{B} \Rightarrow a \in \mathbf{B}$ and if $\mathbf{B} \subset C \Rightarrow a \in C$.

$a \in \mathbf{A} \Rightarrow a \in C$ $\therefore$ If $\mathbf{A} \subset \mathbf{B}$, $\mathbf{B} \subset \mathbf{C}$ then $\mathbf{A} \subset \mathbf{C}$.

iv) It is false. Ex: $\mathbf{A} = \{a, b\}$, $\mathbf{B} = \{c, d\}$, $C = \{a, b, d\}$

Here $\mathbf{A} \not\subset \mathbf{B}$, $\mathbf{B} \not\subset \mathbf{C}$ but $\mathbf{A} \subset \mathbf{C}$.

v)It is false. Let $\mathbf{A} = \{a, b\}$ *and* $\mathbf{B} = \{a, c, d\}$. $b \in \mathbf{A}$ but $b \notin \mathbf{B}$

vi)It is True. Let $\mathbf{A} \subset \mathbf{B}$, then if $a \in \mathbf{A} \Rightarrow a \in \mathbf{B}$ if $a \notin \mathbf{B} \Rightarrow a \notin \mathbf{A}$.

3) Show that if $\mathbf{A} \subset \mathbf{B}$ then C-B $\subset$ C-A.

Sol.: Let $a \in C - B \Rightarrow a \in C \text{ and } a \notin \mathbf{B} \quad \Rightarrow a \in C \text{ and } a \notin \mathbf{A} \Rightarrow a \in C - \mathbf{A}$.

$\therefore C - B \subset C - \mathbf{A}$

4) Show that $\mathbf{A} \cap \mathbf{B} = \mathbf{A} \cap \mathbf{C}$ need not imply B = C

Sol.: Let A = {1,2,3}, B= {2,3,4,5}, C = {2,3,6,7}

$\mathbf{A} \cap \mathbf{B} = \{2, 3\}$, $\mathbf{A} \cap \mathbf{C} = \{2, 3\}$ Here $\mathbf{A} \cap \mathbf{B} = \mathbf{A} \cap \mathbf{C}$ but still $\mathbf{B} \neq \mathbf{C}$.

II 1) Let A, B and C be the sets such that $\mathbf{A} \cup \mathbf{B} = \mathbf{A} \cup \mathbf{C}$ and $\mathbf{A} \cap B = A \cap C$.Show that $\mathbf{B} = \mathbf{C}$.

Sol.: Given $\mathbf{A} \cup \mathbf{B} = \mathbf{A} \cup \mathbf{C}$

$$\Rightarrow (\mathbf{A} \cup \mathbf{B}) \cap \mathbf{C} = (\mathbf{A} \cup \mathbf{C}) \cap C \Rightarrow (\mathbf{A} \cap \mathbf{C}) \cup (\mathbf{B} \cap \mathbf{C}) = \mathbf{C} \Rightarrow (\mathbf{A} \cap \mathbf{B}) \cup (\mathbf{B} \cap \mathbf{C}) = \mathbf{C}$$

Again $\mathbf{A} \cup \mathbf{B} = \mathbf{A} \cup \mathbf{C}$ ...... (1)

$$\Rightarrow (\mathbf{A} \cup \mathbf{B}) \cap \mathbf{B} = (\mathbf{A} \cup \mathbf{C}) \cap \mathbf{B}$$

$$\mathbf{B} = (\mathbf{A} \cap \mathbf{B}) \cup (C \cap B) = (\mathbf{A} \cap \mathbf{B}) \cup (\mathbf{B} \cap \mathbf{C}) \text{......... (2)}$$

From 1 & 2 we get B = C.

2) Show that following four conditions are equivalent.

(i) $\mathbf{A} \subset \mathbf{B}$ ii) $\mathbf{A} - \mathbf{B} = \phi$ iii) $\mathbf{A} \cup \mathbf{B} = \mathbf{B}$ iv) $\mathbf{A} \cap \mathbf{B} = \mathbf{A}$

Sol.: (i) $\Leftrightarrow$ (ii)

$\mathbf{A} \subset \mathbf{B} \Leftrightarrow$ all the elements of A are in B $\Leftrightarrow$ $\mathbf{A} - \mathbf{B} = \phi$

Hence (i) $\Leftrightarrow$ (ii), (ii) $\Leftrightarrow$ (iii)

Now, we have $\mathbf{A} - \mathbf{B} = \phi \Leftrightarrow \mathbf{A} \subset \mathbf{B} \Leftrightarrow \mathbf{A} \cup \mathbf{B} = \mathbf{B}$

Hence (ii) $\Leftrightarrow$ (iii), (iii) $\Leftrightarrow$ (iv)

We have $\mathbf{A} \cup \mathbf{B} = \mathbf{B} \Leftrightarrow \mathbf{A} \subset \mathbf{B} \Leftrightarrow A \cap B = A$

$\therefore (iii) \Leftrightarrow (iv)$

we have $A \cap B = A \Leftrightarrow \mathbf{A} \subset \mathbf{B}$

$\therefore (iv) \Leftrightarrow (i)$

Hence $(i) \Leftrightarrow (ii) \Leftrightarrow (iii) \Leftrightarrow (iv)$

3) Show that for any sets A and B $(\mathbf{A} \cap \mathbf{B}) \cup (\mathbf{A} - \mathbf{B})$ and $\mathbf{A} \cup (\mathbf{B} - \mathbf{A}) = \mathbf{A} \cup \mathbf{B}$

Sol.: $(\mathbf{A} \cap \mathbf{B}) \cup (\mathbf{A} - \mathbf{B}) = (\mathbf{A} \cap \mathbf{B}) \cup (\mathbf{A} \cap \mathbf{B}') = \mathbf{A} \cap (\mathbf{B} \cup \mathbf{B}') = \mathbf{A}$

Again $\mathbf{A} \cup (\mathbf{B} - \mathbf{A}) = A \cup (B \cap A') = (A \cup B) \cap (A \cup A') = (A \cup B) \cap \cup = AUB$.

4) Using properties of sets, show that.

(i) $A \cup (A \cap B) = A$ (ii) $A \cap (A \cup B) = A$

Sol.: : - (i) $A \cup (A \cap B) = (A \cup A) \cap (A \cup B) = A \cap (A \cup B) = A$

(ii) $A \cap (A \cup B) = (A \cap A) \cup (A \cap B) = A \cup (A \cap B) = A$

5) Let A and B be sets . If $A \cap X = B \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X , show that A = B.

Sol.: :- We know that two sets are equal when they are subsets of each other.

Now , we have $A \cup X = B \cup X$ for some set X

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X) \Rightarrow A = (A \cap B) \cup (A \cap X)$

$\Rightarrow A = (A \cap B) \cup \phi = A \cap B \Rightarrow A \subset B$ ......(1)

Again $A \cup X = B \cup X$ or $B \cup X = A \cup X$

$\Rightarrow B \cap (B \cup X) = B \cap (A \cup X)$

$\Rightarrow B = (B \cap A) \cup (B \cap X) = (B \cap A) \cup \phi = B \cap A \Rightarrow B \subset A$ ...... (2)

From 1& 2 we get A= B

6) Find sets A, B and C such that $A \cap B, B \cap C$ and $A \cap C$ are non –empty sets and $A \cap B \cap C = \phi$.

Sol.: :- Let A = {3,4}, B = {4,5}, C= {3,5,6}.

Now $A \cap B = \{3,4\} \cap \{4,5\} = \{4\} \neq \phi$

$B \cap C = \{4,5\} \cap \{3,5,6\} = \{5\} \neq \phi$

$A \cap C = \{3,4\} \cap \{3,5,6\} = \{3\} \neq \phi$ and $A \cap B \cap C = (A \cap B) \cap C = \{4\} \cap \{3,5,6\} = \phi$

Similarly, we can choose some other such type of sets also.

2.RELATIONS AND FUNCTIONS

Exercise 2 (a)

I.1.If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$**, find the values of x and y**

Sol.: $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$

$$\Rightarrow \frac{x}{3}+1=\frac{5}{3}\Rightarrow \frac{x}{3}=\frac{5}{3}-1=\frac{5-3}{3}=\frac{2}{3}\Rightarrow x=2$$

$$y-\frac{2}{3}=\frac{1}{3}\Rightarrow y=\frac{1}{3}+\frac{2}{3}=\frac{3}{3}=1$$

$\therefore$ x = 2, y = 1

2.If the set A has 3 elements and the set B = {3,4,5} then find the number of elements in A × B

Sol.: n(A): =3, n (B) = 3

Number of elements in A x B = n (A x B) = 3 x 3 = 9

3.If G = {7,8} and H = {5,4,2}, find G x H and H x G

Sol.: G = {7,8} , H = {5,4,2}

G x H = {(7,5), (7,4),(7,2),(8,5), (8,4), (8,2)}

H x G = {(5,7), (5,8),(4,7),(4,8), (2,7), (2,8)}

4.State whether each of the following statements are true or false If the statement is false, rewrite the given statement correctly

i) If P = {m,n} and Q = {n,m} then P x Q = {(m,n),(n,m)}

ii) If A and B are non – empty sets, then A x B is a non – empty set of ordered pairs (x,y) such that x $\in$ A and y $\in$ B

iii) If A = {1,2}, B = {3,4}, then A x (B $\cap \phi$) = ϕ

Sol.: (i) P = {m,n}, Q = {n,m}, P x Q = {(m,n), (n,m)} is false

P x Q = {(m,n), (m,m), (n,n), (n,m)}

(ii) The statement is true

(iii) $B\cap\phi=\phi$

$\therefore A\times(B\cap\phi)=\phi$

5.If $A=\{-1,1\}$**, , find A x A x A**

Sol.: $A\times A\times A=\{(-1,-1,-1),(-1,-1,1),(-1,1,-1),(1,-1,-1),(1,1,-1)(1,-1,1),(-1,1,1),(1,1,1)\}$

6.If A x B = $\{(a,x),(a,y),(b,x),(b,y)\}$ find A and B

Sol.: $A\times B=\{(a,x),(a,y),(b,x),(b,y)\}$

A = {a,b}, $B=\{x,y\}$

7.Let A = {1,2} and B = {3,4}. Write A x B. How many subsets will A × B have list them.

Sol.: A = {1,2}, B = {3,4}

A× B = {(1,3),(1,4),(2,3),(2,4)}

A × B will have 2^4 = 16 subsets

Subsets of A × B are $\phi_1\{(1,3)\},\{(1,4)\},\{(2,3)\},\{(2,4)\},\{(1,3),(1,4)\},\{(1,3),(2,3)\}$
$\{(1,3),(2,4)\},\{(1,4),(2,3)\},\{(1,4),(2,4)\},\{(2,3),(2,4)\}$

$\{(1,3),(1,4),(2,3)\},\{(1,3),(1,4),(2,4)\},\{(1,3),(2,3),(2,4)\}$

$\{(1,4),(2,3),(2,4)\}\{(1,3),(1,4),(2,3)(2,4)\}$

8.Let A and B be two sets such that n(SOL.: = 3 and n(B) = 2 If (x ,1), (y,2),(z,1) are in A x B, find A and B, where x , y and z are distinct elements.

Sol.: Given A x B = $\{(x,1),(y,2),(Z,1)\}$ there $x,y,z\in$ A and 1,2∈ B

$\Rightarrow$ $A=\{x,y,z\}$ and B = {1,2}

9.The Cartesian product A × A has 9 elements among which are found (-1,0) and (0,1). Find the set A and the remaining elements of A × A

Sol.: Given (-1,0) ∈ A x A and (0,1) ∈ **A × A**

$\Rightarrow$ A = {-1,0,1}

A x A = {-1,0,1} × {-1,0,1}= $\{(-1,-1),(-1,0),(-1,1),(0,-1),(0,0),(0,1),(1,-1),(1,0),(1,1)\}$

∴ Remaining elements are $\{(-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,1)\}$

II. 1) Let A = {1,2} , B = {1,2,3,4}, C = {5,6} and D = {5,6,7,8} verify that

(i) A × (B ∩C) = (A ×B) ∩ (A ×C)
(ii) A × C is a subset of B × D

Sol.: $A=\{1,2\},B=\{1,2,3,4\},C=\{5,6\},D=\{5,6,7,8\}$

i) $B\cap C=\phi$, $A\times(B\cap C)=\phi$

$A\times B=\{(1,1)(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)\}$

$A\times C=\{(1,5)(1,6),(2,5),(2,6)\}$

$(A\times B)\cap(A\times C)=\phi$

ii) $A\times C=\{(1,5)(1,6),(2,5),(2,6)\}$

$B\times D=\{(1,5)(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),(3,8),(4,5),(4,6),(4,7),(4,8)\}$

$\therefore A\times C\subset B\times D$ (A × C is a subset of B ×D)

Exercise 2 (b)

1.A = {1,2,3,5} and B = {4,6,9} Define a relation R from A to B by R = {(x, y): the difference between x and y is odd $x\in A, y\in B$} write R in roster form.

Sol.: R = {(x, y) : the difference between x and y is odd, x $\in$A, y $\in$ B}

$\Rightarrow$R = {(1,4), (1,6),(2,9), (3,4), (3,6),(5,4),(5,6)}

2.Determine the domain and range of the relation R defined by $R=\{(x,x+5):x\in\{0,1,2,3,4,5\}\}$

Sol.: $R=\{(x,x+5):x\in\{0,1,2,3,4,5\}\}$

$=\{(0,0+5),(1,1+5),(2,2+5),(3,3+5),(4,4+5),(5,5+5)\}$

$=\{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$

Domain = {0,1,2,3,4,5}, Range = {5,6,7,8,9,10}

3.Write the relation $R=\{(x,x^3):x$ is a prime number less than 10$\}$ in roster form

Sol.: $R=\{(x,x^3):x\, is\, a\, prime\, number\, less\, than\, 10\}$ prime number less than 10 are 2,3,5,7

$\therefore R=\{(2,2^3),(3,3^3),(5,5^3),(7,7^3)\}=\{(2,8),(3,27),(5,125),(7,343)\}$

4. Let $A=\{x,y,z\}$ and B = {1,2}. Find the number of relations from A to B

Sol.: Here n(SOL.: = 3, n(B) = 2

Number of relations from A to B $=2^{3\text{x }2}=2^6=64$

5. Let R be the relation on Z defined by R = {(a,b) : a,b $\in$z, a- b is an integer} find the domain and range of R.

Sol.: R = {(a,b) : a, b $\in$ z , a – b is an integer}

The difference of two integers is also an integer

$\therefore$ Domain of R = z, Range of R = z

II.1. Let A = {1,2,3, …..14}. Define a relation R from A to A by

R = {(x, y) : 3x – y = 0 where $x, y\in$A} write down its domain co domain and Range

Sol.: $3x - y = 0 \Rightarrow 3x = y$

$R = \{(x, y) : 3x - y = 0 \text{ where } x, y \in A\} = \{(1,3),(2,6),(3,9),(4,12)\}$

Domain of R = {1,2,3,4} co domain of R = {1,2,3,4, …… , 14}

Range of R = {3,6,9,12}

2. Define a relation R on the set N of natural numbers by R = {(x, y) : y = $x + 5$, x is a natural number less than 4, $x, y \in N$} Depict this relationship using roster form write down the domain and the range.

Sol.: x is natural number less than 4 $\Rightarrow x = 1,2,3$

$R = \{(x, y) : y = x + 5,\ x \text{ is a natural number less than } 4 ;\ x, y \in N\}$

= {(1,6), (2,7), (3,8)}

Domain = {1,2,3}, Range = {6,7,8}

3. The fig 2.7 shows a relationship between the sets P and Q. Write this relation

i) in set – builder form ii) roster form what is its domain and range.

Sol.: i) Set builder form $R = \{(x, y) : y = x - 2,\ x \in P, \text{ and } y \in Q\}$

ii) In roster form , R = {(5,3), (6,4), (7,5)}

Domain = { 5,6,7 } and Range = { 3,4,5 }

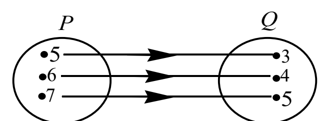


Fig 2.7

4. Let A = {1,2,3,4,6} Let R be the relation on A defined by {(a,b) : a,b $\in$ A , b is exactly divisible by a}

i) Write R in roster form (ii) Find the domain of R (iii) Find the range of R

Sol.: **(i)** R = {(a,b) : a,b $\in$ A , b is exactly divisible by a}

= {(1,1), (1,2), (1,3), (1,4),(1,6),(2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6)}

ii) Domain = {1,2,3,4,6}, iii) Range = {1,2,3,4,6}

Exercise - 2 (c)

1.Find the domain and range of the following real functions (i) f(x) = - $|x|$ (ii) f (x) = $\sqrt{9-x^2}$

Sol.: i) f(x) = - $|x|$, here , $f(x) \le 0, \forall x \in R$

Domain of f is R, Range of R = $(-\infty, 0]$

ii) $f(x) = \sqrt{9-x^2}$

$9 - x^2 > 0 \Rightarrow 3^2 - x^2 > 0 \Rightarrow x^2 - 3^2 \le 0 \Rightarrow (x+3)(x-3) \le 0 \Rightarrow -3 \le x \le 3$

$\therefore$ Domain = [-3,3]

Let $y = \sqrt{9-x^2} \Rightarrow y^2 = 9 - x^2$

$\therefore$ Range = [0,3]

2.A function f is defined by f(x) = 2 x – 5 write down the values of (i) f(0) (ii) f(7) (iii) f (-3)

Sol.: i) f(0) = 2(0) – 5 = - 5

ii) f(7) = 2 (7) – 5 = 14 – 5 = 9

iii) f(-3) = 2 (-3) – 5 = - 6 – 5 = -11

3.Find the range of each of the following functions

(i) f(x) = 2- 3 x , $x \in$ R, $x > 0$

(ii) f(x) = x^2+2, x is real number

(iii) f(x) = x , x is a real number

Sol.: Let y = 2 – 3 x $\Rightarrow 3x = 2 - y \Rightarrow x = \frac{2-y}{3}$ given $x > 0$

$\Rightarrow \frac{2-y}{3} > 0 \Rightarrow 2 - y > 0 \Rightarrow 2 > y$ or y < 2

$\therefore$ Range $(-\infty, 2) = \{y : y \in R \ and \ y < 2\}$

ii) Let y = f(x) = $x^2 + 2 \Rightarrow y = x^2 + 2 \Rightarrow x^2 = y - 2$

$x^2 \geq 0 \Rightarrow y - 2 \geq 0 \Rightarrow y \geq 2$

Range = $[2,\infty)$ or {y : y ∈ R and y≥ 2}

iii) Let y = f(x) = $x \Rightarrow y = x \Rightarrow x = y$

$\therefore x \in R, y \in R$ $\therefore$ Range = { y : $y \in$ R}

II 1.Which of the following relations are functions? Give reasons. If it is a function determine its domain and range

i) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

iii) $\{(1,3),(1,5),(2,5)\}$

Sol.: (i) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$ In this relation no two ordered pairs have the same first components, Hence it is a function.

Domain = {2,5,8,11,14,17} Range = {1}

ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$ In this relation no two ordered pairs have the same first components. Hence it s a function.

Domain = {2,4,6,8,10,12,14}, Range = {1,2,3,4,5,6,7}

iii) {(1,3),(1,5),(2,5)} In this relation (1,3),(1,5) have the same first component, So it is not function.

2. The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(c) = \frac{9c}{5} + 32$ find (i) t(0) ii) t(28) iii) t(-10)

iv) The value of C, when t(c) = 212

Sol.: $t(c)=\frac{9c}{5}+32$

i) $t(0)=\frac{9(0)}{5}+32=32$

ii) $t(28)=\frac{9(28)}{5}+32=\frac{252+160}{5}=\frac{412}{5}$

iii) $t(-10)=\frac{9(-10)}{5}+32=\frac{-90+160}{5}=\frac{70}{5}=14$

iv) $t(c)=212 \Rightarrow \frac{9c}{5}+32=212 \Rightarrow \frac{9c}{5}=212-32=180$

$C=180\times\frac{5}{9}=100$

Exercise 2(d)

1. If $f(x)=x^2$ **find** $\frac{f(1.1)-f(1)}{((1.1)-1)}$

Sol.: $\frac{f(1.1)-f(1)}{(1.1-1)}=\frac{(1.1)^2-1^2}{(1.1-1)}=\frac{(1.1+1)(1.1-1)}{(1.1-1)}$ **= 2.1**

2.Find the domain of the function $f(x)\frac{x^2+2x+1}{x^2-8x+12}$

Sol.: $x^2+2x+1=(x+1)^2$

$x^2-8x+12=x^2-6x-2x+12=x(x-6)-2(x-6)=(x-2)(x-6)$

$\therefore f(x)=\frac{(x+1)^2}{(x-2)(x-6)}$

$\therefore$ Domain of function is set of real numbers except 6 and 2 or R – {2, 6}

3.Find the domain and the range of the real function f defined by $f(x)=\sqrt{(x-1)}$

Sol.: $f(x)=\sqrt{(x-1)}$ is defined only when $x-1\geq 0 \Rightarrow x\geq 1$ $\therefore$ domain of f(x) is $[1,\infty)$

Let $y=f(x)=\sqrt{x-1}\Rightarrow y^2=x-1\Rightarrow x=y^2+1$ then $f(x)=\sqrt{y^2+1-1}=y$ but y can not be negative $\therefore$ Range of f(x) is $[0,\infty)$

4.Find the domain and the range of the real function f defined by f(x) = $|x-1|$

Sol.: f(x) is defined $\forall\ x\in$ R $\therefore$Domain of f(x) is R since f(x) = $|x-1|$, a modulus value is always positive so Range of f(x) is $[0,\infty)$ or $R^+\cup\{0\}$

5.Let $f=\left\{\left(x,\frac{x^2}{1+x^2}\right):x\in R\right\}$ **be a function from R into R determine the range of f**

Sol.: $f=\left\{\left(x,\frac{x^2}{1+x^2}\right):x\in R\right\}$ Let $y=\frac{x^2}{1+x^2}$

Since $1+x^2>x^2\Rightarrow 0\leq y<1$ $\therefore$ Range of f is [0,1)

6.Let R be a relation from N to N defined by R = {(a,b) : a,b $\in$N, and a = b^2} are the following true

(i) (a, Sol.: $\in$R for all a $\in$ N (ii) (a, b) $\in$ R, implies (b, Sol.: $\in$ R (iii) (a, b) $\in$R, (b, c) $\in$ R implies (a, c) $\in$ R justify your answer in each case.

Sol.: **(i)** R = {(a, b) : a, b $\in$ N and a = b^2} It is false, because $a = a^2$ is possible only when a =1 $\in$ N Hence it is not a relation.

(ii) (a, b) $\in$ R $\Rightarrow$(b, Sol.: $\in$ R is false because $a = b^2, b = a^2$ is possible only when a = b and $4 = 2^2$ but $2 \neq 4^2$ hence it is not a relation

(iii) (a, b) $\in$ R, (b, c) $\in$ R $\Rightarrow$ (a, c) $\in$R is false since $a = b^2, b = c^2 \Rightarrow a = \left(c^2\right)^2 = c^4$ Here $a \neq c^2$ so it is not a relation.

7.Let A $A = \{1,2,3,4\}, B = \{1,5,9,11,15,16\}$ **and** $f = \{(1,5),(2,9),(3,1),(4,5),(2,11)\}$ **are the following true**

(i) f is a relation from A to B (ii) f is a function from A to B justify your answer in each case.

Sol.: **(i)** f is a subset of A $\times$ B so it is a relation

(ii) The ordered pairs (2,9) (2,11) have common first component '2' so it is not a function.

8.Let f be the subset of z x z defined by f = {(ab,a + b) : a, b $\in$z} is f a function from z to z ? Justify your answer.

Sol.: Let a = 0, b = 1 then (ab, a + b) = (0,1) $\in$ f

a = 0, b = 2 then (ab, a + b) = (0,2) $\in$ f $\therefore$ f is not a function.

9.Let A = {9,10,11,12,13} and let f : A $\rightarrow$ N be defined by f(n) = the highest prime factor of n. Find the range of f.

Sol.: A = {9,10,11,12,13} and f : A $\rightarrow$ N

For n = 9, 9 = 1 $\times$3 $\times$3 Highest prime factor is 3

For n = 10, 10 = 1 $\times$2 $\times$5 Highest prime factor is 5

For n = 11, 11 = 1 $\times$11 Highest prime factor is 11

For n = 12, 12 = 1 $\times$2 $\times$2 $\times$3 Highest prime factor is 3

For n = 13, 13 = 1 $\times$13 Highest prime factor is 13

$\therefore f = \{(9,3),(10,5),(11,11),(12,3),(13,13)\}$

$\therefore$ Range of f = {3,5,11,13}

II.1.The relation f is defined by $f(x)=\begin{cases}x^2 & 0\le x\le 3\\ 3x & 3\le x\le 10\end{cases}$

The relation g is defined by $g(x)=\begin{cases}x^2 & 0\le x\le 2\\ 3x & 2\le x\le 10\end{cases}$

Show that f is a function and g is not a function

Sol.: $f(x)=\begin{cases}x^2 & 0\le x\le 3\\ 3x & 3\le x\le 10\end{cases}$

at $x = 3$, f(x) = $x^2 = 3^2 = 9$

f(x) = 3(3) = 9

$\therefore$ f(x) is well defined in both cases $\Rightarrow$ f is a function.

$g(x)=\begin{cases}x^2 & 0\le x\le 2\\ 3x & 2\le x\le 10\end{cases}$

at $x = 2$ g(x) = $2^2 = 4$

g(x) = 3(2) = 6

$\therefore$ g(x) is not defined at $x = 2$. So it is not a function.

2.Let f,g : R $\to$ R be defined , respectively by f(x) = x +1, g (x) = 2 x – 3.

Find f + g, f – g and $\frac{f}{g}$

Sol.: f(x) = x + 1, g(x) = 2 x – 3

f + g = x + 1 + 2 x – 3 = 3 x – 2

f- g = x + 1 – (2 x – 3) = - x + 4

$\frac{f}{g}=\frac{x+1}{2x-3}$ when $x\neq\frac{3}{2}$

3. **Let f= {(1,1),(2,3), (0,-1),(-1,-3)} be a function from z to z defined by f(x) = a x + b for some integers a,b. Determine a,b.**

Sol.: f(x) = a x + b

f(1) = 1 $\Rightarrow$ a(1) + b = 1 $\Rightarrow$ a + b = 1

f(0) = -1 $\Rightarrow$ a(0) +b = - 1 $\Rightarrow$ b = -1

a + (-1) = 1 $\Rightarrow$ a = 2

$\therefore$ f(x) = 2 x – 1

III. 1. If f={(4,5),(5,6),(6,-4)} and g = {(4,-4), (6,5),(8,5)} then find (i) f + g (ii) f – g

(iii) 2f + 4g (iv) f + 4 (v) fg (vi) $\frac{f}{g}$.

Sol.: $f=\{(4,5),(5,6),(6,-4)\}\ \ g=\{(4,-4),(6,5),(8,5)\}$

(i) f + g = {(4,5+(-4)} = {(4,1)}
(ii) f – g = {(4,5-(-4)} = {(4,9)}
(iii) 2f + 4g = {(4,2(5) + 4(-4)} = {(4,10-16)} = {(4,-6)}
(iv) f + 4 = {(4,5 + 4), (5,6+4),(6,-4+4)} = {(4,9), (5,10),(6,0)}
(v) fg = {(4,5(-4))} = {(4,-20)}
(vi) $\frac{f}{g}=\left\{\left(4,\frac{5}{-4}\right)\right\}=\left\{\left(4,\frac{-5}{4}\right)\right\}$

2. If f and g are real valued function defined by f(x) = 2 x – 1 and g(x) = x^2 then find

(i) (3f – 2g) (x) (ii) (fg) x (iii) $\left(\frac{f}{g}\right)(x)$ (iv) (f + g+2) x (v) 2f(x) , (iv) 2 + f(x)

Sol.: $f(x)=2x-1, g(x)=x^2$

(i) $(3f-2g)x=3(2x-1)+2(x^2)=2x^2+6x-3$

(ii) $(fg)(x)=f(x)g(x)=(2x-1)x^2=2x^3-x^2$

(iii) $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{2x-1}{x^2}$

(iv) $(f+g+2)(x)=f(x)+g(x)+2=2x-1+x^2+2=x^2+2x+1=(x+1)^2$

(v) $2f(x)=2(2x-1)=4x-2$

(vi) $2+f(x)=2+2x-1=2x+1$

3. If $f(x)=x^2$ and $g(x)=|x|$ find the following functions

(i) f + g (ii) f – g (iii) fg (iv) 2f (v) f + 3 (vi) $\frac{f}{g}$ (for $x\neq 0$)

Sol.: $f(x)=x^2, g(x)=|x|=\begin{cases} x & for\ x>0 \\ -x & for\ x<0 \end{cases}$

(i) $f+g=f(x)+g(x)=x^2+|x|=\begin{cases} x^2+x & for\ x>0 \\ x^2-x & for\ x<0 \end{cases}$

ii) $f-g=f(x)-g(x)=x^2-|x|=\begin{cases} x^2-x & for\ x>0 \\ x^2+x & for\ x<0 \end{cases}$

iii) $fg=f(x)g(x)=x^2|x|=\begin{cases} +x^3 & for\ x>0 \\ -x^3 & for\ x<0 \end{cases}$

iv) $2f = 2f(x) = 2x^2$

v) $f + 3 = f(x) + 3 = x^2 + 3$

vi) $\frac{f}{g} = \frac{f(x)}{g(x)} = \frac{x^2}{|x|} = \begin{cases} \frac{x^2}{x} = x & for\, x > 0 \\ \frac{x^2}{-x} = -x & for\, x < 0 \end{cases}$

4.If the function f is defined by $f(x) = \begin{cases} 3x-2 & , & x > 3 \\ x^2-2 & , & -2 \le x \le 2 \\ 2x+1 & , & x < -3 \end{cases}$ **then find the values, if exist of f(4), f(2.5), f(-2), f(-4), f(0), f(-7), f(1), f(9)**

Sol.: $f(x) = \begin{cases} 3x-2 & , & x > 3 \\ x^2-2 & , & -2 \le x \le 2 \\ 2x+1 & , & x < -3 \end{cases}$

f(4) = 3(4) – 2 = 10, f(2.5) does not exist since 2.5 is not in the domain.

f(-2) = $(-2)^2$ – 2 = 2, f(-4) = 2(-4) + 1 = -7, f(0) = 0^2 – 2 = -2

f(-7) = 2(-7) + 1 = -13, f(1) = 1^2 – 2 = -1, f(9) = 3 (9) – 2 = 25

5.Determine a quadratic function f is defined by

f(x) = a x^2 + b x + c, if f(0) = 6, f(2) = 1, f(-3) = 6

Sol.: f(x) = a x^2 + b x + c ……(1)

f(0) = 6 $\Rightarrow$ a$(0)^2$ + b(0) +c = 6 $\Rightarrow$ c = 6

f(2) = 1 $\Rightarrow$ a$(2)^2$ + b(2) + c = 1 $\Rightarrow$ 4a + 2b + c = 1

4a + 2b + 6 = 1 $\Rightarrow$ 4a + 2b + 5 = 0 …..(2)

f(-3) = 6 $\Rightarrow$ a$(-3)^2$ + b(-3) + c = 6

9a – 3b + 6 = 6 $\Rightarrow$ 9a -3b = 0 ….(3)

$Equation\ 2x3 \rightarrow 12a + 6b + 15 = 0$

$Equation\, 2X3 \rightarrow \underline{18a - 6b + \ \ 0 = 0}$

$30a + 15 = 0 \Rightarrow 30a = -15 \Rightarrow a = \frac{-15}{30} = -\frac{1}{2}$

From (3) 9a – 3b = 0 $\Rightarrow$ 9a = 3b $\Rightarrow$ b=$\frac{9}{3}a = 3\left(\frac{-1}{2}\right) = \frac{-3}{2}$

$\therefore$ Required quadratic function is $f(x) = \frac{-1}{2}x^2 - \frac{3}{2}x + 6$.

3. TRIGONOMETRIC FUNCTIONS

Exercise – 3 (a)

1. Find the radian measures corresponding to the following degree measures:

Sol.: Since $180^0 = \pi$ radians

$$\therefore 1^0 = \frac{\pi}{180^0} \text{ radians, } 1^1 = \left(\frac{1}{60}\right)^0$$

(i) $25^0 = 25\,x\,\frac{\pi}{180} rad = \frac{5\pi}{36}$ radians

(ii) $-47^0 30^1 = -\left(47 + \frac{30}{60}\right)^0 = -\left(47 + \frac{1}{2}\right)^0 = -\left(\frac{94+1}{2}\right)^0$

$= -\frac{95}{2} = -\frac{95}{2}\,x\,\frac{\pi}{180}$ radians $= -\frac{19\pi}{72}$ radians

(iii) $240^0 = 240\,x\,\frac{\pi}{180}$ radians $= \frac{4\pi}{3}$ radians

(iv) $520^0 = 520^0\,x\,\frac{\pi}{180}$ radians $= \frac{26\pi}{9}$ radians.

2.Find the degree measures corresponding to the following radian measures $\left(use\ \pi = \frac{22}{7}\right)$.

Sol.: (i) $\left(\frac{11}{16}\right)^0$ $\left\{\pi \text{ radians} = 180^0 \therefore 1 \text{ radian} = \frac{180^0}{\pi}\right\}$

$$\left(\frac{11}{16}\right)^0 = \left(\frac{11}{16}\,x\,\frac{180}{\pi}\right)^0 = \left(\frac{11}{8}\,x\,\frac{90}{22}\,x\,7\right)^0$$

$$= \left(\frac{45x7}{8}\right)^0 = \left(\frac{315}{8}\right) = 39^0 + \left(\frac{3}{8}\right)^0 = 39^0 + \left(\frac{3}{8}x60^1\right)$$

$$= 39^0 + \left(\frac{45}{2}\right)^1 = 39^0 + 22^1 + \frac{1}{2}^1 = 39^0 22^1 + \frac{1x60^{11}}{2} = 39^0 22^1 30^{11} \left(\because 1^1 = 60^{11}\right)$$

(ii) $-4 = -\left(4x\frac{180}{\pi}\right)^0 = -\left(4x\frac{180}{22}x7\right)^0$

$$= \left(\frac{2x1260}{11}\right)^0 = -\left(\frac{2520}{11}\right)^0 = -\left[229^0 + \left(\frac{1}{11}\right)^0\right] = -\left(229^0 + \frac{1}{11}x60^1\right)\left(\because 1^0 = 60^1\right)$$

$$= -\left[229^0 5^1 + \left(\frac{5}{11}\right)^1\right] = -\left(229^0 5^1 + \frac{5}{11}x60^{11}\right)\left(\because 1^1 = 60^{11}\right)$$

$= -\left(229^0 5^1 27^{11}\right)$(approx)

(iii) $\left(\frac{5\pi}{3}\right)^0 = \left(\frac{5\pi}{3} x \frac{180}{\pi}\right)^0 = 300^0 \left[\because 1^0 = \left(\frac{180}{\pi}\right)^0\right]$

(iv) $\left(\frac{7\pi}{6}\right)^0 = \left(\frac{7\pi}{6} x \frac{180}{\pi}\right)^0 = 210^0$

3. A wheel makes 360 revolutions in one minute, through how many radians does it turn in one second?

Sol.: No. of revolution in one minute = 360^0

$\therefore$ No. of revolution in one second = $\frac{360^0}{60} = 6^0$

$\therefore$ 6 revolutions = 6 x 2 π = 12 radians

Hence, number of radians turned in one second = 12 π

4. Find the degree measure of the angle subtended at the centre of circle of radians 100cm by an arc of length 22 cm $\left(use\pi = \frac{22}{7}\right)$.

Sol.: Given, ℓ = length of arc = 22cm

R = radius of circle = 100cm

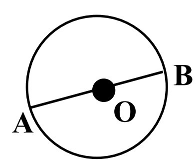
Let θ be the angle subtended at the centre, then $\theta = \frac{\ell}{r}$ radians

$$\theta = \frac{22}{100} radians = \left(\frac{11}{50} x \frac{180^0}{\pi}\right)^0 = \left(\frac{11x18x7}{5x22}\right)^0 = \left(\frac{63}{5}\right)^0$$

$$= 12^0 + \left(\frac{3}{5}\right)^0 = 12^0 + \frac{3}{5} x 60^1 = 12^0 36^1$$

5. In a circle of diameter 40cm, the length of a chord is 20cm. Find the length of minor arc of the chord.

Sol.: Given, diameter = 40 cm



$\therefore$ radius OA = OB = $\frac{diameter}{2} \Rightarrow$ r = $\frac{40}{2} = 20cm$

Also, chord AB = 20cm = ℓ

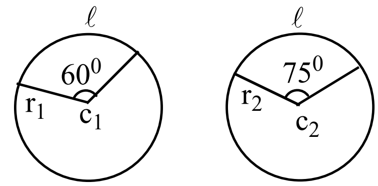
$\therefore$ All the three sides of ΔOAB equal so, it is an equilateral triangle

$$\Rightarrow \theta = \angle AOB = 60^0 = 60 x \frac{\pi}{180} \text{ radians}$$

$$\therefore \theta = \frac{\ell}{r} \Rightarrow 60x\frac{\pi}{180} = \frac{AB}{20} \quad \Rightarrow AB = 60 x 20 x \frac{\pi}{180^0} = \frac{20\pi}{3} cm$$

6. If in two circles, arcs of the same length subtend angles 60^0 and 75^0 at the centre, find the ratio of their radii.

sol: : Let the radii of the two circles be r_1 and r_2 respectively. Also, let the length of arc in each case be ℓ.



For the first circle, $\theta = 60^0 = 60^0 \, x \, \frac{\pi}{180}$

$\theta = \frac{\pi}{3}$ radians

We know that $\ell = r\,\theta \Rightarrow r = \frac{\ell}{\theta}$

$$r_1 = \frac{\ell}{\left(\frac{\pi}{3}\right)} = \frac{3\ell}{\pi} \,\ldots\ldots\ldots(1)$$

For the second circle, $\theta = 75^0 = 75^0 \, x \, \frac{\pi}{180} = \frac{5\pi}{12}$ radians

$$r_2 = \frac{\ell}{\theta} = \frac{\ell}{\frac{5\pi}{12}} = \frac{12\ell}{5\pi} \,\ldots\ldots\ldots\ (2)$$

Dividing eqs. (1) and (2) we get

$$\frac{r_1}{r_2} = \frac{\left(\frac{3\ell}{\pi}\right)}{\left(\frac{12\ell}{5\pi}\right)} = \frac{3\ell}{\pi} \, x \, \frac{5\pi}{12\ell} = \frac{5}{4} \qquad \therefore\ r_1 : r_2 = 5:4$$

7. Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length. (Sol.: 10 cm (b) 15 cm (c) 21 cm

Sol.: : Using formula $\theta = \frac{\ell}{r}$

(a) $\theta = \frac{10}{75} = \frac{2}{15}$ radians, (b) $\theta = \frac{15}{75} = \frac{1}{5}$ radians (c) $\theta = \frac{21}{75} = \frac{7}{25}$ radians

Exercise – 3(b)

I. Find the values of the trigonometric functions in Ex: 1 to 5

1. Sin 765^0

Sol.: $\sin 765^0 = \sin(2 \text{ x } 360 + 45^0) = \sin 45^0 = \frac{1}{\sqrt{2}}$

2. Cosec (-1410^0)

Sol.: $\text{cosec}(-1410^0) = \text{cosec}(4 \text{ x } 360 - 1410) = \text{cosec}[(1440) - 1410] = \text{cosec } 30^0 = 2$

3. Tan $\frac{19\pi}{3}$

Sol.: : $\text{Tan}\frac{19\pi}{3} = \text{Tan}\left(\frac{18\pi + \pi}{3}\right) = \text{Tan}\left(6\pi + \frac{\pi}{3}\right) = \text{Tan}\frac{\pi}{3} = \sqrt{3}$

4. Sin $\left(-\frac{11\pi}{3}\right)$

Sol.: : $\sin\left(-\frac{11\pi}{3}\right) = \sin\left(4\pi - \frac{11\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

5. cot $\left(-\frac{15\pi}{4}\right)$

Sol.: : $\cot\left(\frac{-15\pi}{4}\right) = \cot\left(4\pi - \frac{15\pi}{4}\right) = \cot\frac{\pi}{4} = 1$

II. Find the values of other five trigonometric functions in exercise 1 to 5.

1. Cos x = - $\frac{1}{2}$**, where** x **lies in third quadrant.**

Sol.: Given, $\cos x = -\frac{1}{2}$, where x lies in III Q.

$\therefore$ secx = -2

Now, $\sin^2 x = 1 - \cos^2 x = 1 - \frac{1}{4} = \frac{3}{4}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

since x lies in IIIQ, $\sin x$ will be –ve.

$\therefore \sin x = \frac{-\sqrt{3}}{2}$, $\operatorname{cosec} x = \frac{1}{\sin x} = \frac{-2}{\sqrt{3}}$,

also $\tan x = \frac{\sin x}{\cos x} = \frac{\frac{-\sqrt{3}}{2}}{\frac{-1}{2}} = \sqrt{3}$

and $\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$

2. Sin x = $\frac{3}{5}$**, where x lies in second quadrant**

Sol.: Given, $\sin x = \frac{3}{5}$, x lies in II Quadrant

$\therefore$ cosec x = $\frac{5}{3}$.

Now, $\cos^2 x = 1 - \sin^2 x = 1 - \frac{9}{25} = \frac{16}{25}$ $\Rightarrow \cos x = \pm\frac{4}{5}$

Since x lies in IInd Quadrant, cosx will be negative.

$\cos x = -\frac{4}{5}$, $\sec x = \frac{-5}{4}$, $\tan x = \frac{\sin x}{\cos x} = \frac{-3}{4}$, Cotx = $-\frac{4}{3}$

3. Cot x = $\frac{3}{4}$, where x lies in third quadrant.

Sol.: Given $\cot x = \frac{3}{4}$, x lies in III quadrant

Now, $\text{cosec}^2 x = 1 + \cot^2 x = 1 + \frac{9}{16} = \frac{25}{16}$

$\Rightarrow \text{cosec}\, x = \pm\frac{5}{4}$

Since x lies in III quadrant, cosec x will be be – ve

Cosec $x = -\frac{5}{4}$, $\sin x = \frac{1}{\text{cosec} x} = -\frac{4}{5}$,

$\cos x = -\frac{3}{5}$, $\sec x = \frac{1}{\cos x} = -\frac{5}{3}$, $\tan x = \frac{1}{\cot x} = \frac{4}{3}$

4. Sec x = $\frac{13}{5}$, where x lies in fourth quadrant.

Sol.: Given $\sec x = \frac{13}{5}$, x lies in IV quadrant

Now $\sin^2 x = 1 - \cos^2 x = 1 - \frac{25}{169} = \frac{144}{169}$

Sin $x = \pm\frac{12}{13}$

Since x lies in IVQ, $\sin x$ will be – Ve

Sin $x = -\frac{12}{13}$, $\cos x = \frac{1}{\sec x} = \frac{5}{13}$, cosec $x = \frac{1}{\sin x} = -\frac{13}{12}$

Also, $\tan x = \frac{\sin x}{\cos x} = \frac{-12}{5}$ And $\cot x = \frac{1}{\tan x} = \frac{-5}{12}$

5. Tan x = $\frac{-5}{12}$, where x lies in second quadrant.

Sol.: Given Tan $x = \frac{-5}{12}$, x lies in II quadrant hence ot $x = \frac{1}{\tan x} = \frac{-12}{5}$

Now, $\text{Sec}^2 x = 1 + \tan^2 x = 1 + \frac{25}{144} = \frac{169}{144} \Rightarrow$ Sec $x = \pm\frac{13}{12}$

Since, x lies in II quadrant, sec x will be – ve

$\therefore$ sec $x = -\frac{13}{12}$, $\cos x = \frac{1}{\sec x} = \frac{-12}{13}$

also, $\sin x = \left(\frac{-5}{12}\right)\left(\frac{-12}{13}\right) = \frac{5}{13}$ and $\operatorname{cosec} x = \frac{1}{\sin x} = \frac{13}{5}$

EXERCISE – 3(C)

I. Find the values of

(i) $\sin 75^0$ (ii) Tan 15^0 (iii) cos 75^0 (iv) $\sin 105^0$ (v) Tan75^0 (vi) cot 75^0

(vii) cos 105^0 (viii) Tan105^0 (ix) cot 105^0 (x) cos 15^0 (xi) $\sin 15^0$

Sol.: (i) $\sin 75^0 = \sin(45^0 + 30^0)$ $[\because \sin(A+B) = \sin A \cos B + \cos A \sin B]$

$$= \sin 45^0 \cos 30^0 + \cos 45^0 \sin 30^0 = \frac{1}{\sqrt{2}} x \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} x \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) $\text{Tan}15^0 = \text{Tan}(45^0 - 30^0)$ $\left(\therefore \text{ Tan}(A-B) = \frac{TanA - TanB}{1 + TanA TanB}\right)$

$$= \frac{Tan45^0 - Tan30^0}{1 + Tan45^0 Tan30^0} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

(iii) $\cot 15^0 = \frac{1}{\tan 15^0} = \frac{1}{2-\sqrt{3}} = 2 + \sqrt{3}$ $\left(\cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}\right)$

$$\cot 15^0 = \frac{\cot 45^0 c\text{ot}\, 30^0}{\cot 30^0 - \cot 45^0} = \frac{1.\sqrt{3}+1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{\sqrt{3}-1} x \frac{\sqrt{3}+1}{\sqrt{3}+1} = 2 + \sqrt{3}$$

(iv) $\cos 75^0 = \cos(45^0 + 30^0)$ $[\because \cos(A+B) = \cos A \cos B - \sin A \sin B]$

$= \cos 45^0 \cos 30^0 - \sin 45^0 \sin 30^0$

$$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right) = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

(v) $\sin(105^0) = \sin(75^0 + 30^0) = \sin 75^0 \cos 30^0 + \cos 75^0 \sin 30^0$

$$= \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

=

(or) $\sin 105^0 = \sin(60^0 + 45^0) = \sin 60^0 \cos 45^0 + \cos 60^0 \sin 45^0$

$$= \frac{\sqrt{3}}{2} x \frac{1}{\sqrt{2}} + \frac{1}{2} . \frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}} \; \frac{3 + \sqrt{3} + \sqrt{3} - 1}{4\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(vi) $\text{Tan}\,75^0 = \dfrac{\sin 75^0}{\cos 75^0} = \dfrac{\left(\dfrac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right)} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} = \dfrac{4+2\sqrt{3}}{2} = \dfrac{2\left(2+\sqrt{3}\right)}{2} = 2+\sqrt{3}$

(vii) $\text{Cot}\,75^0 = \dfrac{1}{Tan75^0} = \dfrac{1}{2+\sqrt{3}} = \dfrac{1}{2+\sqrt{3}}\,x\,\dfrac{2-\sqrt{3}}{2-\sqrt{3}} = \dfrac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

$Cot\,75^0 = \dfrac{Cos\,75^0}{Sin75^0} = \dfrac{\dfrac{\sqrt{3}-1}{2\sqrt{2}}}{\dfrac{\sqrt{3}+1}{2\sqrt{2}}} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \dfrac{3+1-2\sqrt{3}}{3-1} = 2-\sqrt{3}$

(viii) $\cos(105^0) = \cos\left(75^0+30^0\right)$ [$\because \cos(A+B) = \cos A\cos B - \sin A\sin B$]

$= \cos 75^0 \cos 30^0 - \sin 75^0 \sin 30^0$

$= \left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right)\left(\dfrac{\sqrt{3}}{2}\right) - \left(\dfrac{\sqrt{3}+1}{2\sqrt{2}}\right)\left(\dfrac{1}{2}\right) = \left(\dfrac{3-\sqrt{3}}{4\sqrt{2}}\right) - \left(\dfrac{\sqrt{3}+1}{4\sqrt{2}}\right) = \dfrac{2-2\sqrt{3}}{4\sqrt{2}} = \dfrac{1-\sqrt{3}}{2\sqrt{2}}$

$\text{Cos}\ 105^0 = \text{Cos}(60^0 + 45^0) = \text{Cos}60^0\ \text{Cos}45^0 - \text{Sin}\ 60^0\ \text{Sin}45^0$

$= \dfrac{1}{2}.\dfrac{1}{\sqrt{2}} - \dfrac{\sqrt{3}}{2}.\dfrac{1}{\sqrt{2}} = \dfrac{1-\sqrt{3}}{2\sqrt{3}} (or) \dfrac{-\sqrt{3}+1}{2\sqrt{2}}$

(ix) $\text{Tan}105^0 = \dfrac{\sin\left(105^0\right)}{\cos\left(105^0\right)}$

$= \dfrac{\left(\dfrac{\sqrt{3}+1}{2\sqrt{2}}\right)}{-\left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right)} = -\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$= -\dfrac{\left(\sqrt{3}+1\right)^2}{3-1} = -\dfrac{\left(3+1+2\sqrt{3}\right)}{2}$

$= -\left(2+\sqrt{3}\right)$

$\text{Tan}105^0 = \tan(60^0 + 45^0)$

$= \dfrac{tan60^0 + \tan 45^0}{1-\tan 60^0 \tan 45^0}$

$= \dfrac{\sqrt{3}+1}{1-\sqrt{3}(1)} = \dfrac{\sqrt{3}+1}{1-\sqrt{3}}\,X\,\dfrac{1+\sqrt{3}}{1+\sqrt{3}}$

$= \dfrac{3+1+2\sqrt{3}}{1-3} = \dfrac{4+2\sqrt{3}}{-2}$

$= -\left(2+\sqrt{3}\right)$

(x) $\text{Cot}105^0 = \dfrac{1}{T\text{an}\,105^0}$

$= \dfrac{1}{-\left(2+\sqrt{3}\right)}$

$= \dfrac{-1}{2+\sqrt{3}}\,X\,\dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \dfrac{\left(2-\sqrt{3}\right)}{4-3} = -\left(2-\sqrt{3}\right)$

$\cot 105^0 = \cot\left(60^0 + 45^0\right)$

$= \dfrac{\cot 60^0 \cot 45^0 - 1}{\cot 45^0 + \cot 60^0}$

$= \dfrac{\dfrac{1}{\sqrt{3}}-1}{1+\dfrac{1}{\sqrt{3}}} = \dfrac{1-\sqrt{3}}{1+\sqrt{3}}\,X\,\dfrac{1-\sqrt{3}}{1-\sqrt{3}}$

$= \dfrac{1+3-2\sqrt{3}}{1-3} = \dfrac{4-2\sqrt{3}}{-2} = -(2-\sqrt{3})$

(xi) $\text{Cos}15^0 = \text{Cos}(45^0 - 30^0)$ [$\because$ Cos(A-B) = CosACosB + SinASinB]

$= \text{Cos}45^0\text{Cos}30^0 + \text{Sin}45^0\text{Sin}30^0$

$$= \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}\right)+\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)=\frac{\sqrt{3}+1}{2\sqrt{2}}$$

(xii) $\text{Sin}15^0 = \text{Sin}(45^0 - 30^0) = \text{Sin}45^0\text{Cos}30^0 - \text{Cos}45^0\text{Sin}30^0$

$$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right)-\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)=\frac{\sqrt{3}-1}{2\sqrt{2}}$$

2. Prove that : Sin(n+1) x Sin(n+2) x +Cos(n+1) x Cos(n+2) x = Cos x.

Sol.: LHS = Sin(n+1) x Sin(n+2) x +Cos(n+2) x Cos(n+1) x

[$\because$ Cos(A-B) = CosACosB + SinA SinB]

= Cos((n+2) x -(n+1) x) = Cos(n x +2 x -n x - x) = Cos x = RHS

II. 1. Prove that $S\text{in}^2\frac{\pi}{6}+C\text{os}^2\frac{\pi}{3}-T\text{an}^2\frac{\pi}{4}=-\frac{1}{2}$

Sol. LHS = $S\text{in}^2\frac{\pi}{6}+C\text{os}^2\frac{\pi}{3}-T\text{an}^2\frac{\pi}{4}=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$

$$=\frac{1}{4}+\frac{1}{4}-1=\frac{1}{2}-1=-\frac{1}{2}=RHS$$

2. Prove that : $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$.

Sol.: LHS = $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=2\left(\frac{1}{2}\right)^2+\operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\cos^2\frac{\pi}{3}$

$$=2\left(\frac{1}{4}\right)+\operatorname{cosec}^2\frac{\pi}{6}\cos^2\frac{\pi}{3}=\frac{1}{2}+(2)^2\left(\frac{1}{2}\right)^2=\frac{1}{2}+1=\frac{3}{2}=RHS$$

3. Prove that : $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$

Sol: LHS = $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=\left(\sqrt{3}\right)^2+\operatorname{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2$

$$= 3 + \text{cosec}\ \frac{\pi}{6}+3\left(\frac{1}{3}\right) = 3 + 2 + 1 = 6 = \text{RHS}.$$

4. Prove that $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$

Sol.: LHS = $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=2\sin^2\left(\pi-\frac{\pi}{4}\right)+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

$$=2\sin^2\frac{\pi}{4}+2\left(\frac{1}{\sqrt{2}}\right)^2+2(2)^2=2\left(\frac{1}{\sqrt{2}}\right)^2+2\left(\frac{1}{2}\right)+8=10=RHS \Rightarrow$$

6. Prove that : $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

Sol.: LHS = $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

[$\because$ cos(A+B) = cosAcosB – sinAsinB]

$$=\cos\left(\frac{\pi}{4}-x+\frac{\pi}{4}-y\right)=\cos\left(\frac{2\pi}{4}-(x+y)\right)$$

$$=\cos\left(\frac{\pi}{2}-(x+y)\right)=\sin(x+y)=RHS$$

7. Prove that : $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

Sol.: LHS $=\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$ $\left[\begin{array}{l}\because Tan(A+B)=\dfrac{TanA+TanB}{1-TanATanB}\\ Tan(A-B)=\dfrac{TanA-TanB}{1+TanATanB}\end{array}\right]$

$$=\frac{\left[\dfrac{Tan\frac{\pi}{4}+Tanx}{1-Tan\frac{\pi}{4}Tanx}\right]}{\left[\dfrac{Tan\frac{\pi}{4}-Tanx}{1+Tan\frac{\pi}{4}Tanx}\right]}=\frac{1+\tan x}{1-\tan x}X\frac{1+\tan x}{1-\tan x}=\left(\frac{1+\tan x}{1-\tan x}\right)^2=RHS$$

8. Prove that : $\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$

Sol.: LHS $=\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\dfrac{(-\cos x)(\cos x)}{(\sin x)(-\sin x)}=\dfrac{\cos^2 x}{\sin^2 x}=\cot^2 x=RHS$

9. Prove that : $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

Sol.: LHS = $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$

[$\because$ cos C – cos D = - 2sin $\left(\frac{C+D}{2}\right)$ sin $\left(\frac{C-D}{2}\right)$]

$$= -2\sin\left(\frac{\frac{3\pi}{4}+x+\frac{3\pi}{4}-x}{2}\right)\sin\left(\frac{\frac{3\pi}{4}+x-\frac{3\pi}{4}+x}{2}\right)$$

$$= -2\sin\left(\frac{3\pi}{4}\right)\sin x = -2\sin\left(\pi-\frac{\pi}{4}\right)\sin x = -\sqrt{2}\sin x = \text{RHS}$$

10. Prove that : $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$.

Sol.: LHS = $\sin^2 6x - \sin^2 4x$ $\qquad$ [$\because \sin^2 A - \sin^2 B = \sin(+b)\sin(A\text{-}B)$]

$= \sin(6x+4x)\sin(6x-4x) = \sin 10x \sin 2x$ = RHS

11. Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Sol.: : LHS = $\cos^2 2x - \cos^2 6x$

$= 1 - \sin^2 2x - (1-\sin^2 6x) \quad = 1 - \sin^2 2x - 1 + \sin^2 6x = \sin^2 6x - \sin^2 2x$

[$\because \sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$]

$= \sin(6x+2x)\sin(6x-2x) \quad = \sin 8x \sin 4x$ = RHS.

12. Prove that : $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\dfrac{\sin 2x}{\cos 10x}$

Sol.: : LHS = $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$, $\quad \sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \frac{-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)} = \frac{-\sin 7x \sin 2x}{\cos 10x \sin 7x} = -\frac{\sin 2x}{\cos 10x} = RHS$$

13. Prove that $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Sol.: LHS = $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$$\left[\begin{array}{l} \because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \\ \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \end{array}\right]$$

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)} = \frac{\sin 4x \cos x}{\cos 4x \cos x} = \tan 4x = RHS$$

14. Prove that : $\dfrac{\sin x-\sin y}{\cos x+\cos y}=\tan\left(\dfrac{x-y}{2}\right)$

Sol.: : LHS = $\dfrac{\sin x-\sin y}{\cos x+\cos y}$ $\quad$ $\sin C-\sin D=2\cos\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$

$$\cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$=\frac{2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}=\frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}=Tan\left(\frac{x-y}{2}\right)=RHS$$

15. Prove that : $\dfrac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

Sol.: LHS = $\dfrac{\sin x+\sin 3x}{\cos x+\cos 3x}$ $\quad$ $\left[\begin{array}{l}\because \sin C+\sin D=2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\\ \cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\end{array}\right]$

$$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}=\frac{\sin\left(\frac{4x}{2}\right)}{\cos\left(\frac{4x}{2}\right)}=\frac{\sin 2x}{\cos 2x}=\tan 2x=\text{RHS.}$$

16. Prove that : $\dfrac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$

Sol.: LHS $=\dfrac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=\dfrac{\sin 3x-\sin x}{\cos^2 x-\sin^2 x}$

$$\left[\begin{array}{l}\because \sin C-\sin D=2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\\ and\ \cos^2 x-\sin^2 x=\cos 2x\end{array}\right]$$

$$=\frac{2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}{\cos 2x}=\frac{2\cos 2x\sin x}{\cos 2x}=2\sin x=RHS$$

17. Prove that : tan4x = $\dfrac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x}$

Sol.: LHS = tan 4x = tan2(2x)

$$=\frac{2\tan 2x}{1-\tan^2 2x}$$

$$= \frac{2\left[\frac{2\tan x}{1-\tan^2 x}\right]}{1-\left[\frac{2\tan x}{1-\tan^2 x}\right]^2} \qquad \left(\because Tan2x = \frac{2Tanx}{1-Tan^2x}\right)$$

$$= \frac{4\tan x}{1-\tan^2 x} X \frac{\left(1-\tan^2 x\right)^2}{\left(1-\tan^2 x\right)^2 - 4\tan^2 x} = \frac{4\tan x\left(1-\tan^2 x\right)^2}{1+\tan^4 x - 2\tan^2 x - 4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)^2}{1-6\tan^2 x+\tan^4 x} = \text{R.H.S.}$$

18. Prove that : $\cos 4x = 1 - 8\sin^2 x\cos^2 x$

Sol.: L.H.S. $= \cos 4x$

$= 1 - 2\sin^2 2x = 1 - 2(\sin 2x)^2$ $\qquad \left(\begin{array}{l}\because \cos 2x = 1 - 2\sin^2 x \\ \sin 2x = 2\sin x\cos x\end{array}\right)$

$= 1 - 2(2\sin x)(\cos x)^2 = 1 - 8\sin^2 x\cos^2 x$ = RHS.

III. 1. Prove that $\cos\left(\frac{3\pi}{2}+x\right)\cos\left(2\pi+x\right)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot\left[2\pi+x\right]\right]=1$

Sol. : LHS $= \cos\left(\frac{3\pi}{2}+x\right)\cos\left(2\pi+x\right)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot\left[2\pi+x\right]\right]$

$= \sin x \cos x (\tan x + \cot x)$ $\qquad \left[\begin{array}{l}\because \cot\left(\frac{3\pi}{2}-\theta\right) = \tan\theta \\ \cos\left(\frac{3\pi}{2}+\theta\right) = \sin\theta\end{array}\right]$

$= \sin x \cos x \left[\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right] = \sin x\cos x\left[\frac{\sin^2 x+\cos^2 x}{\cos x\sin x}\right]$

$= \sin^2 x + \cos^2 x = 1 =$ RHS.

2. Prove that : $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x\sin 4x$

Sol. : LHS $= \sin 2x + 2\sin 4x + \sin 6x = \sin 6x + \sin 2x + 2\sin 4x$

$\because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$= 2\sin\left(\frac{6x+2x}{2}\right)\cos\left(\frac{6x-2x}{2}\right) + 2\sin 2(2x)$

$= 2\sin\left(\frac{8x}{2}\right)\cos\left(\frac{4x}{2}\right) + 2\sin 4x = 2\sin 4x\,(\cos 2x + 1)$

$= 2\sin 4x\,(2\cos^2 x - 1 + 1) = 4\cos^2 x\sin 4x$ = RHS

3. Prove that cot4x (sin5x + sin3x) = cot$x$ (sin5x – sin3x)

Sol. : LHS = cot 4x (sin5x + sin3x)

$$=\cot 4x\left(2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right)$$

$$=\cot 4x\left(2\sin\left(\frac{8x}{2}\right)\cos\left(\frac{2x}{2}\right)\right)$$

$$=\cot 4x\,(2\sin 4x\cos x)=\frac{\cos 4x}{\sin 4x}\left(2\sin 4x\cos x\right)=2\cos 4x\cos x.$$

RHS $=\cot x\,(\sin 5x-\sin 3x)$

$$=\cot x\left(2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right)=\cot x\left(2\cos\left(\frac{8x}{2}\right)\sin\left(\frac{2x}{2}\right)\right)$$

$$=\cot x\,(2\cos 4x\sin x)=\frac{\cos x}{\sin x}\left(2\cos 4x\sin x\right)$$

$=2\cos 4x\cos x$ $\quad\because$ LHS = RHS.

4. Prove that $\dfrac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x$

Sol.: LHS = $\dfrac{\cos 4x+\cos 3x+\cos 2x}{(\sin 4x+\sin 2x)+\sin 3x}=\dfrac{(\cos 4x+\cos 2x)+\cos 3x}{(\sin 4x+\sin 2x)+\sin 3x}$

$$\left[\begin{array}{l}\because \cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\\ \sin C+\sin D=2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\end{array}\right]$$

$$=\frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\sin 3x}=\frac{2\cos 3x\cos x+\cos 3x}{2\sin 3x\cos x+\sin 3x}$$

$$=\frac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)}=\cot 3x=RHS$$

5. Prove that : cot$x$ cot2x – cot2x cot3x – cot3x cot$x$ = 1

Sol.: cot3x = cot(2x + x) $\quad\left(\because \cot(A+B)=\dfrac{\cot A\cot B-1}{\cot A+\cot B}\right)$

$$\Rightarrow \cot 3x=\frac{\cot 2x\cot x-1}{\cot 2x+\cot x}$$

$\Rightarrow$ cot3x (cot2x + cot$x$) = cot2x cot$x$ – 1

$\Rightarrow$ 1 = cot2x cot$x$ – cot3x cot2x – cot3x cot$x$

$\because$ cot$x$ cot2x – cot2x cot3x – cot3x cot$x$ = 1

6. Prove that $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Sol. : LHS $=\cos 6x = \cos 2(3x) = 2\cos^2 3x - 1 = 2(\cos 3x)^2 - 1 \quad \left(\because \cos 2\theta = 2\cos^2\theta - 1\right) =$

$2\left[\left(4\cos^3 x - 3\cos x\right)^2\right] - 1 \qquad \left(\because \cos 3\theta = 4\cos^3\theta - 3\cos\theta\right)$

$= 2\left[16\cos^6 x + 9\cos^2 x - 24\cos^4 x\right] - 1$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$ = R.H.S.

<u>Exercise – 3(d)</u>

I. Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ *and* $\tan\frac{x}{2}$ in each of the following in questions. 1 to 3

1. Tan $x = \frac{-4}{3}$, where x is in second quadrant.

Sol. : Given, $\operatorname{Tan} x = \frac{-4}{3}$ (x is in second quadrant)

i.e., $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

i.e. $\frac{x}{2}$ lies in the I quadrant, so that all trigonometric ratios of $\frac{\pi}{2}$ are +ve.

Now, $\operatorname{Sec}^2 x = 1 + \operatorname{Tan}^2 x = 1 + \frac{16}{9} = \frac{9+16}{9} = \frac{25}{9} \qquad \because \sec x = \pm\frac{5}{3}$

But x is in II quadrant $\because \sec x$ is – ve, i.e., $\sec x = -\frac{5}{3} \Rightarrow \cos x = -\frac{3}{5}$

$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\left(\frac{3}{5}\right)}{2}} = \frac{2}{\sqrt{5}}$, $\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\left(\frac{3}{5}\right)}{2}} = \frac{1}{\sqrt{5}}$

$Tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = 2$

2. $\operatorname{Cos} x = -\frac{1}{3}$, where x is in third quadrant.

Sol. : Given cosx = $-\frac{1}{3}$ x is in III quadrant

i.e., $\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \Rightarrow 90^0 < \frac{x}{2} < 135^0$

i.e. $\frac{x}{2}$ lies in II quadrant, so that $\sin\frac{x}{2}>0$, $\cos\frac{x}{2}<0$ and $\tan\frac{x}{2}<0$.

$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{\left(1+\frac{1}{3}\right)}{2}} = -\sqrt{\frac{2}{3}} \quad \left(\because \cos\frac{x}{2} < 0\right)$

Again, $\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} \quad = - \sqrt{\frac{\left(1-\frac{1}{3}\right)}{2}} = \frac{-1}{\sqrt{3}}$

$$\operatorname{Tan}\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\frac{\sqrt{2}}{\sqrt{3}}}{-\frac{1}{\sqrt{3}}} = -\sqrt{2} \quad \left(\operatorname{Tan}\frac{x}{2}<0\right)$$

3. Sin x = $\frac{1}{4}$, where x is in second quadrant.

Sol. : Given $\sin x = \frac{1}{4}$, x is in II quadrant

i.e. $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

i.e. $\frac{x}{2}$ lies in I quadrant, so that all trigonometric ratios of $\frac{x}{2}$ are positive.

Also, $\cos^2 x = 1-\sin^2 x = 1 - \frac{1}{16} = \frac{15}{16}$

$$\cos x = \pm\frac{\sqrt{15}}{4}$$

But x is in II quadrant and $\cos x < 0$ $\qquad \because \cos x = \frac{-\sqrt{15}}{4}$

$$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} \quad = \sqrt{\frac{1+\left(\frac{\sqrt{15}}{4}\right)}{2}} = \sqrt{\frac{4+\sqrt{15}}{8}}$$

$$\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\left(\frac{\sqrt{15}}{4}\right)}{2}} = \sqrt{\frac{4-\sqrt{15}}{8}}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\sqrt{4+\sqrt{15}}}{\sqrt{4-\sqrt{15}}} = = \frac{\sqrt{4+\sqrt{15}}}{\sqrt{4-\sqrt{15}}} x = \frac{\sqrt{4+\sqrt{15}}}{\sqrt{4+\sqrt{15}}} \quad = 4+\sqrt{15}$$

II. 1. Prove that : $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Sol. LHS = $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

[$\because$ 2cosAcosB = cos(A+B) + cos(A-B)]

$$= \cos\left(\frac{\pi}{13}+\frac{9\pi}{13}\right)+\cos\left(\frac{9\pi}{13}-\frac{\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$= \cos\frac{10\pi}{13}+\cos\frac{8\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$= \cos\left(\pi - \frac{3\pi}{13}\right) + \cos\left(\pi - \frac{5\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= -\cos\frac{3\pi}{13} - \cos\frac{5\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0 = \text{R.H.S.}$$

2. Prove that (sin3 x + sin x)sin x +(cos3 x –cos x)cos x = 0

Sol. : LHS = (sin3 x + sin x)sin x +(cos3 x – cos x)cos x

$$\left[\begin{array}{l} \because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \\ \cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right) \end{array}\right]$$

$$= \left[2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)\right]\sin x + \left[-2\sin\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)\right]\cos x$$

$= 2\sin 2x \ \cos x \ \sin x \ - 2\sin 2x \ \sin x \ \cos x \ = 0 = $ R.H.S.

3. Prove that : (cos x +cosy)^2 + (sinx – siny)^2 = 4cos$^2\left(\frac{x+y}{2}\right)$

Sol. : LHS = (cos x +cosy)^2 + (sin x – siny)^2

$$= \left[2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]^2 + \left[2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]^2$$

$$= 4\cos^2\left(\frac{x+y}{2}\right)\cos^2\left(\frac{x-y}{2}\right) + 4\cos^2\left(\frac{x+y}{2}\right)\sin^2\left(\frac{x-y}{2}\right)$$

$$= 4\cos^2\left(\frac{x+y}{2}\right)\left[\cos^2\left(\frac{x-y}{2}\right) + \sin^2\left(\frac{x-y}{2}\right)\right]$$

$$= 4\cos^2\left(\frac{x+y}{2}\right) \ (1) \ = 4\cos^2\left(\frac{x+y}{2}\right) = \text{R.H.S.}$$

4. Prove that : (cos x – cosy)^2 + (sin x – siny)^2 = 4sin$^2\left(\frac{x-y}{2}\right)$

Sol. : LHS = (cos x – cosy)^2+(sin x – siny)^2

$$= \left[-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]^2 + \left[2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]^2$$

$$= 4\sin^2\left(\frac{x+y}{2}\right)\sin^2\left(\frac{x-y}{2}\right) + 4\cos^2\left(\frac{x+y}{2}\right)\sin^2\left(\frac{x-y}{2}\right)$$

$$= 4\sin^2\left(\frac{x-y}{2}\right)\left[\sin^2\left(\frac{x+y}{2}\right) + \cos^2\left(\frac{x+y}{2}\right)\right]$$

$$= 4\sin^2\left(\frac{x-y}{2}\right)(1) = 4\sin^2\left(\frac{x-y}{2}\right) = RHS$$

5.Prove that : sin3 x +sin2 x – sin x = 4sin x cos $\frac{x}{2}$ cos $\frac{3x}{2}$

Sol. : LHS $= \sin 3x + \sin 2x - \sin x$

$= (\sin 3x - \sin x) + \sin 2x$

$= 2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)+\sin 2x$

$= 2\cos 2x \sin x + 2\sin x \cos x$

$= 2\sin x\left(2\cos\left(\frac{2x+x}{2}\right)\cos\left(\frac{2x-x}{2}\right)\right) = 2\sin x\left(2\cos\frac{3x}{2}\cos\frac{x}{2}\right)$

$= 4\sin x \cos\frac{x}{2}\cos\frac{3x}{2}$ = RHS.

III. 1. Prove that $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \cos 4x$

Sol. LHS $= \sin x + \sin 3x + \sin 5x + \sin 7x$

$= (\sin 7x + \sin x) + (\sin 5x + \sin 3x)$

$\left[\because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$

$= 2\sin\left(\frac{7x+x}{2}\right)\cos\left(\frac{7x-x}{2}\right)+2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)$

$= 2\sin 4x \cos 3x + 2\sin 4x \cos x$

$= 2\sin 4x (\cos 3x + \cos x)$

$= 2\sin 4x\left(2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)\right)$

$= 4\sin 4x \cos 2x \cos x = 4\cos x \cos 2x \cos 4x$ = RHS

2. Prove that $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)} = \tan 6x$

Sol. : LHS $= \frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$

$$= \frac{2\sin\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)+2\sin\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)}{2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)+2\cos\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)}$$

$$= \frac{2\sin 6x\cos x+2\sin 6x\cos 3x}{2\cos 6x\cos x+2\cos 6x\cos 3x}$$

$$= \frac{2\sin 6x(\cos x+\cos 3x)}{2\cos 6x(\cos x+\cos 3x)} = \frac{\sin 6x}{\cos 6x} = \tan 6x = RHS$$

4. COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Exercise – 4(a)

I. Express each of the following complex Numbers given in the form a +ib

1. $5i\left(\frac{-3}{5}\right)i = -3i^2 = 3 = 3+i0 = a+ib.$

2. $i^9 + i^{19} = i^9 + i^{10}i^9 = i^9\left(1+i^{10}\right) = i^9\left(1+\left(i^2\right)^5\right) = i^9\left(1-1\right) = 0 = 0+i0$

3. $i^{-39} = \frac{1}{i^{39}} = \frac{1}{i^{39}} x \frac{i}{i} = \frac{i}{i^{40}} = \frac{i}{\left(i^2\right)^{20}} = \frac{i}{\left(-1\right)^{20}} = i = 0+i\left(1\right) = a+ib$

4. $3(7+i7) + i(7+i7) = 21 + 21i + 7i + 7i^2 = 21 + 28i - 7 = 14 + 28i$

5. $(1-i)-(-1+6i) = 1-i+1-6i = 2-7i$

6. $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right) = \frac{1}{5}+i\frac{2}{5}-4-i\frac{5}{2} = \left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right) = -\frac{19}{5}-\frac{21}{10}i$

7. $\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)-\left(-\frac{4}{3}+i\right) = \left(\frac{1}{3}+4+i\frac{7}{3}+i\frac{1}{3}\right)+\frac{4}{3}-i = \frac{17}{3}+i\frac{5}{3}$

8. $\left(1-i\right)^4 = \left[\left(1-i\right)^2\right]^2 = \left[1+i^2-2i\right]^2 = (-2i)^2 = 4i^2 = -4 = -4+0i$

9. $\left(\frac{1}{3}+3i\right)^3 = \left(\frac{1}{3}\right)^3 + \left(3i\right)^3 + 3\left(\frac{1}{3}\right)\left(3i\right)^2 + 3\left(\frac{1}{3}\right)^2\left(3i\right)$

$= \frac{1}{27}+27i^3+\left(3i\right)^2+\frac{1}{3}\left(3i\right) = \frac{1}{27}-27i+i-9 = -\frac{242}{27}-26i$

10. $\left(-2-\frac{1}{3}i\right)^3 = \left(-2\right)^3 + \left(\frac{-1}{3}i\right)^3 + 3\left(-2\right)\left(-\frac{1}{3}\right)^2 + 3\left(-2\right)^2\left(-\frac{1}{3}i\right)$

$= -8 + \frac{1}{27}i + \frac{2}{3} - 4i = -\frac{22}{3} - i\frac{107}{27} = -\left[\frac{22}{3}+i\frac{107}{27}\right]$

II. Find the multiplicative inverse of the each of the complex Numbers (Qs 11 to 13).

11. 4 – 3i

Let z = 4 – 3i .

The multiplicative inverse is $\frac{1}{z} = \frac{1}{4-3i} = \frac{1}{4-3i} X \frac{4+3i}{4+3i} = \frac{4+3i}{16+9} = \frac{4}{25}+\frac{3i}{25}$

12. $\sqrt{5}+3i$

Sol : Let z = $\sqrt{5}+3i$.

The multiplicative inverse is $\frac{1}{z} = \frac{1}{\sqrt{5}+3i} = \frac{1}{\sqrt{5}+3i} X \frac{\sqrt{5}-3i}{\sqrt{5}-3i} = \frac{\sqrt{5}}{14}-\frac{3i}{14}$

13. – i

Let z = - i

The multiplicative inverse is $\frac{1}{z} = \frac{1}{-i} = \frac{-1}{i} X \frac{i}{i} = \frac{-i}{i^2} = i = 0+i\left(1\right)$

III. Express the following in the terms of a+ib : $\dfrac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-\sqrt{2}i)}$.

Sol. Let Z = $\dfrac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-\sqrt{2}i)} = \dfrac{(3)^2-(i\sqrt{5})^2}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-\sqrt{2}i)}$

$= \dfrac{9+5}{2\sqrt{2}i} = \dfrac{14}{2\sqrt{2}i} X \dfrac{i}{i} = \dfrac{7}{\sqrt{2}i^2} = -\dfrac{7i}{\sqrt{2}} = 0 - i\left(\dfrac{7}{\sqrt{2}}\right)$

EXERCISE – 4(b)

I. 1. Evaluate $\left[i^{18}+\left(\dfrac{1}{i}\right)^{25}\right]^3$.

Sol. : $\left[i^{18}+\left(\dfrac{1}{i}\right)^{25}\right]^3 = \left[\left(i^2\right)^9+\dfrac{1}{i^{25}}\right]^3 = \left[(-i)^9+\dfrac{1}{i\left(i^4\right)^6}\right]^3 = (-1-i)^3 \left(\because i^2=-1\right)$

= $(-1)^3(1+i)^3$= -(1-i+3i-3) = -(-2 + 2i) = 2 – 2i

2. For any two complex numbers z_1 and z_2, prove that Re(z_1, z_2) Re(z_1) Re (z_2) – I_m(Z_1) I_m(Z_2)

Sol.: Let Z_1 = a + ib , Re(Z_1) = a ,I_m(Z_1)= b and Let Z_2 = c + id,Re(Z_2) = c ,I_m(Z_2) = d

Now Z_1 Z_2 = (a +ib)(c+id)= ac +iad+ibc+bdi^2=(ac-bd)+i(ad+bc)

Re(Z_1 Z_2) = ac – bd

Re(Z_1 Z_2) = Re(Z_1) Re(Z_2) - I_m(Z_1) I_m (Z_2)

3. Reduce $\left(\dfrac{1}{1-4i}-\dfrac{2}{1+i}\right)\left(\dfrac{3-4i}{5+i}\right)$ to the standard form

Sol. : We have, $\left(\dfrac{1}{1-4i}-\dfrac{2}{1+i}\right)\left(\dfrac{3-4i}{5+i}\right) = \left[\dfrac{1+i-2(1-4i)}{(1-4i)(1+i)}\right]\left(\dfrac{3-4i}{5+i}\right)$

$= \left[\dfrac{1+i-2+8i}{1+i-4i-4i^2}\right]\left(\dfrac{3-4i}{5+i}\right)$

$= \dfrac{-3+4i+27i+36}{25+5i-15i-3i^2}\left(\because i^2=-1\right)$

$= \dfrac{33+31i}{28-10i} X \dfrac{28+10i}{28+10i}$

$= \dfrac{924+330i+868i+310i^2}{784-100i^2} = \dfrac{614+1198i}{884} = \dfrac{307+599i}{442}$

4. Let z_1 =2-i, z_2 = - 2+i find (i) Re $\left(\dfrac{z_1 z_2}{z_1}\right)$ (ii) $I_m\left(\dfrac{1}{z_1\bar{z}_1}\right)$.

Sol.: Let z_1 =2-i, z_2 =-2+i

$z_1\ z_2$ = (2-i)(-2+i)= -4+2i+2i-i^2=-3+4i

$\overline{z_1} = 2 + i$

$$\therefore \frac{z_1 z_2}{\overline{z_1}} = \frac{-3+4i}{2+i} = \frac{-3+4i}{2+i} x \frac{2-i}{2-i}$$

$$= \frac{-6+3i+8i-4i^2}{5} = \frac{-2+11i}{5} = \frac{-2}{5} + i\frac{11}{5}$$

$$\text{Re}\left(\frac{z_1 z_2}{\overline{z_1}}\right) = -\frac{2}{5}$$

(i) $$\frac{1}{z_1 \overline{z_1}} = \frac{1}{(2-i)(2+i)} = \frac{1}{(2)^2 - i^2} = \frac{1}{4+1} = \frac{1}{5}$$

$$= \frac{1}{5} + i(0)$$

$$I_m\left(\frac{1}{z_1 \overline{z_1}}\right) = 0$$

5. Find the real number of x and y if (x – iy)(3+5i) is the conjugate of (-6-24i)

Sol. : (x-iy)(3+5i) = 3x+5xi – i3y – 5yi^2=(3x +5y)+i(5x – 3y)

Given that (x – iy)(3+5i) = $\left(\overline{-6-24i}\right)$

(3x+5y) + i (5x-3y) = -6+24i

By comparing real and imaginary parts on both sides, we get

3x + 5y = - 6 …..(1) 5x -3y = 24 …..(2)

Solving (1) & (2) $\frac{x}{-120+18} = \frac{y}{30+72} = \frac{1}{-9-25} \Rightarrow \frac{x}{-102} = \frac{y}{102} = \frac{1}{-34}$

X = 3 and y =-3

6. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$**.**

Sol. Let Z = $\frac{1+i}{1-i} - \frac{1-i}{1+i} = \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)} = \frac{(1+i^2+2i)-(1+i^2-2i)}{1-i^2}$

$$= \frac{2i+2i}{2} = \frac{4i}{2} = 2i = 0 + 2i \qquad |Z| = \sqrt{0+2^2} = \sqrt{4} = 2$$

7.Find the number of non – zero integral solutions of the equation $|1-i|^x = 2^x$**.**

Sol. : $|1-i|^x = 2^x \Rightarrow \left(\sqrt{1^2+(-1)^2}\right)^x = 2^x \left(|(a+ib)| = \sqrt{a^2+b^2}\right)$

$$\left(\sqrt{2}\right)^x = 2^x \Rightarrow 2^{\frac{x}{2}} = 2^x$$

Comparing the power of 2 on both sides

$\frac{x}{2} = x \Rightarrow 2x - x = 0 \Rightarrow x = 0$

Hence, number of solutions is zero.

II. 1. If x – iy = $\sqrt{\frac{a-ib}{c-id}}$, then prove that $\left(x^2+y^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$.

Sol. Given, x –iy = $\sqrt{\frac{a-ib}{c-id}} \Rightarrow$ x +i(-y) = $\left(\frac{a-ib}{c-id}\right)^{\frac{1}{2}} \Rightarrow \quad \left|x+i(-y)\right| = \left|\left(\frac{a-ib}{c-id}\right)^{\frac{1}{2}}\right|$

$\sqrt{x^2+(-y)^2} = \left|\frac{a-ib}{c-id}\right|^{\frac{1}{2}}$ (Squaring on both sides)

We get $x^2+y^2 = \left|\frac{a-ib}{c-id}\right| \Rightarrow x^2+y^2 = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$

$$\left(x^2+y^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$$

2.If $Z_1 = 2 - i$, $Z_2 = 1 + i$, find $\left|\frac{Z_1+Z_2+1}{Z_1-Z_2+1}\right|$

Sol.: Let Z_1 = 2-i, Z_2=1+i and $Z_1+Z_2+1 = 2-i+1+i+1 = 4$

$Z_1 - Z_2 + 1 = 2 - i - 1 - i + 1 = 2 - 2i$

$$\left|\frac{Z_1+Z_2+1}{Z_1-Z_2+1}\right| = \left|\frac{Z_1+Z_2+1}{Z_1-Z_2+1}\right| = \frac{|4|}{|2-2i|} = \frac{4}{\sqrt{2^2+(-2)^2}} = \frac{4}{\sqrt{4+4}} = \frac{4}{\sqrt{8}} = \frac{4}{2\sqrt{2}} = \sqrt{2}$$

3.If a + ib = $\frac{(x+i)^2}{2x^2+1}$, prove that $a^2+b^2 = \frac{\left(x^2+1\right)^2}{\left(2x^2+1\right)^2}$

Sol. : a + ib = $\frac{(x+i)^2}{2x^2+1} = \frac{x^2+i^2+2xi}{2x^2+1} = \frac{x^2-1+2xi}{2x^2+1}$

on comparing real and imaginary parts, we obtain $a = \frac{x^2-1}{2x^2+1}$ *and* $b = \frac{2x}{2x^2+1}$.

$$\therefore a^2+b^2 = \left(\frac{x^2-1}{2x^2+1}\right)^2 + \left(\frac{2x}{2x^2+1}\right)^2 = \frac{x^4+1-2x^2+4x^2}{\left(2x^2+1\right)^2} = \frac{x^4+2x^2+1}{\left(2x^2+1\right)^2}$$

$$\therefore a^2+b^2 = \frac{\left(x^2+1\right)^2}{\left(2x^2+1\right)^2}$$

4.If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x}+\frac{v}{y} = 4\left(x^2-y^2\right)$

Sol. : $(x+iy)^3 = u + i\,\vartheta$

$$x^3 + i^3y^3 + 3x^2(iy) + 3x(iy)^2 = u + iv$$

$$x^3 - iy^3 + 3x^2(iy) - 3xy^2 = u + iv$$

$$x^3 - 3xy^2 + i\left(3x^2y - y^3\right) = u + iv$$

$$x\left(x^2 - 3y^2\right) + iy\left(3x^2 - y^2\right) = u + iv$$

Comparing real and imaginary parts on both sides

$$x\left(x^2 - 3y^2\right) = u, y\left(3x^2 - y^2\right) = v \Rightarrow \frac{u}{x} = x^2 - 3y^2, \frac{v}{y} = 3x^2 - y^2$$

$$\Rightarrow \frac{u}{x} + \frac{v}{y} = x^2 - 3y^2 + 3x^2 - y^2 = 4x^2 - 4y^2 = 4\left(x^2 - y^2\right)$$

5.If α and β are different complex numbers with $|\beta|$=1, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$.

Sol. : If Z = x + iy and $\bar{z}$ = x - iy then $|z|^2 = z.\bar{z}$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2 = \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|\left|\overline{\frac{\beta-\alpha}{1-\bar{\alpha}\beta}}\right| = \frac{(\beta-\alpha)(\beta-\bar{\alpha})}{(1-\bar{\alpha}\beta)(1-\alpha\bar{\beta})}$$

$$= \frac{\beta\bar{\beta} - \beta\bar{\alpha} - \alpha\bar{\beta} - \alpha\bar{\alpha}}{1 - \alpha\bar{\beta} - \bar{\alpha}\beta + \alpha\bar{\alpha}\beta\bar{\beta}} = \frac{|\beta|^2 - \beta\bar{\alpha} - \alpha\bar{\beta} - |\alpha|^2}{1 - \alpha\bar{\beta} - \bar{\alpha}\beta + |\alpha|^2|\beta|^2}$$

$$= \frac{1 - \beta\bar{\alpha} - \alpha\bar{\beta} + |\alpha|^2}{1 - \alpha\bar{\beta} - \bar{\alpha}\beta + |\alpha|^2}\left(\because |\beta| = 1\right) = 1$$

$$\therefore \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$$

6.If (a+ib)(c+id)(e+if)(g+ih) = A+iB, then show that

$\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right) = A^2 + B^2$.

Sol.: (a+ib)(c+id)(e+if)(g+ih) = A +iB taking modulus on both sides, we get

$$|a+ib||c+id||e+if||g+ih| = |A+iB|$$

$\sqrt{a^2+b^2}\sqrt{c^2+d^2}\sqrt{e^2+f^2}\sqrt{g^2+h^2} = \sqrt{A^2+B^2}$ squaring on both sides, we get

$$\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right) = A^2 + B^2$$

7.If $\left(\frac{1+i}{1-i}\right)^m$ = 1, then find the least positive integral value of m.

Sol. : $\left(\frac{1+i}{1-i}\right)^m = 1 \Rightarrow \left[\left(\frac{1+i}{1-i}\right)\left(\frac{1+i}{1+i}\right)\right]^m = 1 \Rightarrow \left[\frac{1+i^2+2i}{2}\right]^m = 1$

$$\left(\frac{1-1+2i}{2}\right)^m = 1 \Rightarrow \left(\frac{2i}{2}\right)^m = 1 \Rightarrow (i)^m = 1$$

$$\left(\sqrt{i}\right)^m = 1 \quad \left(\begin{array}{l} \because i^2 = -1 \\ i = \sqrt{-1} \end{array}\right)$$

$$(-1)^{\frac{m}{2}} = 1 \Rightarrow (-1)^{\frac{m}{2}} = (-1)^2 \Rightarrow \frac{m}{2} = 2 \Rightarrow m = 4$$

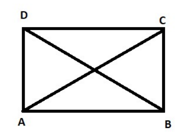
Hence, the least positive integral value of m is 4.

8. Show that the four points in the Argand plane represented by the complex numbers 2 + i, 4 + 3i, 2+5i, 3i are the vertices of a square.

Sol. : Let A(2, 1) B(4,3), C(2, 5) and D(0, 3)

AB = $\sqrt{(2-4)^2+(3-1)^2} = \sqrt{8}$, BC = $\sqrt{(4-2)^2+(3-5)^2} = \sqrt{8}$

CD = $\sqrt{(2-0)^2+(5-3)^2} = \sqrt{8}$, DA = $\sqrt{(2-0)^2+(1-3)^2} = \sqrt{8}$,



AC = $\sqrt{(2-2)^2+(1-5)^2} = \sqrt{16} = 4$ and BD = $\sqrt{(4-0)^2+(3-3)^2} = \sqrt{16} = 4$

Hence AB = BC = CD = DA and AC = BD

$\therefore$ A, B, C,D form a square

9.Show that the points in the argand plane represented by the complex numbers -2 + 7i, -$\frac{3}{2}+\frac{1}{2}i$, 4 – 3i and $\frac{7}{2}(1+i)$ are the vertices of a Rhombus.

Sol. : A (-2, 7) B$\left(\frac{-3}{2},\frac{1}{2}\right)$,C(4, -3) and D $\left(\frac{7}{2},\frac{7}{2}\right)$

AB = $\sqrt{\left(-2+\frac{3}{2}\right)^2+\left(7-\frac{1}{2}\right)^2} = \frac{\sqrt{170}}{2}$, BC = $\sqrt{\left(4+\frac{3}{2}\right)^2+\left(-3-\frac{1}{2}\right)^2} = \frac{\sqrt{170}}{2}$

CD = $\sqrt{\left(4-\frac{7}{2}\right)^2+\left(-3-\frac{7}{2}\right)^2} = \frac{\sqrt{170}}{2}$, DA = $\sqrt{\left(\frac{7}{2}+2\right)^2+\left(\frac{7}{2}-7\right)^2} = \frac{\sqrt{170}}{2}$

AC = $\sqrt{(4+2)^2+(-3-7)^2} = \sqrt{36+100} = \sqrt{136}$,

BD = $\sqrt{\left(\frac{7}{2}+\frac{3}{2}\right)^2+\left(\frac{7}{2}-\frac{1}{2}\right)^2} = \frac{\sqrt{136}}{2}$

$\therefore$ AB = BC =CD = DA and AC $\neq$ BD

$\therefore$ A, B, C, D form a Rhombus.

10.The point P, Q denote the complex numbers Z_1, Z_2 in the argand diagram. O is the origin if $Z_1\overline{Z_2}+\overline{Z_1}Z_2=0$ then show that $\angle POQ=\frac{\pi}{2}$.

Sol. : Let P = (x_1, y_1) $\Rightarrow z_1=x_1+iy_1$, Q = $(x_2, y_2) \Rightarrow z_2=x_2+iy_2$ and O(0,0)

$Z_1=x_1+iy_1 \Rightarrow \overline{Z}_1=x_1-iy_1$, $\quad Z_2=x_2+iy_2 \Rightarrow \overline{Z}_2=x_2-iy_2$

Given that $Z_1\overline{Z}_2+\overline{Z}_1Z_2=0 \Rightarrow (x_1+iy_1)(x_2-iy_2)+(x_1-iy_1)(x_2+iy_2)=0$

$\Rightarrow (x_1x_2+y_1y_2)(x_1x_2+y_1y_2)=0$

$\Rightarrow x_1x_2=-y_1y_2 \Rightarrow \left(\frac{-y_1}{x_1}\right)\left(\frac{-y_2}{x_2}\right)=-1$

$\Rightarrow$ (slope of $\overline{OP}$)(slope of $\overline{OQ}$) = -1

$\Rightarrow \angle POQ=90^0$

11.The complex number Z has argument θ, 0<θ<$\frac{\pi}{2}$ and satisfy te equation $|z-3i|=3$ then prove that $\left(\cot\theta-\frac{6}{z}\right)=i$.

Sol. : Give that 0<θ<$\frac{\pi}{2}$ then as x > 0, y > 0,

$\tan\theta=\frac{y}{x} \Rightarrow \cot\theta=\frac{x}{y}$

Also z = x + iy $\Rightarrow \bar{z}=x-iy$

Then $z\overline{Z}$ = (x + iy)(x-iy) = x^2+y^2

Now, $|z-3i|=3 \Rightarrow |x+iy-3i|=3$ $|x+i(y-3)|=3$

$\sqrt{x^2+(y-3)^2}=3$

Squaring on both sides

$x^2+(y-3)^2=9 \Rightarrow x^2+y^2-6y+9=9 \Rightarrow x^2+y^2=6y$

$\therefore z\bar{z}=6y \Rightarrow \frac{\bar{z}}{y}=\frac{6}{Z}$

$\cot\theta-\frac{6}{Z}=\cot\theta-\left(\frac{\bar{z}}{y}\right)=\frac{x}{y}-\left(\frac{x-iy}{y}\right)=\frac{x}{y}-\frac{x}{y}+i=i$

$\therefore \cot\theta-\frac{6}{Z}=i$.

5. LINEAR INQUALITIES

EXERCISE – 5 (a)

1. Solve 24 x < 100 when (i) x is a natural number (ii) x is an integer.

Sol.: Given 24 x < 100 dividing both sides with 24

$$\frac{24x}{24} < \frac{100}{24}$$

$$x < \frac{50}{12} \Rightarrow x < \frac{25}{06} \Rightarrow x < 4\frac{1}{6}$$

(i) When x is a natural number in this case the solution set of the inequality is { 1, 2, 3, 4}

(ii) When x is an integer, the solution set of the given inequalities is {…..-4, -3, -2, -1, 0, 1, 2, 3,4}

2. Solve -12 x > 30 when (i) x is a natural number (ii) x is an integer.

Sol.: Given -12 $x > 30$, dividing both sides with -12 .

$$\frac{-12x}{-12} < \frac{30}{-1}$$

$$\Rightarrow x < -\frac{5}{2} \Rightarrow x < -2\frac{1}{2}$$

(i) When x is a natural number then there is no solution of given inequality. as natural numbers are positive numbers and there is no positive number which is less than negative number.

(ii) When x is an integer the solution of given inequality is {……….-4,-3} as there are infinite number which are less than - $\frac{5}{2}$.

3. Solve 5x -3 < 7 when (i) x is an integer (ii) x is a real number.

Sol.: Given 5x -3 < 7 adding 3 on both sides

$$5x - 3 + 3 < 7 + 3$$

$$\Rightarrow 5x < 10$$

$$\Rightarrow x < 2$$

(i) When x is an integer , the solution of the given inequality is {…………-1,0,1}.

(ii) When x is a real number, the solution of given inequality is (-∞, 2) i.e all the numbers lying between –∞ and 2 but -∞ and 2 are not included as x < 2.

4. Solve 3x + 8 > 2 when (i) x is an integer (ii) x is a real number.

Sol.: Given 3x + 8 > 2 Adding -8 on both sides

$$3x + 8 - 8 > 2 - 8$$

$$3x > -6 \Rightarrow x > -2$$

(i) When x is an integer the solution of the given inequality is {-1,0,1,2……}

(ii) When x is an real number, the solution of the given inequality is (-2, ∞) i.e. all the numbers lying between -2 and ∞ but -2 and ∞ are not included.

5. Solve the inequality for real x, 4x + 3 < 5x + 7

Sol.: Given 4x + 3 < 5x + 7

$$4x + 3 - 5x < 7$$

$$-x < 7 - 3 \Rightarrow -x < 4 \Rightarrow x > -4$$

By using number line, we can easily look for the numbers greater than -4.

∴ Solution set = (-4, ∞) i.e all the numbers lying between -4 and ∞ but -4 and ∞ are not included as x > - 4.

6. Solve the inequality for real x, $3x - 7 > 5x-1$

Sol.: Given $3x- 7 > 5x -1$

$3x - 5x > -1 + 7$

$-2x > 6 \Rightarrow -x > 3$

$\therefore x < -3$

Solution set = $(-\infty, -3)$ i.e all the numbers between $-\infty$ and -3 but $-\infty$ and -3 are not included as $x < -3$.

7. Solve the inequality $3(x-1) \leq 2(x-3)$ for real x

Sol.: $3(x-1) \leq 2(x-3) \Rightarrow 3x - 2x \leq -6 + 3$

$\therefore x \leq -3$

Solution set = $(-\infty, -3]$

8. Solve $3(2-x) \geq 2(1-x)$ for real x.

Sol.: Given $3(2-x) \geq 2(1-x)$

$6 - 2 \geq -2x + 3x$

$4 \geq x \quad \Rightarrow x \leq 4$

Solution set = $(-\infty, 4]$

9. Solve $x + \frac{x}{2} + \frac{x}{3} < 11$, for real x

Sol.: Given $x + \frac{x}{2} + \frac{x}{3} < 11$

$\frac{6x+3x+2x}{6} < 11 \quad \Rightarrow \frac{11x}{6} < 11$

$\therefore x < 6$

Solution set = $(-\infty, 6)$

10. Solve $\frac{x}{3} > \frac{x}{2} + 1$ for real x

Sol.: Given that $\frac{x}{3} > \frac{x}{2} + 1$

$\frac{x}{3} - \frac{x}{2} > 1 \Rightarrow \frac{2x-3x}{6} > 1 \Rightarrow -x > 6$

$\therefore x < -6$

Solution set = $(-\infty, -6)$

11. Solve $2(2x+3) - 10 < 6(x-2)$ for real x.

Sol.: Given that $2(2x+3) - 10 < 6(x-2)$

$4x - 4 < 6x - 12$

$4x - 6x < -12 + 4$

$-2x < -8 \quad \Rightarrow x > 4$

Solution set = $(4, \infty)$

12. Solve $37 - (3x+5) \geq 9x - 8(x-3)$ for real x.

Sol.: Given that $37 - (3x+5) \geq 9x - 8(x-3)$

$32 - 3x \geq x + 24$

$32 - 24 \geq x + 3x$

$8 \geq 4x$

$\therefore 4x \leq 8$

$x \leq 2$

Solution set = $(-\infty, 2]$

13. Ravi obtained 70 and 75 marks in first two unit tests. Find the maximum marks he should get in the third unit test to have an average of atleast 60 marks.

Sol.: Ravi got x marks in third unit test.

Average marks obtained by Ravi $= \frac{Sum\ of\ marks\ in\ all\ tests}{Number\ of\ tests} = \frac{70+75+x}{3} = \frac{145+x}{3}$

To obtain an average of atleast 60 marks from the given information , we can write the linear inequality as $\frac{145+x}{3} \geq 60$

$145 + x \geq 180$

$x \geq 180 - 145$

$x \geq 35.$

$\therefore$ Ravi should get greater than or equal to 35 marks in third unit test to get an average of atleast 60 marks.

II.**1. Solve the inequality $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$ for real x**

Sol.: Given that $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3} \Rightarrow \frac{3x-6}{5} \leq \frac{10-5x}{3}$

$9x - 18 \leq 50 - 25x$

$34x \leq 68 \Rightarrow x \leq 2$

Solution set = $(-\infty\ \ 2]$

2. Solve the inequality $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x-6)$ for real x

Sol.: $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x-6)$

$\frac{1}{2}\left(\frac{3x+20}{5}\right) \geq \frac{1}{3}(x-6)$

$\left(\frac{3x+20}{10}\right) \geq \frac{x-6}{3}$

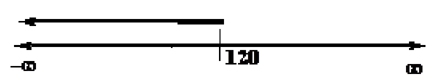
$3(3x + 20) \geq 10(x-6)$

$9x - 10x \geq -60 - 60$

$-x \geq -120$

$\therefore x \leq 120$

Solution set = $(-\infty, 120]$



3. Solve the inequality of $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$ for real x

Sol.: Given $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

$\frac{x}{4} < \frac{5(5x-2)-3(7x-3)}{15}$

$\frac{x}{4} < \frac{25x-10-21x+9}{15}$

$\frac{x}{4} < \frac{4x-1}{15}$

$15x < 4(4x - 1)$

$15x < 16x - 4 \Rightarrow -x < -4$

$x > 4$

Solution set = $(4, \infty)$

4. Solve the inequality of $\frac{2x-1}{3} \geq \left(\frac{3x-2}{4}\right) - \left(\frac{2-x}{5}\right)$ for real x

Sol.: We have $\frac{2x-1}{3} \geq \left(\frac{3x-2}{4}\right) - \left(\frac{2-x}{5}\right)$

$$\frac{2x-1}{3} \geq \frac{5\,(3x-2)-4\,(2-x)}{20}$$

$$\frac{2x-1}{3} \geq \frac{19x-1}{20}$$

$$20\,(2x-1) \geq 3\,(19x-18)$$

$$40x-20 \geq 57x-54$$

$$-17x \geq -34$$

$$\therefore x \leq \frac{-34}{-17} \Rightarrow \quad x \leq 2$$

Solution set = $(-\infty, 2]$

Direction (Q. No. 5 to 8) Solve the inequalities and show their graph of the solution in each case on the number line.

5. Solve 3x -2 < 2x + 1

Sol.: Given $3x - 2 < 2x + 1$

$3x - 2x < 1 + 2$

$x < 3$

All the numbers on the left side of 3 will be less than it

Solution set = $(-\infty, 3)$

6. Solve 5x – 3 ≥ 3x -5

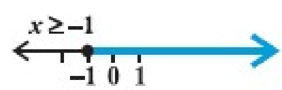
Sol.: Given $5x - 3 \geq 3x - 5$

$5x - 3x \geq -5 + 3$

$2x \geq -2$

$\frac{2x}{2} \geq \frac{-2}{2}$

$x \geq -1$



All the numbers on the right side of -1 will be greater than it.

Solution set = $[-1, \infty)$

7. Solve 3 (1- x) < 2 (x + 4)

Sol.: Given $3\,(1-x) < 2\,(x+4)$

$3 - 3x < 2x + 8$

$-3x - 2x < 8 - 3$

$-5x < 5$

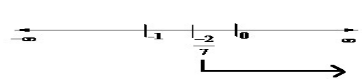
Dividing with – 5

$\frac{-5x}{-5} > \frac{5}{-5}$

$x > -1$

$\therefore$ Solution set $(-1, \infty)$

8. Solve $\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$



Sol.: Given that $\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5} \Rightarrow \frac{x}{2} \geq \frac{125x-10-21x}{15}$

$\frac{x}{2} \geq \frac{4x-1}{15}$

$15x \geq 2\,(4x-1)$

$15x \geq 8x - 2$

$15x - 8x \geq -2$

$7x \geq -2 \Rightarrow x \geq \frac{-2}{7}$

Solution Set = $\left[\frac{-2}{7}, \infty\right)$

9. To receive Grade A in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks) . If Sunita marks in first four examinations are 87, 92, 94, 95 . Find the minimum marks that Sunita must obtain in fifth examination to get Grade A in the course.

Sol.: Let Sunita got x marks in the fifth exam.

Average marks obtained by Sunita

$= \frac{Sum\ of\ marks\ in\ all\ exams}{Number\ of\ exams}$

$= \frac{87+92+94+95}{5} = \frac{368+x}{5}$

Now it is given that Sunita wants to obtain grade A for that her average marks should be greater than or equal to 90.

$\frac{368}{5} \geq 90 \Rightarrow 368 + x \geq 90 \times 5 \Rightarrow x \geq 450 - 368 \Rightarrow x \geq 82$

Sunita should got greater than or equal to 82 marks in fifth exam to get Grade A.

10. Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Sol.: Let the numbers are $2x + 1$ and $2x + 3$.

Given that which are smaller than 10.

$\therefore 2x + 1 < 10$ and $2x + 3 < 10$

$2x < 9$ and $2x < 7$

$x < \frac{9}{2}$ and $x < \frac{7}{2}$ also given that their sum is more than 11.

$(2x + 1) + (2x + 3) > 11 \Rightarrow 4x + 4 > 11 \Rightarrow x > \frac{7}{4}$

$\therefore x \in (\frac{7}{4}, \frac{7}{2})$ in which integer values are $x = 2$ and $x = 3$.

When x = 2 numbers are $(2x + 1, 2x + 3) = (2(2) + 1, 2(2) + 3) = (5, 7)$

When x = 3 numbers are $(2(3) + 1, 2(3) + 3) = (6 + 1, 6 + 3) = (7, 9)$

Required pairs are (5, 7) and (7, 9).

11.Find all pair of consecutive even positive integers both of which are larger than 5 such that their sum is less than 23.

Sol.: Let numbers are $2x$ and $2x + 2$

Given that which are larger than 5.

$\therefore 2x > 5$ and $2x + 2 > 5$

$\Rightarrow x > \frac{5}{2}$ and $x > \frac{3}{2}$ also given that their sum is less than 23.

$\therefore (2x) + (2x + 2) < 23$

$4x < 23 - 2$

$x < \frac{21}{4}$

$\therefore x \in (\frac{5}{2}, \frac{21}{4})$.

In which the integer values are x = 3, 4, 5 .

Pair of numbers = $(2x, 2x + 2)$.

When $x = 3$, pair is $(2(3), 2(3)+2) = (6, 8)$

When x = 4 pair is (2 (4), 2(4) + 2) = (8, 10)
When x = 5 pair is (2 (5), 2 (5) +2) = (10,12)

12. The longest side of a triangle is 3 times the shortest side and the third side is 2 cm, shorter than the longest side if the perimeter of the triangle is atleast 61cms, Find the minimum length of the shortest side.

Sol.: Let the shortest side be x cms

Then according to the given condition

longest side = 3 x cm and third side = (3 x -2) cm.

Given perimeter of the triangle is atleast 61cm

Sum of all sides ≥ 61

$x + (3x) + (3x - 2) \geq 61$

$7x - 2 \geq 61$

$7x \geq 61 + 2$

$7x \geq 63$

$\Rightarrow x \geq \frac{63}{7}$

$\therefore x \geq 9$

$\therefore$ Minimum length at the shortest side = 9 cms

II.

1. A man wants to cut three lengths from a single piece of board of length is 91 cm. The second length is to be 3 cm longer than the shortest and third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be atleast 5cm longer than the second ?

Sol Let the shortest length is x cm.

By given condition, second length = x +3 cms

Third length = $2x$ cm

Given that total length is 91 cm

$\therefore x + (x + 3) + 2x \leq 91$

$\Rightarrow 4x + 3 \leq 91$

$\Rightarrow 4x \leq 91 - 3$

$\Rightarrow 4x \leq 88$

$\Rightarrow x \leq \frac{88}{4}$

$\therefore x \leq 22$ ……………(1)

Given that third piece is to be atleast 5cms longer than second piece.

$\therefore 2x \geq (x + 3) + 5$

$2x - x \geq 3 + 5$

$x \geq 8$ ……….(2)

From equations (1), (2) the length of shortest board should be greater than or equal to 8 but less than or equal to 22.

$\therefore 8 \leq x \leq 22$

EXERCISE – 5 (b)

I.1. Solve the inequality $2 \leq 3x - 4 \leq 5$

Sol.: Given inequality $2 \leq 3x - 4 \leq 5 \Rightarrow 2 + 4 \leq 3x \leq 5 + 4$

$6 \leq 3x \leq 9$

Dividing by 3 in each term

$$\frac{6}{3} \leq \frac{3x}{3} \leq \frac{9}{3} \Rightarrow 2 \leq x \leq 3$$

Solution set = [2, 3]

2. Solve $6 \leq -3(2x - 4) < 12$

Sol.: Given inequality $6 \leq -3(2x - 4) \leq 12$

$6 \leq -6x + 12 \leq 12 \Rightarrow 6 - 12 \leq -6x \leq 12 - 12$

$-6 \leq -6x \leq 0$

Dividing by (-6) to each term

$$-\frac{6}{6} \geq -\frac{61}{-6} \geq \frac{0}{-6}$$

$1 \geq x \geq 0 \Rightarrow 0 \leq x \leq 1$

Solution set = [0, 1]

Note : When we divide the inequalities by negative number, the sign of the inequality will be change.

3. Solve the inequality $-3 \leq 4 - \frac{7x}{2} \leq 18$

Sol.: Given inequality $-3 \leq 4 - \frac{7x}{2} \leq 18$

$-3 - 4 \leq \frac{7x}{2} \leq 18 - 4$ (Adding '- 4' to each term)

$\Rightarrow -7 \leq -\frac{7x}{2} \leq 14 \Rightarrow 14 \leq -7x \leq 28$

$\Rightarrow \frac{-14}{-7} \geq \frac{-7x}{-7} \geq \frac{28}{-7}$ (dividing with '7' each term)

$\Rightarrow 2 \geq x \geq -4 \quad \Rightarrow -4 \leq x \leq 2$

$\therefore$ Solution set = [-4, 2]

4. Solve the inequality $-15 < \frac{3(x-2)}{5} \leq 0$

Sol: $15 \times 5 < 3x - 6 < 0$

$75 < 3x - 6 < 0$

$75 + 6 < 3x \leq 0 + 6$ (Adding '6' to each term)

$\Rightarrow -69 < 3x \leq 6 \Rightarrow \frac{-69}{3} < \frac{3x}{3} \leq 6 \Rightarrow -23 < x \leq 2$ (dividing with '3' to each term)

Solution set = (-23, 2)

5. Solve the inequality $-12 < 4 - \frac{3x}{-5} \leq 2$

Sol: The given inequality

$$-12 < 4 - \frac{3x}{-5} \leq 2$$

$$-12 < 4 + \frac{3x}{-5} \leq 2$$

$$-12 - 4 < \frac{3x}{-5} \leq 2 - 4 \quad \text{(Adding '-4' to each term)}$$

$$-16 < \frac{3x}{-5} \leq -2$$

$$-16 \text{ x } 5 < 3x \leq -2 \text{ x } 5$$

$$-80 < 3x \leq -10$$

$$\therefore \frac{-80}{3} < x \leq -\frac{10}{3} \quad \text{(dividing with '3' to each term)}$$

Solution set = $\left(-\frac{-80}{3}, \frac{-10}{3} \right)$

6. Solve the inequality $7 \leq \frac{3x+11}{2} \leq 11$

Sol: $7 \text{ x } 2 \leq 3x + 11 \leq 22$

$14 - 11 \leq 3x \leq 22 - 11$ Adding '-11' to each term

$3 \leq 3x \leq 11$

$\frac{3}{3} \leq \frac{3x}{3} \leq \frac{11}{3}$

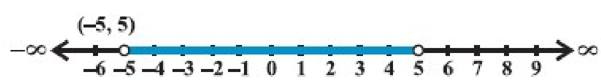
$1 \leq x \leq \frac{11}{3}$

Solution Set = $\left[1, \frac{11}{3} \right]$

II.

1. Solve the inequalities and represent the solution graphically on the number line.

Solve $5x + 1 > -24$, $5x + 1 < 24$



Sol : Given $5x + 1 > -24$

$5x > -24 - 1$, $5x > -25$

$x > \frac{-2}{5} \Rightarrow x > -5$ ....(1)

By (1) and (2)

$\therefore -5 < x < 5$

Solution Set = $(-5, 5)$

Hence the solution of the system are real numbers x lying between -5 and 5 excluding -5 and 5

2. Solve $2(x - 1) < x + 5$ and $3(x + 2) > 2 - x$

Sol: Given $2(x - 1)(x + 5)$

$2x - 2 < x + 5 \Rightarrow 2x - x < 5 + 2$

$x < 7$ ....(1)

Given : $3(x + 2) > 2 - x$

$3x + 6 > 2 - x \Rightarrow 3x + x > 2 - 6$

$4x > -4 \Rightarrow x > -\frac{4}{4}23$

x > -1 …………(2)

By (1) and (2)

$-1 < x < 7$

Solution set = (-1, 7)

Hence solution set of the equations are real numbers x lying between -1 and 7 excluding -1 and 7 .

Solution set = (-1, 7) $-1 < x < 7$

3. Solve $3x - 7 > 2(x - 6)$, $6 - x > 11 - 2x$

Sol: : $3x - 7 > 2(x - 6)$

$3x - 7 > 2x - 12 \Rightarrow \quad 3x - 2x > 12 + 7$

$x > -5$ ……..(1)

Given $6 - x > 11 - 2x$

$x + 2x > 11 - 6$

$x > 5$ ……….(2)

Solution set = (5, ∞)

Hence solution set of the equations are real numbers , 'x' lying on greater than 5 excluding '5'.

Solution set = (5, ∞).

4. Solve $5(2x - 7) - 3(2x + 3) \leq 0$, $2x + 19 \leq 6x$

Sol: Given that $5(2x - 7) - 3(2x + 3) \leq 0$

$10x - 35 - 6x - 9 \leq 0$

$4x - 44 \leq 0 \Rightarrow \quad 4x \leq 44$

$x = \frac{44}{4}$

$x \leq 11$ …………..(1)

Given that $2x - 19 \leq 6x + 4$

$2x - 6x \leq 47 - 19 \Rightarrow \ -4x \leq 28 \Rightarrow \quad -x \leq \frac{28}{4} \Rightarrow \quad -x \leq 7$

$\Rightarrow x \geq -7$ ………(2)

By (1) and (2) $-7 \leq x \leq 11$

Solution set = [-7,11]

Hence, solution set of the inequalities are real numbers x lying between -7 and11 including -7 and 11.

$\therefore 7 \leq x \leq 11$

Solution Set = [-7, 11]

5. A solution is to be kept between 68^0 F and 77^0F what is the range of temperature in degree Celsius (C) if the Celsius / Fahrenheit (F) conversion formula is given by F = $\frac{9}{5}C + 32$?

Sol.: Given that F = $\frac{9}{5}C+32$ $C=\frac{5}{9}(F-32)$

When F = 68, C $=\frac{5}{9}(68-32)$ $=\frac{5}{9}(36)$=20

When F = 77, C = $\frac{5}{9}(77-32)$ = $\frac{5}{9}(45)$ = 25

$\therefore$ The range of temperature in degree Celsius is between 20^0C and 25^0C.

6. IQ of a person is given by the formula IQ = $\frac{MA}{CA}$x100 where MA is metal age and CA is chronological age. If 80 $\le$IQ$\le$140 for a group of 12 years old children, find the range of their mental age.

Sol.: Given that 80 $\le$IQ$\le$140

For a group of 12 years old children CA =12

$\therefore$ IQ = $\frac{MA}{12}$x100 $\Rightarrow \frac{12IQ}{100}=MA$

$\therefore 80\le\frac{MA}{12}X100\le140\Rightarrow\frac{80X12}{100}\le MA\le\frac{140X12}{100}$

$\Rightarrow$9.6 $\le$MA $\le$ 16.8

$\therefore$ Range of mental age of required group of children is [9.6, 16.8].

III. 1. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added ?

Sol.: Let ' x ' be the required solution to be added.

Total amount of boric acid in the mixture = Boric acid in the 8% solution + Boric acid in 2% solution

Total volume of the mixture = 640 + x litres

Resulting mixture is to be between 4% and 6%

$$\Rightarrow 0.04<\frac{0.08(640)+(0.02)x}{640+x}<0.06$$

From 1st part of inequality (0.04)(640+x) < (0.08) (640) + (0.02) x

$\Rightarrow$25.6+0.042<51.2+0.02x

$\Rightarrow$0.02x < 25.6

$x<\frac{25.6}{0.02}$, x < 1280…………………(1)

From 2nd part,

$$\Rightarrow \frac{(0.08)(640)+(0.02)x}{640+x} < 0.06$$

$\Rightarrow 51.2 + 0.02x < 38.4 + 0.06x$

$\Rightarrow 51.2 - 38.4 < 0.06x - 0.02x \Rightarrow 12.8 < 0.04x$

$$\Rightarrow x > \frac{12.8}{0.04} = 320 \Rightarrow x > 320 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From (1) and (2), 320 < x < 1280

$\therefore$ The number of litres of the 2% solution that have to be added is between 320 and 1280.

2.How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Sol.: Let x be the amount of water added to the 1125 lts of 45% acid solution total volume of solution = 1125 + x litres.

The amount of acid in original solution = 0.45 x 1125 = Amount of acid in new solution

Concentration of acid in new solution = $\frac{0.45 x 1125}{1125+x}$

$$0.25 < \frac{0.45 x 1125}{1125+x} < 0.30$$

$$0.25 < \frac{0.45 x 1125}{1125+x} \Rightarrow 0.25(1125+x) < 0.45 X 1125$$

$281.25 + 0.25x < 506.25$

$0.25x < 506.25 - 281.25$

$0.25x < 225$

$$x < \frac{225}{0.25}$$

$x < 900 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$

The second part of inequality

$$\frac{506.25}{1125+x} < 0.30$$

$506.25 < 0.30(1125 + x)$

$506.25 < 337.5 + 0.30x$

$506.25 - 337.5 < 0.30x$

$168.75 < 0.30x$

$$\frac{168.75}{0.30} < x$$

$562.5 < x$ ………………(2)

From (1) and (2), $562.5 < x < 900$

The range of water to be added is (562.5, 900)

MULTIPLE CHOICE QUESTIONS AND ANSWERS

1.Sol.: Let x be the breadth of a rectangle given that length = 4x

Given minimum perimeter of a rectangle is 160 cms

$\therefore 2\,(l + b) \geq 160$

$2\,(4x + x) \geq 160$

$5x \geq 80$

$x \geq 16$

$\therefore$ breadth $\geq$ 16 cms

2.Sol: (1) $x \in (9,\infty)$
Given $-3x + 15 < -12$

$-3x < -12 - 15$

$-3x < -27$

$x > 9$

$\therefore \quad x \in (9,\infty)$

3. Sol : Since it is given that x, y and b are real numbers where x > y and b < 0

$x < y$
$\frac{x}{b} < \frac{y}{b}$ (divide both sides with ‘b’) as b < 0

4.Sol: (3) $x \in (-\infty, -4)$ u $(8, \infty)$

Solution : $|x-2| > 6 \Rightarrow x - 2 < -6$ and $x - 2 > 6 \Rightarrow x < -4$ and $x > 8$

$\therefore x\ \Sigma\ (-\infty, -4) \cup (8, \infty)$

5.Sol : Given $\frac{|x-8|}{x-8} \geq 0$

This is possible if $x - 8 \geq 0$ and $x - 8 \neq 0$

$x \geq 8$ but $x \neq 8$

$x > 8$

$\therefore x \in (8, \infty)$

6. Sol: Given that $|x + 5| \geq 10$

$\Rightarrow$ x + 5 ≤ -10 or x + 5 ≥ 10
$\Rightarrow$ x ≤ -15 or x ≥ 5
x ∈ (-∞ , -15) U (5, ∞)

7.Sol : Given that 4x +4 < 6x + 5

4x -6x < 5-4 $\Rightarrow$ -2x < 1 $\Rightarrow$ $x > -\frac{1}{2}$

$\therefore x \in (-\frac{1}{2}, \infty)$

8.Sol: Given $2x - 17 > 3 - 8x$

$2x + 8x > 3 + 17 \Rightarrow 10x > 20 \Rightarrow x > 2$

$\therefore$ The result is $x > 2$

9.Sol : Let x be the marks obtained in third unit test.

$\Rightarrow$ It is given that the student should have an average of at least 90 marks. We can write linear inequalities as $\frac{90+90\,x}{3} \geq 90$

$180 + x \geq 270$
$x \geq 270 - 180$
$x \geq 90$

10. **Sol :** Given 5x +2 > 12

5x >10 $\Rightarrow$ x > 2
Hence solution is (2, ∞).

11. **Sol :** Given $5x - 2 \leq 3x + 2$

$5x - 3x \leq 2 + 2 \Rightarrow 2x \leq 4$
$x \leq 2$

Solution is {…………-2,-1,0,1,2}

12.Sol: Given that $50x < 150$

$x < 3$
$\therefore$ Solution set is { 1, 2}

13.Sol : Given that $9x > -36$

$\therefore x > -4$
$\therefore$ Least integral value = -3

MULTIPLE CHOIOCE QUESTIONS (KEY)

1) 3	2) 1	3) 3	4) 3	5) 4	6) 3	7) 2	8) 4	9) 1	10) 3
11) 3	12) 2	13) 1							

6. PERMUTATIONS AND COMBINATIONS

Exercise 6(a)

I.1. How many 3 digit numbers can be formed from the digits 1,2,3,4 and 5 assuming that

- **i. Repetition of the digits is allowed?**
- **ii. Repetition of the digits is not allowed?**

Sol:i. When repetition of the digits is allowed,

T H U

Since, there are 5 digits. Therefore, the number of ways to fill each place is 5.

Hence, total number of ways = $5 \times 5 \times 5 = 125$.

ii.When repetition is not allowed,

T H U

Since, there are 5 digits i.e., 1,2,3,4 & 5.

Therefore, the number of ways to fill the unit place is 5.

Number of ways to fill the ten's place = 4

And the number of ways to fill the hundred's place = 3.

Hence, total number of ways = $5 \times 4 \times 3 = 60$.

2. How many 3 digits even numbers can be formed from the digits 1,2,3,4,5,6 if the digits can be repeated?

Sol: For even number we can fill the units place out of the given digits with 2 or 4 or 6.

T H U

The number of ways to fill the unit's place = 3. Here repetition is allowed.
Therefore, the number of ways to fill the ten's place = 6 and the number of ways to fill the hundred's place = 6

Hence, total number of ways = $3 \times 6 \times 6 = 108$

3. How many 4 letters code can be formed using the first 10 letters of the English alphabet, if no letter can be repeated?

Sol:

I II III IV

Number of ways to fill the I Place = 10,

Number of ways to fill the II Place = 9,

Number of ways to fill the III place = 8,

Number of ways to fill the IV place = 7

Hence, total number of ways = 10 × 9 × 8 × 7 = 5040

4 How many 5 digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once?

Sol:

☐	☐	☐	☐	☐
I	II	III	IV	V

Number of ways to fill the III place = 8

Number of ways to fill the IV place = 7

Number of ways to fill the V place = 6

Hence, total number of ways = 8 × 7 × 6 = 336

5. A coin is tossed 3 times and the outcomes are recorded. How many possivle outcomes are there?

Sol: In tossing a coin, there are 2 possible outcomes (Head & Tail).

In tossing a coin second time there are two possible outcomes and in third time there again two possible outcomes.

Hence, total number of outcomes = 2 × 2 × 2 = 8.

6. Given 5 flags of different colors, how many different signals can be generated, if each signal requires the use of 2 flags, one below the other.

Sol: Number of ways to choose the first flag = 5

Number of ways to choose the second flag from the rest of four flags = 4

Hence, the total number of ways = 5 × 4 = 20

Exercise 6(b)

I.

1. Evaluate
i. 8! ii. 4! – 3!

Sol:

i. $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$

ii. $4! - 3! = 4 \times 3 \times 2 \times 1 - 3 \times 2 \times 1 = 24-6 = 18$.

2. Is 3! + 4! = 7!

Sol: $3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$LHS = 3! + 4! = 6 + 24 = 30$

$RHS = 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

$\therefore 3! + 4! \neq 7!$

3. Compute $\frac{8!}{6!\times 2!}$

Sol: $\frac{8!}{6!\times 2!} = \frac{8\times 7\times 6\times 5\times 4\times 3\times 2\times 1}{6\times 5\times 4\times 3\times 2\times 1\times 1\times 2}$

$= \frac{8\times 7}{2\times 1}$ **= 28**

4. If $\frac{1}{6!}+\frac{1}{7!}=\frac{x}{8!}$**, find** x

Sol: Let $\frac{1}{6!}+\frac{1}{7!}=\frac{x}{8!}$

$\Rightarrow \frac{7+1}{7!}=\frac{x}{8X7!} \Rightarrow 8=\frac{x}{8} \therefore x$ **= 64**

5. Evaluate $\frac{n!}{(n-r)!}$ **when i. n = 6, r = 2, ii. n = 9 , r = 5.**

Sol: i. When n = 6, r = 2, then

$$\frac{n!}{(n-r)!}=\frac{6!}{(6-2)!}=\frac{6!}{4!}=\frac{6\times 5\times 4\times 3\times 2\times 1}{4\times 3\times 2\times 1}=6\times 5=30$$

ii. When n = 9, r = 5, then

$$\frac{n!}{(n-r)!}=\frac{9!}{(9-5)!}=\frac{9!}{4!}=\frac{9\times 8\times 7\times 6\times 5\times 4\times 3\times 2\times 1}{4\times 3\times 2\times 1}=9\times 8\times 7\times 6\times 5=15120$$

Exercise 6(c)

Section - I

1. If n_{P_3} **= 1320, find n**

Sol: Let $n_{P_3} = 1320$

$$\Rightarrow \frac{n!}{(n-3)!} = 1320 \Rightarrow \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 1320$$

$$\Rightarrow n(n-1)(n-2) = 1320 \Rightarrow n(n-1)(n-2) = 12\times 11\times 10$$

$$\therefore n = 12$$

2. If n_{P_7} = 42 n_{P_5}, find n

Sol: Let n_{P_7} **= 42** n_{P_5}

$$\Rightarrow \frac{n!}{(n-7)!} = 42\frac{n!}{(n-5)!} \Rightarrow \frac{n!}{(n-7)!} = 42\frac{n!}{(n-5)(n-6)(n-7)!}$$

$$\Rightarrow (n-5)(n-6) = 42 \Rightarrow (n-5)(n-6) = 7\times 6 \Rightarrow n-5 = 7 \Rightarrow n = 7+5 \Rightarrow n = 12$$

3. $If\,(n+1)_{P_5} : n_{P_6} = 2:7, find$ **"n"**

Sol: $Let (n+1)_{P_5} : n_{P_6} = 2:7$

$$\Rightarrow \frac{(n+1)_{P_5}}{n_{P_6}} = \frac{2}{7} \qquad \Rightarrow \frac{\frac{(n+1)!}{(n-4)!}}{\frac{n!}{(n-6)!}} = \frac{2}{7} \qquad \Rightarrow \frac{\frac{(n+1)n!}{(n-4)(n-5)(n-6)!}}{\frac{n!}{(n-6)!}} = \frac{2}{7}$$

$$\Rightarrow \frac{(n+1)}{(n-4)(n-5)} = \frac{2}{7} \qquad \Rightarrow \frac{(n+1)}{(n-4)(n-5)} = \frac{2\times 6}{7\times 6}$$ ($\because$ multiply by 6 to both Numerator & denominator on RHS)

$$\Rightarrow n+1 = 2\times 6 \text{ or } n-4 = 7 \text{ or } n-5 = 6$$

$$\Rightarrow n+1 = 12 \quad \text{or } n = 7+4 \quad \text{or } \ n = 6+5$$

$$\Rightarrow n = 12-1 \quad \therefore n = 11$$

4. If $12_{P_5} + 5\times 12_{P_4} = 13_{P_r}$, find r

Sol: Let $12_{P_5} + 5\times 12_{P_4} = 13_{P_r}$ $\quad\Rightarrow\quad (n-1)_{P_r} + r.(n-1)_{P_{r-1}} = n_{P_r}$

Hence, r = 5.

5. If $18_{P_{r-1}} : 17_{P_{r-1}} = 9:7$ **, Find r ?**

Sol: Let $18_{P_{r-1}} : 17_{P_{r-1}} = 9:7$

$$\Rightarrow \frac{18_{P_{r-1}}}{17_{P_{r-1}}} = \frac{9}{7} \quad \Rightarrow \frac{18!}{(18-(r-1))!}\times\frac{(17-(r-1)!}{17!} = \frac{9}{7}$$

$$\Rightarrow \frac{18\times 17!}{(19-r)!}\times\frac{(18-r)!}{17!} = \frac{9}{7} \quad \Rightarrow \frac{18(18-r)!}{(19-r)!} = \frac{9}{7} \quad \Rightarrow \frac{18(18-r)!}{(19-r)(18-r)!} = \frac{9}{7}$$

$\Rightarrow \dfrac{18}{19-r}=\dfrac{9}{7}$ $\quad\Rightarrow 19-r=14 \;\Rightarrow r=19-4$

$\therefore r=5$

6. Find n, if $(n-1)_{P_3}:n_{P_4}=1:9$

Sol: Let $(n-1)_{P_3}:n_{P_4}=1:9$

$$\Rightarrow \frac{(n-1)_{P_3}}{n_{P_4}}=\frac{1}{9} \qquad \Rightarrow \frac{(n-1)!}{(n-4)!}\times\frac{(n-4)!}{n!}=\frac{1}{9} \qquad \Rightarrow \frac{(n-1)!}{n!}=\frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{n(n-1)!}=\frac{1}{9} \qquad \therefore n=9$$

7. Find r, if (i) $5_{P_r}=2\times 6_{P_r}$ **(ii)** $5_{P_r}=6_{P_{r-1}}$

Sol: (i) Let $5_{P_r}=2\times 6_{P_r} \Rightarrow \dfrac{5!}{(5-r)!}=2\dfrac{6!}{(6-(r-1))!} \Rightarrow \dfrac{5!}{(5-r)!}=\dfrac{2\times6\times5}{(7-r)!}$

$$\Rightarrow \frac{1}{(5-r)!}=\frac{12}{(7-r)(6-r)(5-r)!} \Rightarrow (7-r)(6-r)=12 \Rightarrow 7-r=4 \textbf{ or } 6-r=3$$

$\Rightarrow r=7-4$ or $6-3=r$ $\quad\Rightarrow r=3$ or $r=3$

Hence, **r=3**

(ii) Let $5_{P_r}=6_{P_{r-1}}$

$$\Rightarrow \frac{5!}{(5-r)!}=\frac{6!}{(6-(r-1))!} \Rightarrow \frac{5!}{(5-r)!}=\frac{6\times5!}{(7-r)(6-r)(5-r)!}$$

$\Rightarrow (7-r)(6-r)=6 \Rightarrow (7-r)(6-r)=3\times2$

$\Rightarrow (7-r)=3$ or $(6-r)=2$ $\quad\Rightarrow r=7-3$ or $r=6-2$

$\Rightarrow r=4$ or $r=4$ $\qquad\therefore$ r = 4

8. A man has 4 sons and there are 5 schools within his reach. In how many ways can he admit his sons in the schools so that no two of them will be in the same school.

Sol: The number of ways of admitting 4 sons into 5 schools, if no two of them will be in the same school = 5_{P_4} = 5 ×4 × 3 × 2 × 1 = 120.

9. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

Sol: If no digit is repeated the number of 3 digit numbers can be formed by using the digits 1 to 9 = 9_{P_3} = 9 × 8× 7 = 540 ways.

10. How many 4-digit numbers are there with no digit repeated?

Sol: Let the digits be 0 to 9 and the 4 digit numbers are 10_{P_4}

The number which have 0 in the beginning i.e., 3 – digit numbers out of 9 digits are 9_{P_3}

$\therefore$ 4 – digit numbers which do not have zero in the beginning = $10_{P_4} - 9_{P_3}$ = 4536.

11. How many 3-digit even numbers that can be made using the digits 1,2,3,4,6,7 if no digit is repeated?

Sol: Let 2 be fixed at unit place. Now we have 5 digits and 2 places are to be filled.

$\Rightarrow$ This can be done in 5_{P_2} numbers. When unit place filled up both 4 or 6 again in each case, we have 5_{P_2} numbers.

$\therefore$ 3- digit even numbers are $3 \times 5_{P_2} = 3 \times 5 \times 4 = 60$.

12. Find the number of 4-digit number that can be formed using the digits 1,2,3,4,5 if no digit is repeated. How many of them will be even?

Sol: i. the number of 4-digit numbers can be formed from given 5-digit = 5_{P_4}

$\Rightarrow$ $5_{P_4} = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

ii. When 2 is at units place, then remaining 3 place can be filled in 4_{P_3} ways = 24 ways

When 4 is at units place, then 4-digit numbers are again = 24.

$\therefore$Numbers of even 4 – digit numbers = 24+24 = 48.

13. From a committee 8 persons, in how many ways we can choose a chairman and a vice-chairman assuming one person can not hold more than one position?

Sol: Out of 8 persons chairman can choose in 8 ways. After selecting the chairman, we have to choose a vice-chairman out of 7 persons in 7 ways.

$\therefore$Out of 8 persons, a chairman and a vice-chairman can be chosen in $8 \times 7 = 56$ ways.

14. Find the numbers of 4-digit numbers that can be formed using the digits 1,2,3,4,5,7,8 when repetition is allowed.

Sol: When repetition allowed the number of 4 – digit numbers = $6^4 = 1296$.

15. Find the number of 5 letter words that can be formed using the letters of the word RHYME if each letter can be used any numbers of times.

Sol: When repetition allowed the number of 5 – digit words = $5^5 = 3125$.

16. Find the number of functions from a set A containing 5 elements into a set B containing 4 elements

Sol: Number of functions = $4^5 = 1025$.

17.Find the number of ways arranging the letters of the words.

i. MATHEMATICS **ii. SINGING**

iii. **PERMUTATION** iv. **COMBINATION**

v. **INTERMEDIATE**

Sol: **i**. In the given word MATHEMATICS, there are 2A, 2M and 2T.

Total number of letters = 11.

$\therefore$Number of ways of arrangements $= \dfrac{11!}{2!2!2!}$

ii. $\dfrac{7!}{2!2!2!}$ **iii.** $\dfrac{(11)!}{2!}$ **iv.** $\dfrac{(11)!}{2!2!2!}$ **v.** $\dfrac{(12)!}{3!2!2!}$

18. In how many of distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Sol: In the given word MISSISSIPPI, there are 4 I, 4S, 2P & 1M. Total letters are 11.

Total number of permutations with no restriction = $\dfrac{(11)!}{4!4!2!}$

If we take 4 I's as one letter then total letters become 11 – 4+1 = 8

$\therefore$ The permutations when 4Is are not together = $\dfrac{(11)!}{4!4!2!} - \dfrac{8!}{4!2!}$

= 34650 – 840 = **33810**

19. If there are 25 railway stations on a railway line, how many types of single second-class tickets must be printed so as to enable a passenger to travel from one station to another.

Sol: Number of railway stations on railway line = 25.

$\Rightarrow$ The number of second class tickets must be printed so as to enable a passenger to travel one station to another = $25_{P_2} = 25 \times 24 =$ **600.**

20. Find the number of ways of arranging the letters of the work TRIANGLE so that the relative positions of the vowels and consonants are not disturbed.

Sol: In the word TRIANGLE, number of vowels = 3 and number of consonants = 5

3 vowels can be arranged in their relative position in 3! Ways.

5 consonants can be arranged in their relative position in 5! Ways.

$\therefore$ The number of required arrangements = $3! \times 5! = 6 \times 120 = 710$.

II

1. **How many words with or without meaning can be made from the letters of the word "MONDAY" assuming that no letter is repeated if**
   a. **4 letters are used at a time**
   b. **All letters are used at a time**
   c. **All letters are used but first letter is a vowel,**

Sol: In the word 'MONDAY' all letters are different.

a. Out of 6 different letters 4 letters can be selected in 6_{P_4} ways.

$\therefore$ Required number of words = $\therefore = \dfrac{6!}{(6-4)!} = \dfrac{6!}{2!} = 360.$

b. The word 'MONDAY' has 6 letters.

Number of ways taking 6 letters at a time = $6_{P_6} = \dfrac{6!}{(6-6)!} = \dfrac{6!}{0!} = 720.$

c. First, we fix the vowel.In the word 'MONDAY', they two vowels A and O i.e., first letter can be chosen by 2 ways.

Number of ways taking 5 different letters from remaining 5 letters = 5_{P_5}

$\therefore$ Required number of words = 5_{P_5} = 120.

Hence, total number of ways = 2 × 120 = 240

2. In how many ways can the letters of the word 'PERMUTATIONS' can be arranged if

a. Words start with P and ends with S.
b. Vowels are all together.
c. There are always 4 letters between P & S.

Sol: a. Letters between P & S are 'ERMUTATION'. These 10 letters having 2T's.

These letters can be arranged in $\dfrac{10!}{2!}$ = 1814400 ways.

b. There are 12 letters in the word 'PERMUTATIONS' which has 2Ts and 5 vowels.

Consider these 5 vowels as one unit and these 5 vowels can be arranged in 5! Ways.

Thus, there are 7 letters and 1 unit of 5 vowels with 2Ts.

$\therefore$ Required number of arrangements = $\dfrac{8!}{2!} \times 5! = 24,19,200$

c. There are 12 letters to be arranged in 12 places

1	2	3	4	5	6	7	8	9	10	11	12

P may be filled up at place no. 1,2,3,4,5,6,7 and consequently S may be filled up at place no. 6,7,8,9,10,11,12 leaving four places in between.

Now P & S may be filled in 7 ways. Similarly S & P may be filled in 7 ways.Remaining 10 letters having 2T's.

$\therefore$ There can be arranged in $\dfrac{10!}{2!}$ ways.

Hence, word 'PERMUTATIONS' can be arranged when 4 letters between P & S = $2 \times 7 \times \frac{10!}{2!}$ = 14×1814400 = **25401600.**

3. Find the sum of all 4-digited number that can be formed using the digits 0,2,4,7,8 without repetition

Sol: Given digits are 0,2,4,7,8

If '0' is one digit among the given 'n' digits, then we get the sum of the r-digit numbers that can formed using the given 'n' digits ($0 \le n \le 9$) is

$\{^{(n-1)}p_{(r-1)} \times$ Sum of the given digits $\times$ 111 …… r times} - $\{^{(n-2)}p_{(r-2)} \times$ Sum of the given digits 111 …… (r-1) times}

Here n = 5, r = 4

$\therefore$ Sum of 4 – digit numbers = $\{^{(5-1)}p_{(4-1)} \times (0+2+4+7+8) \times 1111\}$- $\{^{(5-2)}p_{(4-2)} \times (0+2+4+7+8) \times 111\}$ = $(4p_3 \times 21 \times 1111) - (3p_2 \times 21 \times 111)$

= $24 \times 21 \times 1111 - 6 \times 21 \times 111$

= 21 (26664) – 21 (666) = 21 (26664 – 666) = 21 (25998) = **5,45,958.**

4. **Find the number of numbers that are greater than 4000 which can be formed using the digits 0,2,4,6,8 without repetition**.

Sol: Given digits are 0,2,4,6,8.

Zero can not be filled in the first place while forming any digit number.

We can fill the first place with the remaining 4 digits.

All 5-digit numbers are greater than 400.

Number of 5-digit numbers = $4 \times 4!$ = 96.

In the 4-digit numbers the number starting with 4 or 6 or 8 are greater than 4000.

The number of 4-digit numbers which start 4 or 6 or 8 = $3 \times 4p_3 = 3 \times 24 = 72$.

$\therefore$ The number of numbers greater than 4000 = 96 +72 = 168.

III

1. Find the number of ways of arranging 10 students $A_1, A_2, \ldots A_{10}$ in a row such that

a. A_1, A_2, A_3 sit together

b. A_1, A_2, A_3 sit in a specified order,

c. A_1, A_2, A_3 sit together in a specified order.

Sol: $A_1, A_2, \ldots\ldots A_{10}$ are 10 students

a. Let us consider A_1, A_2, A_3 be one unit.
These 3 students can arrange in one unit = 3! Ways.
Then total number of units = 7+1 = 8
These 8 units can be arranged in 8! Ways

$\therefore$ Total number of arrangements = $3! \times 8!$

Hence, the number of ways of arrangements that A1, A2, A3 can sit together = $3! \times 8!$

b. Let A_1, A_2, A_3 sit in a specified order.

A_1, A_2, A_3 can arrange in 10 positions in specified order in $\frac{10_{P_3}}{3!} = \frac{10!}{7! \times 3!}$ ways.

$A_4, A_5, A_6, A_7, A_8, A_9, A_{10}$ these 7 students can arrange remaining 7 places in 7! Ways.

$\therefore$ Number of ways of A1, A2, A3 sit in a specified order = $\frac{10!}{7! \times 3!} \times 7! = \frac{10!}{3!} = 10_{P_3}$

c. Let A_1, A_2, A_3 consider as one unit. Then total number of units = 7+1 = 8

These 8 units can be arranged in 8! Ways.

$\therefore A_1, A_2, A_3$ sit together in a specified order = 8! Ways.

2. Find the sum of 4-digit numbers that can be formed using the digits 1,2,4,5,6 without repetition.

Sol: The sum of all r-digit numbers that can be formed using given 'n' non-zero digits

$(1 \le r \le 9)$ is ${}^{(n-1)}p_{(r-1)} \times$ sum of given digits $\times$ 111….r times.

Given digits are 1,2,4,5,6 i.e., n=5 & r = 4

$\therefore$ The sum of 4-digit number = ${}^{(5-1)}p_{(4-1)} \times (1+2+4+5+6) \times 1111$

$= {}^{4}p_{3} \times 18 \times 1111 = 4,79,952.$

3. 9 different letters of an alphabet are given find the sum of 4 letter words that can be formed using these 9 letters which have

a. No letter is repeated

b. At least one letter is repeated

Sol: The number of 4 letter words that can be formed from 9 different letters of an alphabet when repetition is allowed = $9^4 = 6561$

i. The number of 4 letter words that can be formed 9 different letters of an alphabet when no letter is repeated = $9_{P_4} = 9 \times 8 \times 7 \times 6 = 3024$.

ii. The number of 4 letter words that can be formed from 9 different letter of an alphabet with at least on letter is repeated = $9^4 - 9_{P_4} = 6561 - 3024 = 3537$

4. Find the number of 4-digit numbers that can be formed using the digits 0,1,2,3,4,5 which are divisible by 6 when repetition of the digits is allowed.

Sol:

5 ways	6 ways	6 ways	6 ways

Given digits 0,1,2,3,4,5 these are consecutive digits.

Total number of 4-digit numbers = 5×6^3

Among these total numbers some of the numbers are divisible by 6.

By fixing first places with some fixed digits from 1,2,3,4,5 and 2nd, 3rd places with some fixed digits 0,1,2,3,4,5 we get the following consecutive numbers:

$a_1,a_2,a_3,0$

$a_1,a_2,a_3,1$

$a_1,a_2,a_3,2$

$a_1,a_2,a_3,3$

$a_1,a_2,a_3,4$

$a_1,a_2,a_3,5$

We know that out of any 6 consecutive integers exactly 1 is divisible by 6.

$\therefore$ 1/6th of these 5×6^3 numbers are divisible by 6.

$\therefore$Required number = $\frac{1}{6}\times 5\times 6^3 = 5 \times 36 =$ **180.**

5. Find the number of ways of arranging the letters of the word 'ASSOCIATIONS'. In how many of them

a. All the three S's come together,

b. The two A's do not come together.

Sol: The word "ASSOCIATIONS' has 12 letters in which 2 A's, 3S's, 2O's and 2I's and rest are different.

$\therefore$They can arrange in $\frac{(12)!}{2!3!2!2!}$ ways.

i. Let consider 3S's as one unit.

Then the number of units = 9+1=10. Here 2 A's , 2O's and 2I's.

$\therefore$ They can arrange in $\frac{(10)!}{2!2!2!}$ ways and 3S's can arrange in $\frac{3!}{3!} = 1$ way

$\therefore$ The number of arrangements that all 3S's come together = $\frac{(10)!}{2!2!2!} \times 1$ ways

$= \frac{(10)!}{2!2!2!}$ ways.

ii. 2A's do not come together.

First arranging the remaining 10 letters in which there are 3 S's, 2O's and 2I's are alike and rest are different.

$\therefore$ These can arranged = $\frac{(10)!}{3!2!2!}$ ways

$\Rightarrow$ There are 11 gaps between them.

$\Rightarrow$ The 2A's can be arranged in these 11 gaps in $\frac{11_{P_2}}{2!}$ ways.

$\therefore$ Number of arrangements = $\frac{(10)!}{3!2!2!} \times \frac{11_{P_2}}{2!}$

Exercise 6(d)

I)

1. If ${}^nc_8 = {}^nc_2$, find ${}^nc_2$

Sol: Since ${}^nc_r = {}^nc_{n-r}$, Let ${}^nc_8 = {}^nc_2$

Now ${}^nc_2 = {}^nc_{n-2} \Rightarrow {}^nc_8 = {}^nc_{n-2} \Rightarrow$ 8 = n – 2 $\Rightarrow$ n = 8 + 2 = 10.

Now, ${}^nc_2 = {}^{10}c_2 = \frac{10!}{2!8!} = \frac{9\times10}{1\times2} = 45$

2. Determine n, if

a. ${}^{2n}c_3 : {}^nc_3 = 12:1$

b. ${}^{2n}c_3 : {}^nc_3 = 11:1$

Sol: a. Given ${}^{2n}c_3 : {}^nc_3 = 12:1$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = \frac{12}{1}$$

$$\Rightarrow \frac{2.2.n(n-1)(2n-1)}{n(n-1)(n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{(2n-1)}{(n-2)} = \frac{3}{1} \Rightarrow 2n-1 = 3n-6 \Rightarrow -1+6 = 3n-2n \Rightarrow \quad n = 5$$

b. Given ${}^{2n}c_3 : {}^nc_3 = 11:1$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = \frac{11}{1}$$

$$\Rightarrow \frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{11}{1} \Rightarrow \frac{4(2n-1)}{n-2} = \frac{11}{1} \Rightarrow 8n-4 = 11n-22$$

$$\Rightarrow -4+22 = 11n-8n \Rightarrow 18 = 3n \quad \Rightarrow n = \frac{18}{3} = 6.$$

3. How many chords can be drawn through 21 points on a circle

Sol: We get a chord by gaining two points, if P is the number of chords from 21 points,

then $\Rightarrow$ P = ${}^{21}c_2 = \frac{(21)!}{2!(21-2)!} = \frac{(21)!}{2!(19)!} = \frac{21\times20\times(19)!}{2\times1\times(19)!}$ = 210 chords.

4. In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Sol: 3 boys out of 5 boys can be selected in ${}^5c_3$ ways.

3 girls out of 4 girls can be selected in ${}^4c_3$ ways.

$\therefore$No. of ways in which 3 boys and 3 girls are selected = ${}^5c_3 \times {}^4c_3$ ways

= 10 ×4 = 40 ways.

5. If ${}^nc_4$ = 210, find n.

Sol: Let ${}^nc_4 = 210$

$$\Rightarrow \frac{n!}{4!(n-4)!} = 210 \Rightarrow n(n-1)(n-2)(n-3) = 210 \times 24$$

$$\Rightarrow n(n-1)(n-2)(n-3) = 10 \times 9 \times 8 \times 7$$

$\therefore$n = 10

6. If ${}^{12}c_r$ = 495, find the possible values of r

Sol: Let ${}^{12}c_r = 495 = 11 \times 9 \times 5$

$$\Rightarrow {}^{12}c_r = \frac{12 \times 11 \times 9 \times 5 \times 2}{12 \times 2} = \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} = \frac{(12)!}{8!4!} = {}^{12}c_8 \text{ or } {}^{12}c_4$$

$\therefore$r = 4 or 8

7. If 10 ${}^nc_2$ = 3 ${}^{(n+1)}c_3$, find *n*

Sol: Let $10\,{}^nc_2 = 3\ {}^{(n+1)}c_3$

$$\Rightarrow 10 \times \frac{n!}{2!(n-2)!} = 3 \times \frac{(n+1)!}{3!(n+1-3)!}$$

$\Rightarrow$ 10 × n(n-1) = (n+1)n(n-1) $\Rightarrow$ 10 = n+1 $\Rightarrow$ n = 10-1 $\Rightarrow$ n=9

8. If ${}^{15}c_{2r-1} = {}^{15}c_{2r+4}$, find r

Sol: Since, if ${}^nc_r = {}^nc_s \Rightarrow$ n = r + s **or r = s**

Let ${}^{15}c_{2r-1} = {}^{15}c_{2r+4} \Rightarrow$ (2r – 1) + (2r+4) = 15

$\Rightarrow$ 4r +3 = 15 $\Rightarrow$ 4r = 15 – 3 $\Rightarrow$ 4r = 12 $\Rightarrow$ r = $\frac{12}{4}$ = **3**

9. If ${}^{12}c_{r+1} = {}^{12}c_{3r-5}$, find r

Sol: Let ${}^{12}c_{r+1} = {}^{12}c_{3r-5}$

$\Rightarrow$ r + 1 = 3r – 5 $\Rightarrow$ r – 3r = -5 – 1 $\Rightarrow$ -2r = -6 $\Rightarrow$ **r = 3**

10. If $^{n}c_5 = {}^{n}c_6$, then find $^{13}c_n$

Sol: Let $^{n}c_5 = {}^{n}c_6$

$\Rightarrow$ n = 5 + 6 = 11

Now $^{13}c_n = {}^{13}c_{11} = \dfrac{13!}{2!(11)!} = \dfrac{13X12}{2X1} =$ **78.**

II)

1. **Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls. If each selection consists of 3 balls of each colour**

Sol: The no. of ways of selecting 3 red balls out of 6 red balls $= {}^{6}C_3$.

The no. of ways of selecting 3 white balls out of 5 white balls $= {}^{5}C_3$

The no. of ways of selecting 3 blue balls out of 5 blue balls $= {}^{5}C_3$

$\therefore$ The no. of ways selecting 3 balls of each colour $= {}^{6}C_3 \times {}^{5}C_3 \times {}^{5}C_3$

$= 20 \times 10 \times 10 =$ **2000.**

2. **Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.**

Sol: One ace will be selected from four aces and four cards will be selected from 52-4 = 48 cards. If P is the required no. of ways, then

$\Rightarrow$ P $= {}^{4}C_1 \times {}^{48}C_4 = 4 \times \dfrac{48!}{4!(48-4)!} = \dfrac{48!}{4!(44)!} = 4 \times \dfrac{48\times 47\times 46\times 45\times (44)!}{4\times 3\times 2\times 1\times (44)!}$

$\therefore$ P = 778320 ways.

3. **In how many ways can select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 5 bowlers?**

Sol: Total players = 17, Bowlers = 5 & Non bowlers = 12

Team of 11 players is to be selected having exactly 4 bowlers.

$\therefore$ No. of ways $= {}^{5}C_4 \times {}^{12}C_7$

$= 5 \times 792$

= 3960.

4. **A bag contain 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.**

Sol: The no. of ways in which 2 black balls out of 5 black balls are selected = $^5C_2$

The no. of ways in which 3 red balls out of 6 red balls are selected = $^6C_3$

$\therefore$ Total no. of ways in which 2 black balls and 3 red balls can be selected is

$\Rightarrow$ $^5C_2 \times {}^6C_3 = 10 \times 20 =$ **200.**

5. In how many ways can a student choose a program of 5 courses, if 9 courses are available and 2 specific courses are compulsory for every student ?

Sol: Total no. of available courses = 9

Out of these, 5 courses have to be chosen. But it is given that 2 courses are compulsory for every student i.e., you have to choose only 3 courses instead of 5, out of 7 instead of 9.

$\therefore$ It can be done in $^7C_3$ ways $= \dfrac{7!}{3!4!} = \dfrac{7\times6\times5}{6} = 35$ ways.

6. Prove that for $3 \le r \le n$, $^{(n-3)}C_r + 3\,{}^{(n-3)}C_{(r-1)} + 3\,{}^{(n-3)}C_{(r-2)} + {}^{(n-3)}C_{(r-3)} = {}^nC_r$

Sol: Since $^nC_{(r-1)} + {}^nC_r = {}^{(n+1)}C_r$, Given can be taken as

$$^{(n-3)}C_r + {}^{(n-3)}C_{(r-1)} + 2\left[{}^{(n-3)}C_{(r-1)} + {}^{(n-3)}C_{(r-2)}\right] + {}^{(n-3)}C_{(r-2)} + {}^{(n-3)}C_{(r-3)}$$

$$= {}^{(n-2)}C_r + {}^{2(n-2)}C_{(r-1)} + {}^{(n-2)}C_{(r-2)}$$

$$= {}^{(n-2)}C_r + {}^{(n-2)}C_{(r-1)} + {}^{(n-2)}C_{(r-1)} + {}^{(n-2)}C_{(r-2)}$$

$$= {}^{(n-1)}C_r + {}^{(n-1)}C_{(r-1)}$$

$$= {}^{(n-1+1)}C_r = {}^nC_r$$

7. Simplify $^{34}C_5 + \sum_{r=0}^{4} {}^{38-r}C_4$

Sol: Since $^nC_{(r-1)} + {}^nC_r = {}^{(n+1)}C_r$

Now $^{34}C_5 + \sum_{r=0}^{4} {}^{(38-r)}C_4 = {}^{34}C_5 + {}^{34}C_4 + {}^{35}C_4 + {}^{36}C_4 + {}^{37}C_4 + {}^{38}C_4$

$$= {}^{35}C_5 + {}^{35}C_4 + {}^{36}C_4 + {}^{37}C_4 + {}^{38}C_4$$

$$= {}^{36}C_5 + {}^{36}C_4 + {}^{37}C_4 + {}^{38}C_4$$

$$= {}^{37}C_5 + {}^{37}C_4 + {}^{38}C_4$$

$$= {}^{38}C_5 + {}^{38}C_4 = {}^{39}C_5$$

8. In a class there are 30 students. If each student plays chess game with each of the other student, then find the total number of chess games played by them.

Sol: Total no. of students = 30

Each student plays chess game, then the total no. of chess games played by them = ${}^{30}C_2$ = **435.**

9. Find the number of ways of selecting 3 vowels and 2 consonants from the letters of the word “EQUATION”

Sol: No. of Vowels in the word “EQUATION” is 5 (i.e., A,E,I,O,U) and no. of consonants

is 3 (i.e., Q,T,N)

$\therefore$ 3 vowels can be selected from 5 vowels in ${}^{5}C_3$ = 10 ways

2 consonants can be selected from 3 consonants in ${}^{3}C_2$ = 3 ways

Hence, the required no. of ways of selecting vowels and 2 consonants

= 10 $\times$3 = 30ways

10. Find the number of diagonals of a polygon with 12 sides.

Sol: The no. of diagonals of a polygon with ‘n’ sides = $\dfrac{n(n-3)}{2}$

Put n = 12

$\therefore$ No. of diagonals of a polygon with 12 slides = $\dfrac{12(12-3)}{2}$ = **54**

11. If n persons are sitting in a row, find the number of ways of selecting two persons, who are sitting adjacent to each other.

Sol: $\Rightarrow$ The no. of ways of selecting 2 persons out of ‘n’ persons sitting in a row, who are

sitting adjacent to each other = (n-1)

$$\left(\therefore \overbrace{a_1, a_2}^{1} \underset{2}{\,} \overbrace{a_3, a_4}^{3}, \ldots \overbrace{a_{n-1}, a_{n-2}}^{n-1} \right) \ldots\ldots\ldots a_{n-1}, a_n$$

III

1. Prove that $\dfrac{{}^{4n}c_{2n}}{{}^{2n}c_n} = \dfrac{1.3.5\ldots\ldots\ldots(4n-1)}{\{1.3.5\ldots\ldots\ldots(2n-1\}^2}$

Sol: Since ${}^{n}c_r = \dfrac{n!}{r!(n-r)!}$

$$\Rightarrow \frac{{}^{4n}c_{2n}}{{}^{2n}c_n} = \frac{\frac{(4n)!}{(2n)!(2n)!}}{\frac{2n!}{n!n!}} = \frac{(4n)!}{(2n)!(2n)!} \times \frac{n!n!}{(2n)!}$$

$$= \frac{4n(4n-1)(4n-2)......3.21}{\{2n(2n-1)(2n-2)......3.21\}} \times \frac{n!n!}{(2n)!}$$

$$= \left\{ \frac{(4n(4n-2)....2)(4n-1)(4n-3)....3.1}{\{2n(2n-2)....4.2\}^2 \{(2n-1)(2n-3)....3.1\}^2} \right\} \times \frac{n!n!}{(2n)!}$$

$$= \frac{\{2.^n 2n(2n-1)(2n-2)....2.1\}\{(4n-1)(4n-3)....3.1\}}{\{2^n.n(n-1)(n-2)....2.1\}^2 \{(2n-1)(2n-3).....3.1\}^2} \times \frac{n!n!}{(2n)!}$$

$$= \frac{2^{2n}(2n)!\{(4n-1)(4n-3).....3.1\}}{2^{2n}.(n!)^2 \{(2n-1)(2n-3).....3.1\}^2} \times \frac{(n!)^2}{(2n)!}$$

$$= \frac{(4n-1)(4n-3)....3.1}{\{(2n-1)(2n-3)....3.1\}^2}$$

$$\therefore \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{1.3.5.....(4n-1)}{\{1.3.5.....(2n-1)\}^2}$$

2. If a set A has 12 elements , find the number of subset of a having
(i) 4 elements (ii) At least 3 elements (iii) At most 3 elements.

Sol: The no.of elements in a set A = 12

(i)The number of subsets of A with exactly 4 elements = $12_{C_4} = 495$

(ii) The required subsets contains at least3 elements . Number of subsets of A with exactly O elements= 12_{C_o}

No. of subsets of A with exactly 1 element = 12_{C_1}

No. Of subsets of A with exactly 2 element = 12_{C_2}

Total No. Of subsets of A formed $= 2^{12}$

$\therefore$The no. of subsets of A with at least 3 elements

$$= 2^{12} - \left(12_{C_0} + 12_{C_1} + 12_{C_2}\right) = 4096 - (1 + 12 + 66) = 4096 - 79 = 4017 .$$

iii) The required subsets contains at most 3 elements.

No. of subsets of A with exactly 0 elements = 12_{C_0} , No. of subsets of A with exactly 1 elements = 12_{C_1}

No. of subsets A with exactly 2 elements = 12_{C_2}

No. Of subsets A with exactly 3 elements = 12_{C_3} .

$\therefore$ The no. of subsets of A with at most 3 elements

$= 12_{C_0} + 12_{C_1} + 12_{C_2} + 12_{C_3} = 1 + 12 + 66 + 220 = 299$

3. There are 8 railway stations along a railway line . In how many ways can a train be stopped at 3 of these stations such that no two of them are consecutive.

Sol: We can select 3 stations out of 8 stations in 8_{C_3} ways =56 ways

No. Of ways of selecting 3 out of 8 stations such that 3 are consecutive = 6.

Number of ways selecting 3 out of 8 stations such that 2 of them are consecutive

$= 2 \times 5 + 5 \times 4 = 10 + 20 = 30$

$\therefore$ No. of ways of a train to be stopped at 3 of 8 stations such that no two of them are consecutive $= 56 - (6 + 30) = 20$.

(or)

No. of ways = $(n - r + 1)_{C_r}$. Here $n = 8$, $r = 3$

No. Of ways = $(8 - 3 + 1)_{C_3}$

$= 6_{C_3} = \dfrac{6!}{3!3!} = \dfrac{4 \times 5 \times 6}{3 \times 2 \times 1} = 20.$

4. A question paper is divided into 3 sections A, B,C containing 3,4,5 questions respectively. Find the number of ways of attempting 6 questions. Choosing at least one from each section.

Sol: The selection of question may be of the following

Sec A(3)	Sec B (4)	Sec C(5)
3	2	1
3	1	2
2	3	1
2	2	2
2	1	3
1	4	1
1	3	2
1	2	3
1	1	4

Total number of ways

$$= {}^3C_3.{}^4C_2.{}^5C_1 + {}^3C_3.{}^4C_1.{}^5C_2 + {}^3C_2.{}^4C_3.{}^5C_1 + {}^3C_2.{}^4C_2.{}^5C_2 + {}^3C_2.{}^4C_1.{}^5C_3 +$$
$${}^3C_1.{}^4C_4.{}^5C_1 + {}^3C_1.{}^4C_3.{}^5C_2 + {}^3C_1.{}^4C_2.{}^5C_3 + {}^3C_1.{}^4C_1.{}^5C_4 = 805$$

5. A Class contain 4 boys and g girls. Every Sunday, 5 students with at least 3 boys go for a picnic. A different group is being sent every week. During the picnic, the class teacher gives each girl in the group doll. If the total number of dolls distributed is 85 find g.

Sol: Since the group contains 5 students with at least 3 boys, the students of the group may be of the following two types.

1) 3 boys and 2 girls

2) 4 boys and 1 girl.

In 1st case the no. of ways of select $= 4_{C_3} \times g_{C_1} = 2g(g-1)$

In 2nd case the no. of ways to select $= 4_{C4} \times g_{C_1} = g$

The total no. of dolls given by class teacher

$= 2\left[2g(g-1)\right] + [g] = 4g^2 - 4g + g = 4g^2 - 3g$.

Given that $4g^2 - 3g = 85 \Rightarrow 4g^2 - 3g - 85 = 0 \Rightarrow (g-5)(4g+17) = 0$

$\therefore g = 5$

<u>Exercise 6(e)</u>

I.

1. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consist of (i) exactly 3 girls (ii) at least 3 girls (iii) almost 3 girls?

Sol: (i) No. of ways of for Ming a committee with exactly 3 girls $= 4_{C_3} \times 9_{C_4} = 504$ ways.

(ii) Atleast 3 girls

Boys (9)	Girls (4)	Total (7)
4	3	7
3	4	7

$\therefore$ Required no. of ways $= 9_{C_4} \times 4_{C_3} + 9_{C_3} \times 4_{C_4} = 504 + 84 = 588$.

(iii) Required no. of ways $= 9_{C_7} \times 9_{C_0} + 9_{C_6} \times 9_{C_1} + 9_{C_5} \times 9_{C_2} + 9_{C_4} \times 9_{C_3}$ ways $= 1632$ ways .

2. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

Sol: Total numbers = 5 men +4 women = 9 members and 9 places

Even places = 4 women in 4 every places in 4_{P_4} ways = $4!$ ways .

and we arrange 5 men in remaining 5 places in 5_{P_5} ways $= 5!$ ways.

By the fundamental principal of counting, total no. of seating arrangements $= 4! \times 5!$

$= 24 \times 120 = 2880$.

3. In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's. are together.

Sol: In the word ASSASSINATION, consider 4S's as one unit. Then total no. of units are 10. In these 3A, 2I, 2N and remain each of one letters.

$\therefore$ No. of arrangements $= \dfrac{(10)!}{3!2!2!}$ =151200 ways.

II

1. How many words, with or without meaning , each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER?

Sol: The word DAUGHTER consists of 3 vowels and 5 consonants.

2 vowels and 3 consonants may be selected in $3_{C_2} \times 5_{C_3} = \dfrac{3}{1} \times \dfrac{5 \times 4}{2 \times 1} = 30$ ways.

Also 5 letters can be arranged in 5! Ways i.e 120.

$\therefore$No. of words formed by 2 vowels and 3 consonants are selected from the word DAUGHTER $= 30 \times 120 = 3600$.

2. How many words, with or without meanings can be formed using all the letters of the word EQUATION at a time so that the vowels and consonants occurs together?

Sol: In the word EQUATION, no. of vowels are 5 and consonants are 3.

5 vowels can be arranged in 5! Ways

3 consonants can be arranged in 3! Ways

These two blocks can be arranged in 2! Ways.

$\therefore$Required no. of ways $= 5! \times 3! \times 2! = 1440$ ways .

3. If the different permutations of all the letter of the word EXAMINATION are listed as in a dictionary, how many words are there in this list before the first word starting with E?

Sol: The letters of given word are, A,A,E,I,I,M,N,N,O,T,X…i.e. Words starting with A are formed with the letters $2I's, 2N's, A, E, X, M, T, O$ (total 10 letters)

$\therefore$ No. of words formed by these letters $= \dfrac{(10)!}{2!2!} = 907200$ ways.

4. How many 6-digits numbers can be formed from the digits 0,1,3,5,7 and 9 which are divisible by 10 and no digit is repeated ?

Sol:

					0

remaining 5 places can be filled by U

the five digits 1,3,5,7, and 9 in 5! Ways = 120.

$\therefore$ Required numbers = 120.

5. The English alphabet has 5 vowels and 21 consonants, how many words with two different vowels and two different consonants can be formed from alphabet ?

Sol: 2 vowels can be select from 5 in 5_{C_2} ways .

2 consonants can be select from 21 in 21_{C_2} ways

$\therefore$ No.of words formed $= 5_{C_2} \times 21_{C_2} = 2100$

In these 2100 selections has 4 letters which can be arranged in 41 ways .

$\therefore$ Required no. of different words $= 2100 \times 4! = 50400$.

III

1. In an examination, a question paper consists of 12 questions divided into two parts i.e Part I and Part II, containing 5 and 7 questions respectively. A student is required to attempt 8 questions in all selecting at least 3 from each part. In how many ways can students select the questions?

Sol: Question paper consists of 12 questions.

A student is required to attempt 8 questions in all selecting at least 3 from each part.

Part- I (5)	Part-II (7)	Questions attempt (8)
3	5	8
4	4	8
5	3	8

$\therefore$ Total no. of ways = $5_{C_3} \times 7_{C_5} + 5_{C_4} \times 7_{C_4} + 5_{C_5} \times 7_{C_3} = 210 + 175 + 35 = 420$

2. Determine the number of 5 cards combining out of deck of 52 cards if each selection of 5 cards has exactly one king.

Sol: One king selected from 4 cards of kings is 4_{C_1}

Remaining 4 cards should be select from 52-4 = 48 cards. These are in 48_{C_4} ways.

Hence total no. of ways $= 4_{C_1} \times 48_{C_4}$

3. From a class of 25 students, 10 are to be chosen for an excursion party, There are 3 students, who decide that either all of them will join on none of them will join. In how many ways can the excursion party be chosen ?

Sol: Here we have two cases

Case : (i) If all the three students will join party then, we have to select 7 students out of 22 students .

Case: (ii)If all the three students will not go for the party then we have to select 10 students out of 22 students. These can be 22_{C_7} ways in case (i) or $22_{C_{10}}$ ways in case (ii) .

$\therefore$ Total no. of ways = $22_{C_7} + 22_{C_{10}}$.

Multiple choice questions

1) **Sol**.: A student has 5 pants and 8 shirts. Then the number of ways in which he can wear the dress in different combinations as $5 \times 8 = 40$.
2) **Sol:**

$$\Rightarrow n(n-1)(n-2)(n-3) = 8 \times 7 \times 6 \times 5$$

$$\therefore n = 8$$

3) **Sol:**

If $n_{P_r} = 1320$

$\Rightarrow n_{P_r} = 12 \times 11 \times 10 = 12_{P3}$

$r = 3$

4) **Sol:**
A man has 3 sons and 6 schools within his reach. Then he can send has sons to school in 6_{P_3} ways. Since no two of his sons read in the same school.
5) **Sol:**The letters {T,U,E,S,D,A,V} are 7. These can arranged in $7! = 5040$ ways.

6) **Sol:** The expansion of $a^4b^3c^5$ is aaaa bbb cccc $a's-4, b's-3, c's-5$ and total – 12 letters.

$\therefore$ No. of arrangements = $\dfrac{(12)!}{4!3!5!}$ ways.
7) **Sol:** When repetition is allowed, the number of 4- digit numbers from 1,2,3,4,5,6 are 6^4 .
8) **Sol:** 8 men + 4 women = 12 members can arrange $(12)!$ in row.
9) **Sol:**

$1080 = 2^3 \times 3^3 \times 5'$
$\therefore$ No. of divisors $= (3+1)(3+1)(1+1)$ = 32.
10) **Sol:**

The no. of ways selecting 4 boys out of $8 = 8_{C_4}$.

The no. of ways selecting 3 girls out of $5 = 5_{C_3}$.

$\therefore$ Total no. of ways $= 8_{C_4} \times 5_{C_3}$

11) **Sol:**

If ${}^{12}c_{r+1} = {}^{12}c_{2r-5} \Rightarrow s+1 = 2s-5 \Rightarrow s = 6$

12) **Sol:**

No. of diagonals $= \dfrac{10(10-3)}{2} = 35.$

13) **Sol:**

$$\frac{n(n-3)}{2} = 54$$
$$\therefore n = 12$$

MULTIPLE CHOICE QUESTIONS (KEY)

1) 4	2) 2	3) 1	4) 1	5) 2	6) 4	7) 3	8) 3	9) 3	10) 2
11) 4	12) 1	13) 4							

****

7. Binomial Theorem

Exercise – 7(a)

I. Expand each of the expressions.

1. $(1-2x)^5$

Sol.: $(1-2x)^5 = 5_{c_o}(1)^5(-2x)^0 + 5_{c_1}(1)^4(-2x)^1 + 5_{c_2}(1)^3(-2x)^2 + 5_{c_3}(1)^2(-2x)^3 + 5_{c_4}(1)^1(-2x)^4 + 5_{c_5}(1)^0(-2x)^5$

$= 1+5(-2x)+10(4x^2)+10(-8x^3)+5(16x^4)+1(-32x^5)$

$(1-2x)^5 = 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

2. $\left(\frac{2}{x}-\frac{x}{2}\right)^5$

Sol.

$$\left(\frac{2}{x}-\frac{x}{2}\right)^5 = 5_{c_0}\left(\frac{2}{x}\right)^5\left(-\frac{x}{2}\right)^0 + 5_{c_1}\left(\frac{2}{x}\right)^4\left(-\frac{x}{2}\right)^1 + 5_{c_2}\left(\frac{2}{x}\right)^3\left(-\frac{x}{2}\right)^2$$
$$+5_{c_3}\left(\frac{2}{x}\right)^2\left(-\frac{x}{2}\right)^3 + 5_{c_4}\left(\frac{2}{x}\right)^1\left(-\frac{x}{2}\right)^4 + 5_{c_5}\left(\frac{2}{x}\right)^0\left(-\frac{x}{2}\right)^5$$

$$=1\left(\frac{32}{x^5}\right) + 5\left(\frac{16}{x^4}\right)\left(-\frac{x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(-\frac{x^2}{4}\right) + 10\left(\frac{4}{x^2}\right)\left(-\frac{x^3}{8}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right) + 1\left(-\frac{x^5}{32}\right)$$

$$\therefore \left(\frac{2}{x}-\frac{x}{2}\right)^5 = \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5x^3}{8} - \frac{x^5}{32}$$

3. $(2x-3)^6$

Sol.: $(2x-3)^6 = 6_{c_0}(2x)^6(-3)^0 + 6_{c_1}(2x)^5(-3)^1 + 6_{c_2}(2x)^4(-3)^2 + 6_{c_3}(2x)^3(-3)^3 + 6_{c_4}(2x)^2(-3)^4 + 6_{c_5}(2x)^1(-3)^5 + 6_{c_6}(2x)^0(-3)^6$

$= 1(64x^6) + 6(32x^5)(-3) + 15(16x^4)(9) + 20(8x^3)(-27) + 15(4x^4)(81) + 6(2x)(-243) + 1(729)$

$= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729.$

4. $\left(\frac{x}{3}+\frac{1}{x}\right)^5$

Sol.:

$$\left(\frac{x}{3}+\frac{1}{x}\right)^5 = 5_{c_0}\left(\frac{x}{3}\right)^5\left(\frac{1}{x}\right)^0 + 5_{c_1}\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right)^1 + 5_{c_2}\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2$$
$$+5_{c_3}\left(\frac{x}{3}\right)^2\left(-\frac{1}{x}\right)^3 + 5_{c_4}\left(\frac{x}{3}\right)^1\left(-\frac{1}{x}\right)^4 + 5_{c_5}\left(\frac{x}{3}\right)^0\left(-\frac{1}{x}\right)^5$$

$$= 1\left(\frac{x^5}{243}\right) + 5\left(\frac{x^4}{81}\right)\left(\frac{1}{x}\right) + 10\left(\frac{x^3}{27}\right)\left(\frac{1}{x^2}\right) + 10\left(\frac{x^2}{9}\right)\left(\frac{1}{x^3}\right) + 5\left(\frac{x}{3}\right)\left(\frac{1}{x^4}\right) + 1\left(\frac{1}{x^5}\right)$$

$$= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^5}$$

5. $\left(x+\frac{1}{x}\right)^6$

Sol: $\left(x+\frac{1}{x}\right)^6 = 6_{c_0}(x)^6\left(\frac{1}{x}\right)^0 + 6_{c_1}(x)^5\left(\frac{1}{x}\right)^1 + 6_{c_2}(x)^4\left(\frac{1}{x}\right)^2 + 6_{c_3}(x)^3\left(\frac{1}{x}\right)^3$

$$+ 6_{c_4}(x)^2\left(\frac{1}{x}\right)^4 + 6_{c_5}(x)^1\left(\frac{1}{x}\right)^5 + 6_{c_6}((x)^0\left(\frac{1}{x}\right)^6$$

$$= 1(x^6) + 6(x^5)\left(\frac{1}{x}\right) + 15(x^4)\left(\frac{1}{x^2}\right) + 20(x^3)\left(\frac{1}{x^3}\right) + 15(x^2)\left(\frac{1}{x^4}\right) + 6(x)\left(\frac{1}{x^5}\right) + 1\left(\frac{1}{x^6}\right)$$

$$\therefore \left(x+\frac{1}{x}\right)^6 = x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

6. $(4x+5y)^7$

Sol: $(4x+5y)^7 = 7_{c_0}(4x)^7(5y)^0 + 7_{c_1}(4x)^6(5y)^1 + 7_{c_2}(4x)^5(5y)^2 + 7_{c_3}(4x)^4(5y)^3$

$$+ 7_{c_4}(4x)^3(5y)^4 + 7_{c_5}(4x)^2(5y)^5 + 7_{c_6}(4x)^1(5y)^6 + 7_{c_7}(4x)^0(5y)^7$$

$$= \sum_{r=0}^{7} 7_{c_r}(4x)^{7-r}(5y)^r$$

7. $\left(\frac{2}{3}x+\frac{7}{4}y\right)^5$

Sol: $\left(\frac{2}{3}x+\frac{7}{4}y\right)^5 = 5_{c_0}\left(\frac{2x}{3}\right)^5\left(\frac{7y}{4}\right)^0 + 5_{c_1}\left(\frac{2x}{3}\right)^4\left(\frac{7y}{4}\right)^1 + 5_{c_2}\left(\frac{2x}{3}\right)^3\left(\frac{7y}{4}\right)^2$

$$+ 5_{c_3}\left(\frac{2x}{3}\right)^2\left(\frac{7y}{4}\right)^3 + 5_{c_4}\left(\frac{2x}{3}\right)^1\left(\frac{7y}{4}\right)^4 + 5_{c_5}\left(\frac{2x}{3}\right)^0\left(\frac{7y}{4}\right)^5$$

$$= \sum_{r=0}^{5} 5_{c_r}\left(\frac{2x}{3}\right)^{5-r}\left(\frac{7y}{4}\right)^r$$

8. $\left(\frac{2p}{5}-\frac{3q}{7}\right)^6$

Sol: $\left(\frac{2p}{5}-\frac{3q}{7}\right)^6 = 6_{c_0}\left(\frac{2p}{5}\right)^6\left(-\frac{3q}{7}\right)^0 + 6_{c_1}\left(\frac{2p}{5}\right)^5\left(-\frac{3q}{7}\right)^1 + 6_{c_2}\left(\frac{2p}{5}\right)^4\left(-\frac{3q}{7}\right)^2 + 6_{c_3}$

$$\left(\frac{2p}{5}\right)^3\left(-\frac{3q}{7}\right)^3 + \ 6_{c_4}\left(\frac{2p}{5}\right)^2\left(-\frac{3q}{7}\right)^4 + \ 6_{c_5}\left(\frac{2p}{5}\right)^1\left(-\frac{3q}{7}\right)^5 + \ 6_{c_6}$$

$$\left(\frac{2p}{5}\right)^0\left(-\frac{3q}{7}\right)^6$$

$$= \sum_{r=0}^{6} 6_{c_r}\left(\frac{2p}{5}\right)^{6-r}\left(-\frac{3q}{7}\right)^r$$

9. Write down and simplify

i) 6^{th} term in $\left(\frac{2x}{3}+\frac{3y}{2}\right)^9$

Sol: $T_6 = T_{5+1} = 9_{c_5}\left(\frac{2x}{3}\right)^{9-5}\left(\frac{3y}{2}\right)^5$

$$= 126\left(\frac{2x}{3}\right)^4\left(\frac{3y}{2}\right)^5$$

$$= 126\left(\frac{2^4}{3^4}\right)\left(\frac{3^5}{2^5}\right)\left(x^4y^5\right)$$

$$=126\left(\frac{3}{2}\right)\left(x^4y^5\right)$$

$$=189\ x^4y^5$$

ii) 7^{th} term in $(3x-4y)^{10}$

Sol: $T_7 = T_{6+1} = 10_{c_6}(3x)^{10-6}(-4y)^6$

$$= 210(3x)^4(-4y)^6$$

$$= 210\ (3^4)\ (4^6)\ \left(x^4\ y^6\right)$$

iii) 10^{th} term in $\left(\frac{3p}{4}-5q\right)^{14}$

Sol: $T_{10} = T_{9+1} = 14_{c_9}\left(\frac{3p}{4}\right)^{14-9}\left(-5^2\right)^9$

$$= 2002\left(\frac{3p}{4}\right)^5\left(-5^2\right)^9$$

$$= \frac{-2002(3^5)(5^9)}{4^5}\left(p^5q^9\right)$$

iv) r^{th} term in $\left(\frac{3a}{4}+\frac{56}{7}\right)^8$, $(1 \le r \le 8)$

Sol: $T_r = T_{r-1+1} = 8_{c_{r-1}}\left(\frac{3a}{5}\right)^{8-(r-1)}\left(\frac{56}{7}\right)^{r-1}$

$$= 8_{c_{r-1}}\left(\frac{3a}{5}\right)^{9-r}\left(\frac{56}{7}\right)^{r-1}$$

v) Write the middle term in $\left(3-\frac{x^3}{6}\right)^7$

Sol: Middle terms are $\left(\frac{7+1}{2}\right)^{th}$ term, $\left(\frac{7+1}{2}+1\right)^{th}$ term i.e T_4, T_5 are middle terms.

i) $T_4 = T_{3+1} = 7_{c_3}(3)^4\left(-\frac{x^3}{6}\right)^3$

ii) $T_5 = T_{4+1} = 7_{c_4}(3)^3\left(-\frac{x^3}{6}\right)^4$

vi) Write the middle term in $\left(\frac{x}{3}+9y\right)^{10}$

Sol.: $\left(\frac{10}{2}+1\right)^{th} = 6^{th}$ term is middle term i.e T_6

$T_6 = T_{5+1} = 10_{c_5}\left(\frac{x}{3}\right)^5(9y)^5$

vii) Write the 4^{th} term in $(x-2y)^{12}$

$T_4 = T_{3+1} = 12_{c_3}(x)^9(-2y)^3$

viii) Write the 13^{th} term in $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$, $x \neq 0$

$T_{13} = T_{12+1} = 18_{c_{12}}(9x)^6\left(-\frac{1}{3\sqrt{x}}\right)^{12}$

10) Find the number of terms in the expansion of i) $\left(\frac{3a}{4}+\frac{b}{2}\right)^9$**, ii)** $(3p+4q)^{14}$

Sol.: i) No. of terms = 9+1=10

ii) No. of terms = 14+1=15

II

1) Using Binomial theorem, evaluate the following $(96)^3$

Sol.: $(96)^3 = (100-4)^3$

$= 3_{c_0}(100)^3(-4)^0 + 3_{c_1}(100)^2(-4)^1 + 3_{c_2}(100)^1(-4)^2 + 3_{c_3}(100)^0(-4)^3$

$= (100)^3 - 3(100)^2(4) + 3(100)(4)^2 - (4)^3$

$= 1000000 - 120000 + 4800 - 64 = 884736$

2) Using Binomial theorem, evaluate the following $(102)^5$

Sol.: $(102)^5 = (100+2)^5$

$= 5_{c_0}(100)^5(2)^0 + 5_{c_1}(100)^4(2)^1 + 5_{c_2}(100)^3(2)^2 + 5_{c_3}(100)^2(2)^3$

$+ 5_{c_4}(100)^1(2)^4 + 5_{c_5}(100)^0(2)^5$

$= (100)^5 + 5(100)^4(2) + 10(100)^3(2)^2 + 10(100)^2(2)^3 + 5(100)(2)^4 + (2)^5$

$= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32 = 11040808032$

3) Using Binomial theorem, evaluate the following $(101)^4$

Sol.: $(101)^4 = (100+1)^4$

$= 4_{c_0}(100)^4(1)^0 + 4_{c_1}(100)^3(1)^1 + 4_{c_2}(100)^2(1)^2 + 4_{c_3}(100)^1(1)^3 + 4_{c_4}(100)^0(1)^4$

$= (100)^4 + 4(100)^3 + 6(100)^2 + 4(100)^1 + 1$

$= 100000000 + 4000000 + 60000 + 1$

$= 104060401$

4) Using Binomial theorem, evaluate the following $(99)^5$

Sol.: $(99)^5 = (100-1)^5$

$= 5_{c_0}(100)^5(-1)^0 + 5_{c_1}(100)^4(-1)^1 + 5_{c_2}(100)^3(-1)^2 + 5_{c_3}(100)^2(-1)^3$

$+ 5_{c_4}(100)^1(-1)^4 + 5_{c_5}(100)^0(-1)^5$

$= (100)^5 - 5(100)^4 + 10(100)^3 - 10(100)^2 + 5(100) - 1$

$= 10000000000 - 500000000 + 10000000 - 100000 + 500 - 1$

$= 10010000500 - 500100001$

$= 9509900499$

5) Using Binomial theorem, indicate which is larger $(1.1)^{10000}$ or 1000.

Sol: $(1.1)^{1000} = (1+0.1)^{10000}$

$= 10000_{C_0} + 10000_{C_1}(0.1)^1 +$ Other positive terms.

$= 1 + 10000 \text{ X } 0.1 +$ Other positive terms $= 1001$ other positive terms.

$= >1000$

Hence, $(1.1)^{10000} > 1000$

6) Find $(a+b)^4 - (a-b)^4$. Hence, evaluate $(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4$

Sol: Using Binomial Theorem, the expressions, $(a+b)^4 - (a-b)^4$, can be expanded as $(a+b)^4 - (a-b)^4$

$= (a+b)^4 - (a-b)^4 = 4_{C_0}a^4b^0 + 4_{C_1}a^3b^1 + 4_{C_2}a^2b^2 + 4_{C_3}a^1b^3 + 4_{C_4}a^0b^4$

$$-[\,4_{C_0}a^4b^0 - 4_{C_1}a^3b^1 + 4_{C_2}a^2b^2 - 4_{C_3}a^1b^3 + 4_{C_4}a^0b^4\,]$$

$$= 2[\,4_{C_1}a^3b^1 + 4_{C_3}a^1b^3\,]$$

$$= 2[\,4a^3\,b + 4ab^3\,]$$

$$=8ab\,(a^2+b^2)$$

Now putting a $=\sqrt{3}$ and b $=\sqrt{2}$, we have

$$(\sqrt{3}+\sqrt{2}\;)^4 - (\sqrt{3}-\sqrt{2}\;)^4 = 8\left(\sqrt{3}\right)\left(\sqrt{2}\right)\left[\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2\right]$$

$$= 8\sqrt{6}\,(3+2) = 40\sqrt{6}$$

7) Find $(x+1)^6 + (x-1)^6$. Hence or otherwise evaluate $\left(\sqrt{2}+1)\right)^6+\left(\sqrt{2}-1\right)^6$

Sol: Using Binomial Theorem, the expressions, $\left(x+1\right)^6+\left(x-1\right)^6$, can be expanded as

$$\left(x+1\right)^6+\left(x-1\right)^6$$

$$\left(x+1\right)^6+\left(x-1\right)^6 = 6_{C_0}x^6 1^0 + 6_{C_1}x^5 1^1 + 6_{C_2}x^4 1^2 + 6_{C_3}x^3 1^3 + 6_{C_4}x^2 1^4 + 6_{C_5}x^1 1^5 + 6_{C_6}x^0 1^6$$

$$+$$

$$\left[6_{C_0}x^6 1^0 - 6_{C_1}x^5 1^1 + 6_{C_2}x^4 1^2 - 6_{C_3}x^3 1^3 + 6_{C_4}x^2 1^4 - 6_{C_5}x^1 1^5 + 6_{C_6}x^0 1^6\right]$$

$$= 2\left[\,6_{C_0}x^6 + 6_{C_2}x^4 + 6_{C_4}x^2 + 6_{C_6}\,\right]$$

$$=2(x^6+15x^4+15x^2+1)$$

Now putting $x=\sqrt{2}$, we have

$$\left(\sqrt{2}+1)\right)^6+\left(\sqrt{2}-1\right)^6 = 2\left[\left(\sqrt{2}\right)^6+15\left(\sqrt{2}\right)^4+15\left(\sqrt{2}\right)^2+1\right]$$

$$= 2(8+60+30+1)$$

$$= 198.$$

8) Show that $9^{n+1}-8n-9$ is divisible by 64, whenever n is a positive integer.

Sol: Using Binomial Theorem,

$$\left(1+x\right)^{n+1} = n+1_{C_0}x^0 + n+1_{C_1}x^1 + n+1_{C_2}x^2 + \ldots\ldots\ldots\ldots\ldots + n+1_{C_{n+1}}x^{n+1}$$

Now putting x=8, we have

$$\left(1+8\right)^{n+1} = n+1_{C_0}8^0 + n+1_{C_1}8^1 + n+1_{C_2}8^2 + \ldots\ldots\ldots\ldots + n+1_{C_{n+1}}8^{n+1}$$

$$\Rightarrow 9^{n+1} = 1+(n+1)(8) + 8^2\left[n+1_{C_2} + \ldots\ldots\ldots + n+1_{C_{n+1}}8^{n-1}\right]$$

$$\Rightarrow 9^{n+1}-1-8\left(n+1\right) = 64\left[n+1_{C_2} + \ldots\ldots\ldots + n+1_{C_{n+1}}8^{n-1}\right]$$

$$\Rightarrow 9^{n+1} - 8n - 9 = 64\left[n+1_{C_2} + \ldots\ldots\ldots + n+1_{C_{n+1}}8^{n-1}\right]$$

Now, RHS is divisible by 64 for all possible integers n. So, LHS is also divisible by 64.

Hence, 9^{n+1} - 8n – 9 is divisible by 64, whenever n is a positive integer.

9) Prove that $\sum_{r=0}^{n} 3^r n_{C_r} = 4^n$

Sol: LHS = $\sum_{r=0}^{n} 3^r n_{C_r}$

$$= 3^0 n_{C_0} + 3^1 n_{C_1} + 3^2 n_{C_2} + \ldots\ldots\ldots\ldots + 3^{n-1} n_{C_{n-1}} + 3^n n_{C_n}$$

$= n_{C_0}3^0 + n_{C_1}3^1 + n_{C_2}3^2 + ... + n_{C_{n-1}}3^{n-1} + n_{C_n}3^n$

$= (1+3)^n = 4^n$

= RHS

10) Using Binomial Theorem prove that $50^n - 49n - 1$ is divisible by 49^2 for all positive integers n.

Sol: $50^n - 49n - 1 = (49+1)^n - 49n - 1$

$$= \left[n_{C_0}49^n + n_{C_1}49^{n-1} + ... + n_{C_{n-2}}49^2 + n_{C_{n-1}}49^1 + n_{C_n}49^0\right] - 49n - 1$$

$$= n_{C_0}49^n + n_{C_1}49^{n-1} + ... + n_{C_{n-2}}49^2 + n_{C_{n-1}}49^1 + 1 - 49n - 1$$

$$= 49\left[n_{C_0}49^{n-1} + n_{C_1}49^{n-2} + ... + n_{C_{n-2}}49^1\right] + 49n + 1 - 49n - 1$$

$$= 49^2\left[n_{C_0}49^{n-2} + n_{C_1}49^{n-3} + ... + n_{C_{n-2}}\right]$$

This is divisible by 49^2 for all n.

$\therefore 50^n - 49n - 1$ is divisible by 49^2 for all positive integers n.

Exercise – 7(b)

I

1. If a and b are distinct integers, prove that a – b is a factor of $a^n - b^n$, whenever n is a positive integer. [Hint write $a^n = (a - b + b)^n$ and expand]

Sol: Here, $a^n = (a - b + b)^n = [(a - b) + b]^n$

By using Binomial Theorem, the expression $[(a - b) + b]^n$ can be expanded as

$$n_{C_0}(a-b)^n b^0 + n_{C_1}(a-b)^{n-1}b^1 + n_{C_2}(a-b)^{n-2}b^2 + ... + n_{C_{n-1}}(a-b)^1 b^{n-1} + n_{C_n}(a-b)^0 b^n$$

$$= (a-b)^n + n_{C_1}(a-b)^{n-1}b + n_{C_2}(a-b)^{n-2}b^2 + ... + n_{C_{n-1}}(a-b)^1 b^{n-1} + b^n$$

$$\Rightarrow a^n = (a-b)\left[(a-b)^{n-1} + n_{C_1}(a-b)^{n-2}b + n_{C_2}(a-b)^{n-3}b^2 + ... + n_{C_{n-1}}b^{n-1}\right] + b^n$$

$$\Rightarrow a^n - b^n = \left[(a-b)^{n-1} + n_{C_1}(a-b)^{n-2}b + n_{C_2}(a-b)^{n-3}b^2 + ... + n_{C_{n-1}}b^{n-1}\right]$$

Now, a – b is a factor of RHS, so, a – b is also a factor of LHS.

Hence, a – b is a factor of $a^n - b^n$, whenever n is a positive integer.

2. Evaluate $\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6$

Sol: $\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6 = 6_{C_0}\left(\sqrt{3}\right)^6 + 6_{C_1}\left(\sqrt{3}\right)^5\left(\sqrt{2}\right) + 6_{C_2}\left(\sqrt{3}\right)^4\left(\sqrt{2}\right)^2 + 6_{C_3}\left(\sqrt{3}\right)^3\left(\sqrt{2}\right)^3$

$+6_{C_4}\left(\sqrt{3}\right)^2\left(\sqrt{2}\right)^4 + 6_{C_5}\left(\sqrt{3}\right)^1\left(\sqrt{2}\right)^5 + 6_{C_6}\left(\sqrt{3}\right)^0\left(\sqrt{2}\right)^6$ -

$\left[6_{C_0}\left(\sqrt{3}\right)^6 - 6_{C_1}\left(\sqrt{3}\right)^5\left(\sqrt{2}\right) + 6_{C_2}\left(\sqrt{3}\right)^4\left(\sqrt{2}\right)^2 - 6_{C_3}\left(\sqrt{3}\right)^3\left(\sqrt{2}\right)^3\right.$

$\left.+6_{C_4}\left(\sqrt{3}\right)^2\left(\sqrt{2}\right)^4 - 6_{C_5}\left(\sqrt{3}\right)^1\left(\sqrt{2}\right)^5 + 6_{C_6}\left(\sqrt{3}\right)^0\left(\sqrt{2}\right)^6\right]$]

$$= 2\left[6_{C_1}\left(\sqrt{3}\right)^5\left(\sqrt{2}\right) + 6_{C_3}\left(\sqrt{3}\right)^3\left(\sqrt{2}\right)^3 + 6_{C_5}\left(\sqrt{3}\right)^1\left(\sqrt{2}\right)^5\right]$$

$$= 2\left[6x\,9\sqrt{6} + 20x\,6\sqrt{6} + 6x\,4\sqrt{6}\right]$$

$= 2[54 + 120 + 24]\ \sqrt{6}\ = 396\sqrt{6}$

3. Evaluate $\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4$

A) $\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4=$

$$4_{C_0}\left(a^2\right)^4+4_{C_1}\left(a^2\right)^3\left(\sqrt{a^2-1}\right)^1+4_{C_2}\left(a^2\right)^2\left(\sqrt{a^2-1}\right)^2$$

$$+4_{C_3}\left(a^2\right)^1\left(\sqrt{a^2-1}\right)^3+4_{C_4}\left(a^2\right)^0\left(\sqrt{a^2-1}\right)^4+$$

$$\left[4_{C_0}\left(a^2\right)^4-4_{C_1}\left(a^2\right)^3\left(\sqrt{a^2-1}\right)^1+4_{C_2}\left(a^2\right)^2\left(\sqrt{a^2-1}\right)^2\right.$$

$$\left.-4_{C_3}\left(a^2\right)^1\left(\sqrt{a^2-1}\right)^3+4_{C_4}\left(a^2\right)^0\left(\sqrt{a^2-1}\right)^4\right]$$

$$=2\left[4_{C_0}\left(a^2\right)^4+4_{C_2}\left(a^2\right)^2\left(\sqrt{a^2-1}\right)^2+4_{C_4}\left(a^2\right)^0\left(\sqrt{a^2-1}\right)^4\right]$$

$= 2\ [a^8 + 6a^4\ (a^2 – 1) + (a^2 – 1)^2]$
$= 2\ [a^8 + 6a^6 - 6a^4 + a^4+1 – 2a^2]$
$= 2\ [a^8 + 6a^6 - 5a^4 – 2a^2 + 1]$
$= 2a^8 + 12a^6 - 10a^4 - 4a^2+2$

II

1. Find a approximation of $(0.99)^5$ using the first three terms of its expansion.

Sol .: $(0.99)^5 = (1-0.01)^5$

$$=5_{C_0}\left(-0.01\right)^0+5_{C_1}\left(-0.01\right)^1+5_{C_2}\left(-0.01\right)^2+.........$$

$= 1 – 5(0.01) + 10(0.0001) + ……$
$\approx 1 – 0.05 + 0.001 = 0.951$

2. Expand using Binomial Theorem $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4$ **, $x\neq 0$**

Sol: Here, $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4=\left[\left(1+\frac{x}{2}\right)-\frac{2}{x}\right]^4$

Using Binomial Theorem, the expression $\left[\left(1+\frac{x}{2}\right)-\frac{2}{x}\right]^4$ can be expressed as

$$=4_{C_0}\left(1+\frac{x}{2}\right)^4$$

$$+4_{C_1}\left(1+\frac{x}{3}\right)^3\left(-\frac{2}{x}\right)+4_{C_2}\left(1+\frac{x}{2}\right)^2\left(-\frac{2}{x}\right)^2+4_{C_3}\left(1+\frac{x}{2}\right)\left(-\frac{2}{x}\right)^3+4_{C_4}\left(-\frac{2}{x}\right)^4$$

$$=\left(1+\frac{x}{2}\right)^4-4\left(1+\frac{x}{2}\right)^3\left(\frac{2}{x}\right)+6\left(1+\frac{x^2}{4}+x\right)\left(\frac{4}{x^2}\right)-4\left(1+\frac{x}{2}\right)\left(\frac{8}{x^3}\right)+\frac{16}{x^4}$$

$$=\left(1+\frac{x}{2}\right)^4-\frac{8}{x}\left(1+\frac{x}{2}\right)^3+6\left(\frac{4}{x^2}+1+\frac{4}{x}\right)-4\left(\frac{8}{x^3}+\frac{4}{x^2}\right)+\frac{16}{x^4} \quad \ldots(1)$$

Again applying Binomial Theorem, We have

$$\left(1+\frac{x}{2}\right)^4=4_{C_0}\left(\frac{x}{2}\right)^0+4_{C_1}\left(\frac{x}{2}\right)^1+4_{C_2}\left(\frac{x}{2}\right)^2+4_{C_3}\left(\frac{x}{2}\right)^3+4_{C_4}\left(\frac{x}{2}\right)^4$$

$$= 1 + 4\ x\ \frac{x}{2} + 6\ x\ \frac{x^2}{4} + 4\ x\ \frac{x^3}{8} + \frac{x^4}{16} = 1 + 2x + \frac{3x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}$$

And $\left(1+\frac{x}{2}\right)^3 = 4_{C_0}\left(\frac{x}{2}\right)^0+4_{C_1}\left(\frac{x}{2}\right)^1+4_{C_2}\left(\frac{x}{2}\right)^2+4_{C_3}\left(\frac{x}{2}\right)^3+4_{C_4}\left(\frac{x}{2}\right)^4$

$$= 1+3x\frac{x}{2}+3x\frac{x^2}{4}+1x\frac{x^3}{8}$$

$$= 1+\frac{3x}{2}+\frac{3x^2}{4}+\frac{x^3}{8}$$

Now putting the value of $\left(1+\frac{x}{2}\right)^4$ and $\left(1+\frac{x}{2}\right)^3$ in (1), we have

$$\left[\left(1+\frac{x}{2}\right)-\frac{2}{x}\right]^4=\left[1+2x+\frac{3x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}\right]-\frac{8}{x}\left[1+\frac{3x}{2}+\frac{3x^2}{4}+\frac{x^3}{8}\right]+6\left(\frac{4}{x^2}+1+\frac{4}{x}\right)-4\left(\frac{8}{x^3}+\frac{4}{x^2}\right)+\frac{16}{x^4}$$

$$= 1+2x+\frac{3x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}-\frac{8}{x}-12-6x-x^2+\frac{24}{x^2}+6+\frac{24}{x}-\frac{32}{x^3}-\frac{16}{x^2}+\frac{16}{x^4}$$

$$=\left[\frac{16}{x^4}-\frac{32}{x^3}+\frac{8}{x^2}+\frac{16}{x}-4x+\frac{x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}-5\right]$$

3. Find the expansion of $\left(3x^2-2ax+3a^2\right)^3$ using Binomial Theorem.

Sol : Here, $\left(3x^2-2ax+3a^2\right)^3=\left[\left(3x^2-2ax\right)+3a^2\right]^3$

Using Binomial Theorem, the expression $\left[\left(3x^2-2ax\right)+3a^2\right]^3$ can be expressed as

$$= 3_{C_0}\left(3x^2-2ax\right)^3+3_{C_1}\left(3x^2-2ax\right)^2\left(3a^2\right)+3_{C_2}\left(3x^2-2ax\right)^1\left(3a^2\right)^2+3_{C_2}\left(3a^2\right)^3$$

$$=\left(3x^2-2ax\right)^3+3\left(9x^4+4a^2x^2-12ax^3\right)\left(3a^2\right)+3\left(3x^2-2ax\right)\left(9a^4\right)+\left(27a^6\right)$$

$$= \left(3x^2-2ax\right)^3+\left(81a^2x^4+36a^4x^2-108a^3x^3\right)+\left(81a^4x^2-54a^5x\right)+27a^6$$

………..(1)

Again applying Binomial Theorem, we have

$$\left(3x^2-2ax\right)^3=3_{C_0}\left(3x^2\right)^3+3_{C_1}\left(3x^2\right)^2\left(-2ax\right)^1+3_{C_2}\left(3x^2\right)\left(-2ax\right)^2+3_{C_3}\left(-2ax\right)^3$$

$$=27x^6-3x\,18ax^5+3x\,12a^2x^4-8a^3x^3$$

$$=27x^6-54ax^5+36a^2x^4-8a^3x^3$$

Now putting the value of $\left(3x^2-2ax\right)^3$ in (1), we have

$$\left(3x^2-2ax+3a^2\right)^3= [27x^6-54ax^5+36a^2x^4-8a^3x^3]+$$
$$\left(81a^2x^4+36a^4x^2-108a^3x^3\right)+\left(81a^4x^2-54a^5x\right)+27a^6$$

$$=27x^6-54ax^5+117a^2x^4-116a^3x^3+117a^4x^2-54a^5x+27a^6$$

4. Prove that $c_0+3_{C_1}+5_{C_2}+.......+\left(2n+1\right)C_n=\left(2n+2\right)2^{n-1}$

Sol: Let S $=c_0+3_{C_1}+5_{C_2}+.......+\left(2n+1\right)C_n$ ….(1)

$$S=\left(2n+1\right)C_n+\left(2n+1\right)C_{n-1}+\left(2n-3\right)C_{n-2}+.....+C_0$$

$$S = \left(2n+1\right)C_0+\left(2n-1\right)C_1+\left(2n-3\right)C_2+.....+C_n \quad ….(2)$$

$\left(\because n_{C_r}=n_{C_{n-r}}\right)$

By adding (1) and (2), we get $2S=\left(2n+2\right)C_0+\left(2n+2\right)C_1+.....+\left(2n+2\right)C_n$

$$2S=\left(2n+2\right)\left(C_0+C_1+.....+C_n\right)\left[\therefore C_0+C_1+C_2+.....C_n=2^n\right]$$

$$S=\left(2n+2\right)2^{n-1}$$

5. Prove that for any real numbers a, d;

$$a.C_0+\left(a+d\right)C_1+\left(a+2d\right)C_2+.....+\left(a+nd\right)C_n=\left(2a+nd\right)2^{n-1}$$

Sol: Let S= $a.C_0+\left(a+d\right)C_1+\left(a+2d\right)C_2+.....+\left(a+nd\right)C_n=\left(2a+nd\right)2^{n-1}$ ……(1)

$$S=\left(a+nd\right)c_n+[a+\left(n-1\right)d]c_{n-1}+\left[a+\left(n-1\right)d\right]c_{n-2}+.....+a_{c_0}$$

$$S=\left(a+nd\right)c_0+[a+\left(n-1\right)d]c_1+\left[a+\left(n-1\right)d\right]c_2+.....+a_{c_0} \quad …….(2) \quad \left[\therefore {}^nC_r={}^nC_n-r\right]$$

By adding (1) and (2) we get

$$2S = \left(2a+nd\right)c_0+\left(2a+nd\right)c_1+\left(2a+nd\right)c_2+.....+\left(2a+nd\right)\,c_n$$

$$2S = \left(2a+nd\right)(c_0+c_1+c_2+.....+c_n)$$

$$2S = \left(2a+nd\right)2^n$$

$\left[\because C_0 + C_1 + C_2 + ...C_n = 2^n\right]$

$\therefore$ S$=\left(2a+nd\right)2^{n-1}$

6. If n is a positive integer then prove that: $c_0 + \frac{c_1}{2} + \frac{c_2}{3} + ... + \frac{c_n}{n+1} = \frac{2^{n-1}-1}{n+1}$

Sol: Let S$=c_0 + \frac{c_1}{2} + \frac{c_2}{3} + ... + \frac{c_n}{n+1}$

$$S = n_{c_0} + \frac{1}{2}n_{c_1} + \frac{1}{3}n_{c_2} + ... + \frac{1}{n+1}n_{c_n}$$

$$(n+1)\,S = \frac{n+1}{1}n_{c_0} + \frac{n+1}{2}n_{c_1} + \frac{n+1}{3}n_{c_2} + ... + \frac{n+1}{n+1}n_{c_n}$$

$$= (n+1)_{C_1} + (n+1)_{C_2} + (n+1)_{C_3} + ... + (n+1)_{C_{(n+1)}} \quad \left[\because \frac{n+1}{r+1}n_{c_r} = (n+1)_{C_{r+1}}\right]$$

$$= [(n+1)_{C_0} + (n+1)_{C_1} + (n+1)_{C_2} + ... + (n+1)_{C_{(n+1)}}] - (n+1)_{c_{n+1}}$$

$$(n+1)S = 2^{n+1} - (n+1)_{C_0} = 2^{n+1} - 1$$

$$\therefore S = \frac{2^{n+1}-1}{n+1}$$

7. If n is a positive integer and x is any non-zero real number, then prove that:

$$_{C_0} + _{C_1}\frac{x}{2} + _{C_2}\frac{x^2}{3} + _{C_3}\frac{x^3}{4} + ... + _{C_n}\frac{x^n}{n+1} = \left(\frac{(1+x)^{n+1}-1}{(n+1)x}\right)$$

Sol: Let S $= {}_{C_0} + {}_{C_1}\frac{x}{2} + {}_{C_2}\frac{x^2}{3} + {}_{C_3}\frac{x^3}{4} + ... + {}_{C_n}\frac{x^n}{n+1}$

$$\Rightarrow S = n_{C_0} + n_{C_1}\frac{1}{2}x + n_{C_2}\frac{1}{3}x^2 + n_{C_3}\frac{1}{4}x^3 + ... + \frac{1}{n+1}n_{C_n}x^n$$

$$\Rightarrow (n+1)xS = \frac{n+1}{1}n_{c_0}x + \frac{n+1}{2}n_{c_1}x^2 + \frac{n+1}{3}n_{c_2}x^3 + ... + \frac{n+1}{n+1}n_{c_n}x^{n+1}$$

$$\Rightarrow (n+1)xS = n+1_{c_1}x + n+1_{c_2}x^2 + n+1_{c_3}x^3 + ... + (n+1)_{c_{(n+1)}}x^{n+1}$$

$$\Rightarrow (n+1)xS = \left[(n+1)_{c_0}x + (n+1)_{c_1}x + (n+1)_{c_2}x^2 + ... + (n+1)_{c_{n+1}}x^{n+1}\right] - (n+1)_{c_0}$$

$$\Rightarrow (n+1)xS = (1+x)^{n+1} - 1$$

$$\therefore S = \left(\frac{(1+x)^{n+1}-1}{(n+1)x}\right)$$

8. Prove that $2C_0 + 5C_1 + 8C_2 + ... + (3n+2)C_n = (3n+4)2^{n-1}$

Sol: Let S $= 2C_0 + 5C_1 + 8C_2 + ... + (3n+2)C_n$ -------------- (1)

$$S = (3n+2)C_n + (3n-1)C_{n-1} + (3n-4)C_{n-2} + ... + 2C_0$$

S $= (3n+2)C_0 + (3n-1)C_1 + (3n-4)C_2 + ... + 2C_n$ --------------- (2)

By adding (1) & (2), we get

$2S = (3n+4)C_0 + (3n+4)C_1 + ... + (3n+4)C_n$

$2S = (3n+4)\,(C_0 + C_1 + ... + C_n)$

$2S = (3n+4)\,2^n$

$2S = (3n+4)\,2^{n-1}.\,2$

$S = (3n+4)2^{n-1}$

9. Prove that $C_0 + 2C_1 + 4C_2 + 8C_3 + ... + 2^n C_n = 3^n$

Sol: Since $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + ... + C_nx^n$

Put $x = 2$

$$(1+2)^n = C_0 + C_1 2 + C_2 2^2 + C_3 2^3 + ... + C_n 2^n$$

$$3^n = C_0 + 2C_1 + 4C_2 + 8C_3 + ... + 2^n C_n$$

$$\therefore C_0 + 2C_1 + 4C_2 + 8C_3 + ... + 2^n C_n = 3^n$$

10. If $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n.}x^{2n}$ **, then prove that:**

i) $a_0 + a_1 + a_2 + ... + a_{2n} = 3^n$

ii) $a_0 + a_2 + a_4 + ... + a_{2n} = \dfrac{3^n + 1}{2}$

iii) $a_1 + a_3 + a_5 + ... + a_{2n-1} = \dfrac{3^n - 1}{2}$

iv) $a_0 + a_3 + a_6 + a_9 + ... = 3^{n-1}$

Sol: Let $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n.}x^{2n}$ ----------- (1)

i) Putting x=1, in (1) , we get

$(1+1+1^2) = a_0 + a_1.1 + a_2.1^2 + ... + a_{2n.}.1^{2n}$

$\therefore a_0 + a_1 + a_2 + ... + a_{2n} = 3^n$

ii) Since from (1), we have

$a_0 + a_2 + a_4 + ... + a_{2n} = 3^n$ ---------------- (1)

Putting x = - 1, in (1), we get

$a_0 - a_1 + a_2 - .... + a_{2n} = 1$ -------------- (2)

By adding (1) & (2) we get

$2a_0 + 2a_2 + ... + 2a_{2n} = 3^n + 1$

$2(a_0 + a_2 + a_4 + ... + a_{2n}) = 3^n + 1$

$\therefore a_0 + a_2 + a_4 + ... + a_{2n} = \dfrac{3^n + 1}{2}$

iii) By subtracting (2) from (1), we get

$2a_1 + 2a_3 + ... + 2a_{2n-1} = 3^n - 1$

$2(a_1 + a_3 + ... + a_{2n-1}) = 3^n - 1$

$\therefore a_1 + a_3 + a_5 + ... + a_{2n-1} = \dfrac{3^n - 1}{2}$

iv) Putting $x=1$, in (1) , we get

$$a_0 + a_2 + a_4 + ... + a_{2n} = 3^n \text{ ------------- (1)}$$

Putting $x=\omega$, in (1) , we get

$$a_0 + a_1\omega + a_2\omega + ... + a_{2n}\omega^{2n} = 0 \text{ ------------- (2)} \qquad \left(\because 1+\omega+\omega^2 = 0\right)$$

Putting $x = \omega^2$ in (1) , we get

$$a_0 + a_1\omega^2 + a_2\omega^4 + ... + a_{2n}\omega^{4n} = 0 \text{ ------------ (3)}$$

(1) + (2) + (3), we get

a_0 (1+1+1) + a_1 $(1+\omega+\omega^2)$ + a_2 $(1+\omega^2+\omega^4)$ + a_3 $(1+\omega^3+\omega^6)$ + …………+a_{2n} ($1+\omega^{2n}+\omega^{4n}$) = 3^n

$$\therefore a_0 + a_3 + a_6 + a_9 + ... = 3^{n-1}$$

11. If P and Q are the sum of odd terms and the sum of even terms respectively in the expansion of $(x+a)^n$ then prove that:

i) $P^2 - Q^2 = \left(x^2 - a^2\right)^n$

ii) $4PQ = (x+a)^{2n} - (x-a)^{2n}$

Sol.: $\left(x^2 - a^2\right)^n = n_{C_0}x^n + n_{C_1}x^{n-1} + n_{C_2}x^{n-2}a^2 + n_{C_3}x^{n-3}a^3 + ... + n_{C_{n-1}}xa^{n-1} + n_{C_n}a^n$

$= \left[n_{C_0}x^n + n_{C_2}x^{n-2}a^2 + ... + n_{C_n}a^n\right] + \left[n_{C_1}x^{n-1}a + n_{C_3}x^{n-3}a^3 + ... + n_{C_{n-1}}xa^{n-1}\right]$

= Sum of odd terms + Sum of even terms

$\left(x^2 - a^2\right)^n$ = P + Q

Where P = $n_{C_0}x^n + n_{C_2}x^{n-2}a^2 + ... + n_{C_n}a^n$

Q = $n_{C_1}x^{n-1}a + n_{C_3}x^{n-3}a^3 + ... + n_{C_{n-1}}xa^{n-1}$

$$\left(x^2 - a^2\right)^n = n_{C_0}x^n - n_{C_1}x^{n-1}a + n_{C_2}x^{n-2}a^2 - n_{C_3}x^{n-3}a^3 + ... + (-1)^n n_{C_n}a^n$$

i) $P^2 - Q^2$ = (P + Q) (P – Q)

$$= \left[n_{C_0}x^n + n_{C_2}x^{n-2}a^2 + ... + n_{C_1}x^{n-1}a + n_{C_3}x^{n-3}a^3 + ...\right]$$

$$\left[n_{C_0}x^n + n_{C_2}x^{n-2}a^2 + ... - n_{C_1}x^{n-1}a - n_{C_3}x^{n-3}a^3 - ......\right]$$

$$= \left[n_{C_0}x^n + n_{C_1}x^{n-1}a + n_{C_2}x^{n-2}a^2 + ...\right] - \left[n_{C_0}x^n - n_{C_1}x^{n-1}a + n_{C_2}x^{n-2}a^2 - ....\right]$$

$$= (x+a)^n (x-a)^n$$

$$P^2 - Q^2 = \left((x+a)(x-a)\right)^n$$

$$\therefore P^2 - Q^2 = \left(x^2 - a^2\right)^n$$

ii) $4PQ = (P+Q)^2 - (P-Q)^2$

$$= \left[n_{C_0} x^n + n_{C_1} x^{n-1} a + n_{C_2} x^{n-2} a^2 + \ldots\ldots\ldots\right]^2 -$$
$$\left[n_{C_0} x^n - n_{C_1} x^{n-1} a + n_{C_2} x^{n-2} a^2 - \ldots\ldots\right]^2$$

$$= \left[(x+a)^n\right]^2 - \left[(x-a)^n\right]^2$$

$\therefore$ 4PQ $= (x+a)^{2n} - (x-a)^{2n}$

12. If $\left(1+3x-2x^2\right)^{10}$ = $a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{20}x^{20}$, then prove that

i) $a_0 + a_1 + a_2 + \ldots\ldots + a_{20} = 2^{10}$

ii) $a_0 - a_1 + a_2 - a_3 + \ldots\ldots\ldots + a_{20} = 4^{10}$

Sol.: Let $\left(1+3x-2x^2\right)^{10}$ = $a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{20}x^{20}$ -------------- (1)

i) Putting $x = 1$ in (1) , we get

$(1+3-2)^{10}$ = $a_0 + a_1 + a_2 + \ldots\ldots + a_{20}$

$\therefore$ $a_0 + a_1 + a_2 + \ldots\ldots + a_{20} = 2^{10}$

ii) Putting $x =$ -1 in (1) , we get

$(1-3-2)^{10}$ = $a_0 - a_1 + a_2 - \ldots\ldots + a_{20}$

$(-4)^{10}$ = $a_0 - a_1 + a_2 - \ldots\ldots + a_{20}$

$\therefore a_0 - a_1 + a_2 - a_3 + \ldots\ldots\ldots + a_{20} = 4^{10}$

13. Show that the middle term in the expansion (1+ x)^{2n} is $\dfrac{1.3.5\ldots\ldots..(2n-1)}{n\,!}2nx^n$**.**

Sol.: The middle term in the expansion $(1+x)^{2n}$ is $\left(\dfrac{2n}{2}+1\right)$ i.e (n+1)th term.

$$T_{n+1} = 2n_{c_n} x^n$$

$$= \frac{(2n)!}{(2n-n)!n!} x^n$$

$$= \frac{(2n)!}{n!n!} x^n$$

$$= \frac{2n(2n-1)(2n-2).........4.3.2.1}{n!\text{n}!}x^n$$

$$= \frac{\left[2n(2n-2)......4.2\right]\left[(2n-1)(2n-3).....3.1\right]}{n!\text{n}!}x^n$$

$$= \frac{2^n\left[n(2n-2)(n-2)..........2.1\right]\left[(2n-1)(2n-3).....3.1\right]}{n!\text{n}!}x^n$$

$$= \frac{2^n\text{n}!\ \left[(2n-1)(2n-3).....3.1\right]}{n!\text{n}!}x^n$$

$$= \frac{2^n\left[(2n-1)(2n-3).....3.1\right]}{n!}x^n$$

Middle term $T_{n+1} = \dfrac{1.3.5....(2n-1)}{n!}(2x)^n$

14 If the second, third and fourth terms in the binomial expansion (a+x)^n are 240,720 and 1080 respectively. Find the values of x, a and n.

Sol.: In the given expansion (a+x) ^{n}

$T_2 = 240 \rightarrow T_{1+1} = 240$

$\Rightarrow n_{c_1}a^{n-1}x = 240$ ----------------------- (1)

$T_3 = 720 \rightarrow T_{2+1} = 720$

$\Rightarrow n_{c_2}a^{n-2}x^2 = 720$--------------------- (2)

And

$T_3 = 1080 \rightarrow T_{3+1} = 1080$

$\Rightarrow n_{c_3}a^{n-3}x^3 = 1080$------------------- (3)

$$\frac{(2)}{(1)} \rightarrow \frac{n_{c_2}a^{n-2}x^2}{n_{c_1}a^{n-1}x} = \frac{(720)}{(240)} \rightarrow \frac{\frac{n(n-1)}{2}a^{n-1}a^{-1}xx}{na^{n-1}x} = 3$$

$\Rightarrow \dfrac{n(n-1)}{2} = 3 \rightarrow$(n-1) x = 6a ----------- (4)

$$\frac{(3)}{(2)} \rightarrow \frac{n_{c_3}a^{n-3}x^3}{n_{c_2}a^{n-2}x^2} = \frac{(1080)}{(720)} \Rightarrow \frac{\frac{n(n-1)(n-2}{6}a^{n-2}a^{-1}x^2x}{\frac{n(n-1)}{2}a^{n-2}x^2} = \frac{3}{2}$$

$$\Rightarrow \frac{(n-2)x}{3a} = \frac{3}{2}$$

$\Rightarrow\ 2(n-2)x = 9a$ ---------------- (5)

$$\frac{(5)}{(4)} \Rightarrow \frac{2(n-2)x}{(n-1)x} = \frac{9a}{6a}$$

$\Rightarrow$2(2n-4) = 3(n-3)$\Rightarrow$4n - 8 = 3n – 3 $\Rightarrow$n = 5

Put n = 5 in (4), we get 4x = 6a

$$\Rightarrow \frac{x}{3} = \frac{a}{2}$$

Put $x = 3$, a = 2, n = 5 in (1)

$$5_{c_1} 2^{5-1} 3 = 240$$

$$5 \times 16 \times 3 = 240$$

$$240 = 240$$

$$\therefore n = 5,\ x = 3,\ a = 2$$

8. SEQUENCES AND SERIES

Exercise 8(a)

1. Write the first five terms of the sequence whose n^{th} term is a_n = n (n+2)

Sol.: Given that a_n = n (n+2)

Substituting n = 1, 2, 3, 4 and 5 we have

$a_1 = 1\ (1+2) = 3$, $a_2 = 2\ (2+2) = 8$, $a_3 = 3(3+2) = 15$,

$a_4 = 4\ (4+2) = 24$, $a_5 = 5\ (5+2) = 35$

Hence the required terms are 3, 8, 15, 24, and 35.

2. **Write the first five terms of the sequence whose n^{th} term is $a_n = \frac{n}{n+1}$**

Sol.: Given that $a_n = \frac{n}{n+1}$

Substituting n = 1, 2, 3, 4 and 5 we have

$a_1 = \frac{1}{1+1} = \frac{1}{2}$, $a_2 = \frac{2}{2+1} = \frac{2}{3}$, $a_3 = \frac{3}{3+1} = \frac{3}{4}$,

$a_4 = \frac{4}{4+1} = \frac{4}{5}$, $a_5 = \frac{5}{5+1} = \frac{5}{6}$

Hence the required terms are $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\ and\ \frac{5}{6}$

3. Write the first five terms of the sequence whose n^{th} term is $a_n = 2^n$.

Sol.: Given that $a_n = 2^n$

Substituting n = 1, 2, 3, 4 and 5 we have

$a_1 = 2^1 = 2$, $a_2 = 2^2 = 4$, $a_3 = 2^3 = 8$, $a_4 = 2^4 = 16$, $a_5 = 2^5 = 32$

Hence the required terms are 2, 4, 8, 16 and 32.

4. Write the first five terms of the sequence whose n^{th} term is $a_n = \frac{2n-3}{6}$

Sol.: Given that $a_n = \frac{2n-3}{6}$, Substituting n = 1, 2, 3, 4 and 5 we have

$a_1 = -\frac{1}{6}$, $a_2 = \frac{1}{6}$, $a_3 = \frac{1}{2}$, $a_4 = \frac{5}{6}$, $a_5 = \frac{7}{6}$

Hence the required terms are $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}\ and\ \frac{7}{6}$

5. **Write the first five terms of the sequence whose n^{th} term is $a_n = (-1)^{n-1}\ 5^{n+1}$**

Sol.: Given that $a_n = (-1)^{n-1}\ 5^{n+1}$, Substituting n = 1, 2, 3, 4 and 5 we have

$a_1 = (-1)^{1-1}\ 5^{\ 1+1} = 5^2 = 25$

$a_2 = (-1)^{2-1}\ 5^{\ 1+1} = -\ 5^3 = -\ 125$

$a_3 = (-1)^{3-1}\ 5^{\ 1+1} = 5^4 = 625$

$a_4 = (-1)^{4-1}\ 5^{\ 1+1} = -5^5 = -\ 3125$

$a_5 = (-1)^{5-1}\ 5^{\ 1+1} = 5^6 = 15625$

Hence the required terms are 25, -125, 625, -3125 and 15625.

6. Write the first five terms of the sequence whose nth term is $a_n = n\left(\dfrac{n^2+5}{4}\right)$

Sol.: Given that $a_n = n\left(\dfrac{n^2+5}{4}\right)$, Substituting n = 1, 2, 3, 4 and 5 we have

$a_1 = \dfrac{3}{2}$, $a_2 = \dfrac{9}{2}$, $a_3 = \dfrac{21}{2}$, $a_4 = 21$, $a_5 = \dfrac{75}{2}$

Hence the required terms are $\dfrac{3}{2}, \dfrac{9}{2}, \dfrac{21}{2}, 21\ and\ \dfrac{75}{2}$

7. Find the indicated terms of the sequence whose nth term is a_n **= 4n – 3 ; a_{17}, a_{24}.**

Sol.: Given that a_n = 4n – 3, Substituting n = 17 , we have

$a_{17} = 4(17) - 3 = 68 - 3 = 65$

Substituting n = 24 we have

$a_{24} = 4(24) - 3 = 96 - 3 = 93$

8. Find the indicated terms of the sequence whose nth term is $a_n = \dfrac{n^2}{2^n}$ **;** a_7

Sol.: Given that $a_n = \dfrac{n^2}{2^n}$, Substituting $n = 7$ we have

$a_7 = \dfrac{7^2}{2^7} = \dfrac{49}{128}$

9. Find the indicated terms of the sequence whose nth term is a_n **= (-1)$^{n-1}$** $n^3; a_9$

Sol.: Given that a_n = $(-1)^{n-1}\ n^3$, Substituting n = 9 we have , $a_9 = (-1)^{9-1}\, 9^3 = 9^3 = 729$

10. Find the indicated terms of the sequence whose nth term is $a_n = \dfrac{n(n-2)}{n+3}; a_{20}$

Sol.: Given that $a_n = \dfrac{n(n-2)}{n+3}$, Substituting n = 20, we have

$$a_{20} = \frac{20(20-2)}{20+3} = \frac{20x18}{23} = \frac{360}{23}$$

II. 1. Write the first five terms of the sequence and obtain the corresponding series :

$a_1 = 3a_n = 3a_{n-1} + 2$ **for all** $n > 1$.

Sol.: Given that $a_1 = 3$, $a_n = 3\,a_{n-1} + 2$ for all n > 1 , Substituting n = 2, 3, 4 and 5, we have

$$a_2 = 3\,a_{2-1} + 2 = 3\,a_1 + 2 = 3 = 3 \times 3 + 2 = 11$$
$$a_3 = 3\,a_{3-1} + 2 = 3\,a_2 + 2 = 3 = 3 \times 11 + 2 = 35$$
$$a_4 = 3\,a_{4-1} + 2 = 3\,a_3 + 2 = 3 = 3 \times 35 + 2 = 107$$
$$a_5 = 3\,a_{5-1} + 2 = 3\,a_4 + 2 = 3 = 3 \times 107 + 2 = 323$$

Therefore the required terms are 3, 11, 35, 107 and 323.

Hence the corresponding series is 3 + 11+ 35 + 107 + 323 + ……

2. Write the first five terms of the sequence and obtain the corresponding series :

a_1 **= -1,** $\qquad a_n = \frac{a_{n-1}}{n}; n \geq 2$

Sol.: Given that a_1 = -1, $a_n = \frac{a_{n-1}}{n}; n \geq 2$, Substituting n = 2, 3, 4 and 5, we have

$$a_2 = \frac{a_{2-1}}{2} = \frac{a_1}{2} = -\frac{1}{2}, \qquad a_3 = \frac{a_{3-1}}{3} = \frac{a_2}{3} = \frac{-\frac{1}{2}}{3} = -\frac{1}{6}$$

$$a_4 = \frac{a_{4-1}}{4} = \frac{a_3}{4} = \frac{\frac{-1}{6}}{4} = -\frac{1}{24}, \quad a_5 = \frac{a_{5-1}}{5} = \frac{a_4}{5} = \frac{-\frac{1}{24}}{5} = -\frac{1}{120}$$

Hence the required terms are -1, $-\frac{1}{2}$, $-\frac{1}{6}$, - $\frac{1}{24}$ and - $\frac{1}{120}$

Hence the corresponding series is 1, $-\frac{1}{2}$, $\frac{1}{6}$, - $\frac{1}{24}$ and - $\frac{1}{120}$….

3. Write the first five terms of the sequence and obtain the corresponding series :

a_1 **=** a_2 **= 2,** $\quad a_n$ **=** a_{n-1} **-1 , n > 2**

Sol.: Given that $a_1 = a_2 = 2$, $a_n = a_{n-1}$ -1 , n > 2, Substituting n = 3, 4 and 5, we have

$a_3 = a_{3-1}$ -1 = a_2 -1 = 2 – 1 = 1 , $a_4 = a_{4-1}$ -1 = a_3 -1 = 1 – 1 = 0, $a_5 = a_{5-1}$ -1 = a_4 - 1= 0 –1 = -1

Hence the required terms are 2, 2, 1, 0 and -1.

Hence the corresponding series is 2 + 2 + 1 + 0 + (-1)…..

4. The 4th term of a **G. P. is square of its second term, and the first term is -3. Determine its 7th term .**

Sol.: Let a be the first term and r be the common ratio of the G. P. .

$\therefore$ a = - 3.

We know that, $a_n = a\,r^{n-1}$, $\therefore$ $□^4 = a\,r^3 = (-3)\,r^3$ and $a^2 = a\,r^1 = (-3)r$

According to the question, $[-3)\,r^3 = [\,(-3)\,r]^2 \Rightarrow -3r^3 = 9r^2 \Rightarrow$ r = - 3.

Therefore , $a_7 = a\,r^{7-1} = a\,r^6 = (-3)\,(-3)^6 = (-3)^7 = -2187$

Hence, the seventh term of the G. P is -2187.

5. Which term of the following sequences :

(Sol.: 2, 2 $\sqrt{2}$,4 ………..is 128 **?**

(b)$\sqrt{3}$, 3, 3 $\sqrt{3}$….. is 729 ?

(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ …… is $\frac{1}{19683}$ **?**

Sol.: (a) 2, 2 $\sqrt{2}$, 4……….

Here a = 2 and $r = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Let $a_n = 128 \Rightarrow a\,r^{n-1} = 128 \Rightarrow 2\,(\sqrt{2})^{n-1} = 2^7 \Rightarrow (\sqrt{2})^{n-1} = 2^6$

$\Rightarrow 2^{\frac{n-1}{2}} = 2^6 \Rightarrow \frac{□-1}{2} = 6 \quad \Rightarrow n = 13$

Hence the 13th term of the given sequence is 128.

(b) $\sqrt{3}$, 3, 3 $\sqrt{3}$…..

Here $a = \sqrt{3}$ and $r = \frac{3}{\sqrt{3}} = \sqrt{3}$

Let $a_n = 729 \Rightarrow a\,r^{n-1} = 729 \Rightarrow \sqrt{3}\,(\sqrt{3})^{n-1} = 3^6 \Rightarrow (\sqrt{3})^n = 3^6 \Rightarrow (3)^{\frac{□}{2}} = 3^6$

$\Rightarrow \frac{□}{2} = 6 \quad \Rightarrow n = 12.$

Hence the 12th term of the given sequence is 729.

(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ …..

Here $a = \frac{1}{3}$ and $r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{3}{9} = \frac{1}{3}$

Let $a_n = \frac{1}{19683} \Rightarrow a\,r^{n-1} = \frac{1}{19683} \Rightarrow \frac{1}{3}\left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^9 \Rightarrow \left(\frac{1}{3}\right)^n = \left(\frac{1}{3}\right)^9 \Rightarrow n = 9$

Hence, the 9th terms of the given sequence is $\frac{1}{19683}$.

6. For what values of x, the numbers $-\frac{2}{7}$, x, $-\frac{7}{2}$ are in G.P ?

Answer : Here, $-\frac{2}{7}$, x, $-\frac{7}{2}$ are in G. P.

$$\Rightarrow \frac{x}{\left(\frac{-2}{7}\right)} = \frac{\left(\frac{-7}{2}\right)}{x} \Rightarrow x^2 = \frac{2}{7} \times \frac{7}{2} = 1 \quad \Rightarrow x = \pm 1$$

Hence , for $x = \pm 1$, the given numbers will be in G. P.

II. 1. Find the sum to indicated number of terms in the following geometric progressions :

0.15,0.015,0.0015 ……. 20 terms.

Sol.: Here a = 0.15 and r = $\frac{0.015}{0.15}$ = 0.1 and $S_n = \frac{a(1-r^n)}{1-r}$

$S_{20} = \frac{0.15(1-(0.1)^{20})}{1-0.1} = \frac{0.15(1-(0.1)^{20})}{0.9} = \frac{1}{6}[1-(0.1)^{20}]$

2. Find the sum to indicated number of terms in the following geometric progressions :

$\sqrt{7}, \sqrt{21}, 3\sqrt{7}$…… n terms

Sol Here, a = $\sqrt{7}$and $r = \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3} = \sqrt{3}$ and $S_n = \frac{a(r^n-1)}{r-1}$

$$\Rightarrow S_n = \frac{\sqrt{7}\,[(\sqrt{3})^n - 1}{\sqrt{3}-1} = \frac{\sqrt{7}\,[(\sqrt{3})^n - 1}{\sqrt{3}-1} \text{ x } \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{\sqrt{7}[(\sqrt{3}^n)-1}{\sqrt{3}-1} \text{ x } \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{7}(\sqrt{3}+1)[3^{\frac{n}{2}}-1]}{2}$$

3. Find the sum to indicated number of terms in the following geometric progressions :

1, $-a$, a^2, $-a^3$ ………..n terms (if a $\neq$-1)

Sol.: Here a = 1 and r = $\frac{-a}{1} = -a$

$S_n = \frac{a(r^n-1)}{r-1}$

$\Rightarrow S_n = \frac{1[1-)(-a)^n-1)}{1-(-a)} = \frac{1-(-a^n)}{1+a}$

4. Find the sum to indicated number of terms in the following geometric progressions :

x^3, x^5, x^7 ……………n terms (if $x \neq \pm 1$)

Sol.: Here a = x^3 and r = $\frac{x^5}{x^3} = x^2$

$S_n = \frac{a(r^n-1)}{r-1} \Rightarrow S_n = \frac{x^3[(x^2)^n-1]}{x^2-1} = \frac{x^3[x^{2n}-1]}{x^2-1}$

5. Evaluate $\sum_{k=1}^{11}(2+3^k)$

Sol.: $\sum_{k=1}^{11}(2+3^k) = (2+3^1)+(2+3^2)+(2+3^3)\ .....(2+3^{11})$

$= 2\times 11 + (3^1+3^2+3^3+ \ldots\ldots\ldots\ldots + 3^{11}) = 22 + \frac{3(3^{11}-1)}{3-1} = 22 + \frac{3}{2}(3^{11}-1)$

6. The sum of first three terms of a G. P is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms

Sol.: Let a, ar and ar^2 be the three terms of GP.

According to question, sum of first three terms $a + ar\ ar^2 = \frac{39}{10}$ ....(1)

Product of terms a x ar x $ar^2 = 1 \Rightarrow a^3r^3 = 1$

$\Rightarrow ar = 1$

Dividing equation (2) by (1) we have

$\frac{a+ar+a^2}{ar} = \frac{39}{10}$

$\Rightarrow \frac{1+r+r^2}{r} = \frac{39}{10} \Rightarrow 10 + 10r + 10r^2 = 39r$

$\Rightarrow 10r^2 - 29r + 10 = 0 \Rightarrow (5r-2)(2r-5) = 0$

$\Rightarrow r = \frac{2}{5}$ or $\frac{5}{2}$

If $r = \frac{2}{5}$ from (1), we have a $x\ \frac{2}{5} = 1$

$\Rightarrow$ a $= \frac{5}{2}$

Therefore, the three terms a, ar and ar^2 are given $\frac{5}{2}, \frac{5}{2}\ x\ \frac{2}{5}$ and $\frac{5}{2}\ x\ \left(\frac{2}{5}\right)^2$

$\Rightarrow \frac{5}{2}$, 1 and $\frac{2}{5}$

If $r = \frac{5}{2}$ from (1) we have a x $\frac{5}{2} = 1 \Rightarrow$ a $= \frac{2}{5}$

Therefore the three terms a ar and ar^2 are given by $\frac{2}{5}, \frac{2}{5}\ x\ \frac{5}{2}$ and $\frac{2}{5}\ x\ (\frac{5^2}{2})$

$\Rightarrow \frac{2}{5}$, 1 and $\frac{5}{2}$

7. How many terms of G. P . 3, 3^2, 3^3.......are needed to give the sum 120 ?

Sol.: The given G. P is 3, 3^2, 3^3 ....

Here a = 3 and r = $\frac{3^2}{3}$ = 3

Let n terms of this G. P be required to obtain the sum as 120.

$S_n = \frac{3\,[3^n - 1]}{3-1} = 120 = 120 \Rightarrow 3\,(3^n - 1) = 240 \Rightarrow 3^n - 1 = 80 \Rightarrow 3^n = 3^4 \Rightarrow n = 4$

Hence, four terms of the given G. P. are required to obtain the sum as 120.

8. The sum of first three terms of a G. P is 16 and the sum of the next three terms is 128. Determine the first term the common ratio and the sum of n terms of the G. P.

Sol.: Let the G. P be a, ar, ar^2, ar^3 ……………

The sum of the first three terms of a G. P is 16, therefore a + ar + ar^2 = 16

$\Rightarrow a\,(1 + r + r^2) = 16$ ……………(1)

The sum of the next three terms is 128, so $ar^3 + ar^4 + ar^5 = 128$

$\Rightarrow ar^3\,(1 + r + r^2) = 128$ ……………..(2)

Dividing equation (2) by (1) we obtain

$\frac{ar^3[1+r+r^2]}{a[1+r+r^2} = \frac{128}{16} \Rightarrow r^3 = 8 \Rightarrow r = 2$

Putting the value of r in equation (1), we have

$a\,(1 + 2 + 2^2) = 16 \Rightarrow 7a = 16 \Rightarrow a = \frac{16}{7}$

Now $S_n = \frac{a\,(r^n - 1)}{r-1} \Rightarrow S_n = \frac{\frac{16}{7}[2^n - 1]}{2-1} = \frac{16}{7}(2^n - 1)$

9. Given a G. P with a = 729 and 7th term 64, determine S_7.

Sol.: Given that a = 729 and 7th term 64.

$\Rightarrow a_7 = ar^{7-1} = 64$

$\Rightarrow 729\,r^6 = 64 \Rightarrow r^6 = \frac{64}{729} \Rightarrow r^6 = (\frac{2}{3})^6$

$\Rightarrow r = \frac{2}{3}$

Now $S_n = \frac{a(1 - r^n)}{1-r}$ $\quad S_n = \frac{729[1 - \left(\frac{2}{3}\right)^7}{1 - \frac{2}{3}} = \frac{729\left[1 - \left(\frac{2}{3}\right)^7\right]}{\frac{1}{3}} = 729 \text{ x } 3 = \left[1 - \frac{128}{2187}\right]$

$= 2187\left[\frac{2187 - 128}{2187}\right] = 2187 - 128 = 2059$

10.Find a G. P for which sum of the first two terms is -4 and the fifth term is 4 times, the third term.

Sol.: : Let the first term of the given G. P is a and the common ratio is r.

Sum of the first two terms is -4 , therefore $a_1 + a_2 = -4$

$\Rightarrow$ a + ar = - 4 ....(1)

The fifth term is 4 times the third term, therefore $a_5 = 4a_3$

$\Rightarrow ar^4 = 4 \text{ x } ar^2 \Rightarrow r^2 = 4 \Rightarrow r^2 = \pm 2$

If r = 2 from (1), we have a + a x 2 = - 4

$\Rightarrow$ 3a = - 4

$\Rightarrow a = -\frac{4}{3}$

Therefore, the required GP a, ar, ar^2 ....is given by $-\frac{4}{3}$, $- -\frac{4}{3}$ x 2, $-\frac{4}{3}$ x 2^2 .... or $\frac{4}{3}$, $-\frac{8}{3}$, $-\frac{16.}{3}$............

If r = -2, from (1) we have a + a x (-2) = - 4

$\Rightarrow$ a = - 4 $\Rightarrow$ a = 4.

Therefore the required GP a, ar, ar^2 .... is given by 4, 4 x (-2) x 4 x (-2)3 ....or 4, -8, 16.........

11.If the 4th, 10th and 16th terms of a G.P are x , y and z respectively. Prove that x, y, z are in G.P

Sol.: According to the given condition ,

$a_4 = ar^3 = x$ ... (1),$a_{10} = ar^9 = y$ ...(2) ,$a_{16} = ar^{15} = z$ ....(3)

Dividing (2) by (1) we have

$\frac{ar^9}{ar^3} = \frac{y}{x} \Rightarrow r^6 = \frac{y}{x}$ ....(4)

Dividing (3) by (2) we have

$\frac{ar^{15}}{ar^9} = \frac{z}{y} \Rightarrow r^6 = \frac{z}{y}$ ....(5)

From (4) and (5) we have

$\frac{y}{x} = \frac{z}{y} \Rightarrow$ x, y, z are in G. P

12.Find the sum to n terms of the sequence 8, 88, 888, 8888............

Sol.: The given sequence is 8, 88, 888, 8888.......

This sequence is not a G. P , However, it can be changed to G. P by writing the terms as

S_n = 8, 88, 888, 8888……. n terms

$\Rightarrow \frac{8}{9}$ [9 + 99+ 999+ 9999 + …….to n terms]

$\Rightarrow \frac{8}{9}$ [(10-1) + (100-1)+ (1000-1)+ (10000-1)+ …….to n terms]

$\Rightarrow \frac{8}{9}[(10^1 + 10^2 + 10^3 + 10^4 + \ldots\ldots\ldots + 10^n) - (1 + 1 + 1 + 1 + \ldots$ to n terms$)]$

$\Rightarrow \frac{8}{9}[(10^1 + 10^2 + 10^3 + 10^4 + \ldots\ldots\ldots + 10^n) - n]$

$\Rightarrow \frac{8}{9}\left[\frac{10\,(10^n - 1)}{10-1} - n\right] = \frac{8}{9}\left[\frac{10\,(10^n-1)}{9}\right] - \frac{8}{9}n = \frac{80}{81}(10^n - 1) - \frac{8}{9}n$

13.Find the sum of the products of the corresponding terms of the sequence 2, 4, 8, 16, 32 and 128, 32, 8, 2, $\frac{1}{2}$

Sol.: The required sum of the product : $2 \times 128 + 4 \times 32 + 8 \times 8 + 16 \times 2 + 32 \times \frac{1}{2}$

= 256 + 128 + 64 + 32 + 16 which is a GP with first term 256 and the common ratio $\frac{1}{2}$

Now $S_n = \frac{a\,(1 - r^n)}{1 - r}$

$$= S_5 = \frac{256[1 - \left(\frac{1}{2}\right)^5}{1 - \frac{1}{2}} = \frac{256[1 - \frac{1}{32}]}{\frac{1}{2}} - 512\left[\frac{32-1}{32}\right] = 16 \times 31 = 496$$

14.Show that the products of the corresponding terms of the sequences a, ar, ar^2, ….ar^{n-1} and A, AR, AR^2………. AR^{n-1} form a G. P and find the common ratio.

Sol.: The required product of two series $a \times A + ar \times AR + ar^2 \times AR^2 + \ldots + ar^{n-1}$

$\times AR^{n-1}$

$= aA + aArR + aAr^2R^2 + \ldots\ldots\ldots + aAr^{n-1}R^{n-1}$, which form a GP as the common ratio is same.

Common ratio $= \frac{Second\ term}{First\ Term} = \frac{aArR}{aA} = r$R

III.

1. Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the 4th by 18.

Sol.: Let a be the first term and r be the common ratio of the G.P.

$a_1 = a$, $a_2 = ar$, $a_3 = ar^2$, $a_4 = ar^3$

By the given condition,

$a_3 = a_1 + 9 \Rightarrow a\,r^2 = a + 9 \Rightarrow a\,(r^2 - 1) = 9$ ....(1)

and $a_2 = a_4 + 18 \Rightarrow a\,r = a\,r^3 + 18 \Rightarrow a\,r\,(1 - r^2) = 18$.........(2)

From (1) and (2) we $\frac{ar\,(1-r^2)}{a(r^2-1)} = \frac{18}{9} \Rightarrow r = -2$

Substituting the value of r in (1) we obtain, $4a = a + 9 \Rightarrow 3a = 9 \Rightarrow a = 3$

Hence the first four numbers of the G. P are 3, 3 (-2), $3\,(-2)^2$ and $3\,(-2)^3$ i.e., 3, -6, 12 and -24.

2. If the p^{th}, q^{th} and r^{th} terms of a G. P are a, b and c, respectively. Prove that $a^{q-r}\,b^{r-p}\,c^{p-q} = 1$

Sol.: Let A be the first term and R be the common ratio of the G. P

$a_p = AR^{p-1} = a$ ....(1), $a_q = AR^{q-1} = b$ ....(2), $a_r = AR^{r-1} = c$ .....(3)

Now, LHS = $a^{q-r}\,b^{r-p}\,c^{p-q} = (AR^{p-1})^{q-r}\left(AR^{q-1}\right)^{r-p}\left(AR^{r-1}\right)^{p-q}$

$$= A^{q-r} R^{(p-1)(q-r)} A^{r-p} R^{(q-1)(r-p)} A^{(p-q)} R^{(r-1)(p-q)}$$

$$= (A^{q-r+r-p+p-q}) \times (R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)+(r-1)(p-q)})$$

$$= A^0 R^0 = 1 = \text{RHS}.$$

3. If the first and the n^{th} term of a GP are *a and b* respectively and if P is the product of *n* terms, prove that $p^2 = (ab)^n$

Sol.: Here, a is the first term . Let the common ratio be r .

$b = a\,r^{n-1}$

Now $P = a \times ar \times ar^2 \times \ldots\ldots \times ar^{n-1} = a^n \times (r.\,r^2,\,r^3 \ldots .r^{n-1}) = a^n\,r^{1+2+3+\ldots\ldots+(n-1)}$

$$= a^n r^{\frac{n-1}{2}[2\times1+(n-1-1)1]}$$

$$\Rightarrow P = a^n r^{\frac{(n-1)n}{2}}$$

Therefore, $p^2 = \left[a^n r^{\frac{(n-1)n}{2}}\right]^2 = \left[a^2 r^{n-1}\right]^n = \left[a.a\,r^{n-1}\right]^n = \left[ab\right]^n$

4. Show that the ratio of the sum of first n terms of a G. P to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{\square}{\square^{\square}}$.

Sol.: Let a be the first term and the common ratio be r.

Now, the ratio of the sum of first n terms of a G. P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is given by

$$\frac{S_n}{S_{2n}-S_n} = \frac{\frac{a(r^n-1)}{r-1}}{\frac{a(r^{2n}-1)}{r-1} - \frac{a(r^n-1)}{r-1}}$$

$= \frac{r^n - 1}{(r^{2n}-1)-(r^n-1)} = \frac{r^n-1}{r^{2n}-r^n} \quad = \frac{r^n-1}{r^n(r^n-1)} \quad = \frac{1}{r^n}$

5. If a, b, c and d are in G. P. show that (a^2 + b^2 + c^2) (b^2 + c^2 +d^2) = (ab + bc+cd)^2.

Sol.: Let the common ratio be r.

According to the question, b = ar, c= ar^2 and d = ar^3

LHS = (a^2 + b^2 + c^2) (b^2 + c^2 + d^2) = [a^2 + (ar)^2 + (ar)$^{2\ 2}$] [(ar)^2 + (ar^2)^2 + (ar^3)^2]

$$= a^2\left[1+r^2+r^4\right]a^2r^2\left[1+r^2+r^4\right]$$

$$= a^4r^2\left[1+r^2+r^4\right]^2 \text{ ....(1)}$$

RHS = (ab+ bc + cd)^2 = [a(ar) + (ar) (ar)^2)+ (ar)^2 + (ar^2)^2 + (ar^3)^2]

= [a^2 r (1+ r^2 + r^4] ^{2} = a^4 r^2[1+ r^2 + r^4]^2 ....(2)

From (1) and (2), we get (a^2 + b^2 + c^2) (b^2 + c^2 + d^2) = (ab + bc+ cd)^2

6. Insert two number between 3 and 81 so that the resulting sequence is G. P.

Sol.: Let G_1 and G_2 be the two numbers between 3 and 81 such that the series, 3, G_1, G_2 , 81 forms a G. P.

Let a be the first term and r be the common ratio of the G. P.

Given that 81 = (3) (r)^3 $\Rightarrow$ r^3 = 27 $\Rightarrow$ r = 3 (Taking real roots only)

For $r = 3$, G_1 = ar = (3) (3) = 9 and G_2 = ar^2 = (3) (3)^2 = 27

Therefore the required two numbers are 9 and 27.

7.Find the value of *n* so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between *a* and *b*.

Sol.: **Given that**, $\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab}$

$$\Rightarrow a^{n+1}+b^{n+1} = a^{\frac{1}{2}}b^{\frac{1}{2}}\left(a^n+b^n\right) \Rightarrow a^{n+1} - a^{n+\frac{1}{2}}b^{\frac{1}{2}} + b^{n+1} - a^{\frac{1}{2}}b^{n+\frac{1}{2}} = 0$$

$$\Rightarrow \left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right)\left(a^{n+\frac{1}{2}} - b^{n+\frac{1}{2}}\right) = 0 \Rightarrow \left(a^{n+\frac{1}{2}} - b^{n+\frac{1}{2}}\right) = 0$$

$$\Rightarrow \frac{a^{n+\frac{1}{2}}}{b^{n+\frac{1}{2}}} = 1 \Rightarrow \left(\frac{a}{b}\right)^{n+\frac{1}{2}} = 1 = \left(\frac{a}{b}\right)^0$$

$$\therefore n = -\frac{1}{2}$$

8.The sum of two numbers is 6 times their geometric means, show that numbers are in the ratio of $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

Sol.: Let a and b are the two numbers whose geometric means is $\sqrt{ab}$.

Given that, $a+b=6\sqrt{ab}$ ....(1)

$\Rightarrow (a + b)^2 = 36ab \;\Rightarrow\; a^2 + b^2 + 2ab = 36ab \;\Rightarrow\; a^2 + b^2 - 2ab = 32ab$

$\Rightarrow (a - b)^2 = 32ab \Rightarrow a - b = 4\sqrt{2}\sqrt{ab}$ ....(2)

From (1) and (2) we get $2a = 6\sqrt{ab} + 4\sqrt{2}\sqrt{ab} \;\Rightarrow a = 2\left(3 + 2\sqrt{2}\right)\sqrt{ab}$

And $2b = 6\sqrt{ab} + 4\sqrt{2\sqrt{ab}} \;\Rightarrow\; b = 2(3 - 2\sqrt{2}))\sqrt{ab}$

Therefore $\dfrac{a}{b} = \dfrac{2\left(3+2\sqrt{2}\right)\sqrt{ab}}{2\left(3-2\sqrt{2}\right)\sqrt{ab}} = \dfrac{3+2\sqrt{2}}{3-2\sqrt{2}}$.

9.If A and G be A. M and G. M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.

Sol.: It is given that A and G are A. M and G. M between two positive numbers.

Let the two positive numbers be a and b.

$AM = A = \frac{a+b}{2} \;\Rightarrow\; a + b = 2A$ ....(1)

$GM = G = \sqrt{ab} \;\Rightarrow\; ab = G^2$ ....(2)

We know that $(a-b)^2 = (a + b)^2 - 4ab$

$\Rightarrow (a- b)^2 = 4A^2 - 4G^2 = 4(A^2 - G^2) = 4(A- G)(A + G)$

$\Rightarrow a - b = 2\sqrt{(A-G)(A+G)}$ ....(3)

From (1) and (2) we get $2a = 2A + 2\sqrt{(A-G)(A+G)} \Rightarrow a = A + \sqrt{(A-G)(A+G)}$

Substituting the value of a in (1) we get $b = A - \sqrt{(A-G)(A+G)}$

Therefore the two numbers a and b are $A \pm \sqrt{(A-G)(A+G)}$

10. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour , 4th hour and nth hour. ?

Sol.: It is given that the number of bacterial doubles every hour.
Therefore, the number of bacteria after every hour will form a G. P.
Here $a = 30$ and $r = 2$.

The number of bacteria at the end of 2nd hour $(a_3) = ar^2 = (30)(2)^2 = 120$

The number of bacteria at the end of the 4th hour $(a_5) = ar^4 = (30)(2)^4 = 480$

Hence number of bacteria at the end of nth hour $(a_{n+1}) = ar^n = (30)2^n$

11. What will Rs.500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually ?

Sol.: The amount deposited in the bank is Rs.500.

At the end of first year, amount = Rs.500 $\left(1+\frac{1}{10}\right)$ = Rs. 500 (1.1)

At the end of 2^{nd} year, amount = Rs. 500 (1.1) (1.1)

At the end of 3^{rd} year, amount = Rs. 500 (1.1)(1.1)(1.1) and so on

∴ Amount at the end of 10 years = Rs. 500 (1.1) (1.1)……… (10 times)= Rs. 500 $(1.1)^{10}$

12. If A. M and G. M of roots of a quadratic equation are 8 and 5, respectively then obtain the quadratic equation.

Sol.: Let the root of the quadratic equation be a and b

According to the question, $AM = \frac{a+b}{2} = 8 \Rightarrow a + b = 16$ ……………..(1)

$GM = \sqrt{ab} = 5 \Rightarrow ab = 25$ …………………(2)

The quadratic equation is given by $x^2 - x$ (*Sum of roots*) + (*Product of roots*) = 0

$\Rightarrow \; x^2 - x(a+b) + (ab) = 0 \Rightarrow x^2 - 16x + 25 = 0$

Hence the required quadratic equation is $x^2 - 16x + 25 = 0$

Exercise – 8 (C)

I.

1. Find the 20^{th} term of the series 2 x 4 + 4 x 6 + 6 x 8 + …. + n terms

Sol.: The given series is 2 x 4 + 4 x 6 + 6 x 8 + …. + n terms

Here n^{th} term = $a_n = 2n \times (2n + 2) = 4n^2 + 4n$

$a_{20} = 4\,(20)^2 + 4\,(20) = 4\,(400) + 80 = 1600 + 80 = 1680$

Hence the 20^{th} term of the series is 1680.

2.The sum of some terms of G. P. is 315 whose first term and the common ratio are 5 and 2 respectively. Find the last term and number of terms.

Sol.: It is given that the first term a is 5 and common ratio r is 2.

Let the sum of n terms be 315.

$\Rightarrow 5\,(2^n - 1) = 315 \Rightarrow 2^n - 1 = 63 \Rightarrow 2^n = 64 \Rightarrow 2^n = 2^6 \Rightarrow n = 6$

∴ The last term of the G. P. is 6^{th} term = $ar^{6-1} = (5)\,(2)^5 = (5)\,(32) = 160$

3. The first term of a G. P is 1. The sum of the third term and fifth term is 90. Find the common ratio of G. P.

Sol.: Let a and r be the first term and the common ratio of the G. P respectively.

Given that $a_1 = 1,\; a_3 = ar^2 = r^2,\; a_5 = a\,r^4 = r^4$ and $r^2 + r^4 = 90$

$\Rightarrow r^4 + r^2 - 90 = 0 \Rightarrow r^2 = \frac{-1 \pm \sqrt{1+360}}{2} = \frac{-1 \pm 19}{2} \Rightarrow r = \pm 3$

[Considering only real roots]
Hence the common ratio of the G. P. is ± 3

II.

1. The sum of three numbers in G. P is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

Sol.: Let the three numbers in G. P are a, ar and ar^2.

From the given condition , $a + ar + ar^2 = a(1 + r + r^2) = 56$. …………..(1)
And also given that $a-1$, $ar - 7$, $ar^2 - 21$ are in A. P.
$\Rightarrow (ar - 7) - (a - 1) = (ar^2 - 21) - (ar - 7)$
$\Rightarrow ar^2 - 2ar + a = 8$
$\Rightarrow a(r^2 + 1 - 2r) = 8$ ………….(2)
From (1) by (2) we get $\dfrac{a(1+r+r^2)}{a(r^2+1-2r)} = \dfrac{56}{8} \Rightarrow \dfrac{(1+r+r^2)}{(r^2+1-2r)} = 7 \Rightarrow 6r^2 - 15r + 6 = 0$

$\Rightarrow 2r(r - 2) - (r - 2) = 0 \Rightarrow (2r - 1)(r - 2) = 0 \Rightarrow r = 2$ or $\frac{1}{2}$
When $r = 2$ from the equation (1) we get $a = 8$, $ar = 16$ and $ar^2 = 32$
When $r = \frac{1}{2}$ from the equation (1), we have $a = 32$ $ar = 16$ and $ar^2 = 8$
Therefore when $r = \frac{1}{2}$, the three numbers in G. P are 32, 16 and 8.
Therefore the three required numbers are 8, 16 and 32.

2. A G. P consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.
Sol.: Let the G. P be T_1, T_2, T_3…….. T_{2n} and the number of terms = $2n$
Let the G. P be a, ar, ar^2………………
Given condition is $T_1 + T_2 + T_3 + \ldots\ldots + T_{2n} = 5[T_1 + T_2 + T_3 + \ldots\ldots + T_{2n-1}]$
$\Rightarrow T_1 + T_2 + T_3 + \ldots\ldots + T_{2n} - 5[T_1 + T_3 + \ldots\ldots + T_{2n-1}] = 0$
$\Rightarrow T_2 + T_4 + \ldots\ldots\ldots\ldots\ldots + T_{2n} = 4[T_1 + T_3 + \ldots\ldots + T_{2n-1}]$

$$\Rightarrow \frac{ar[(r^2)^n - 1]}{r^2 - 1} = 4 \text{ x } \frac{a\left[(r^2)^n - 1\right]}{r^2 - 1}$$

$\Rightarrow ar = 4a \quad \Rightarrow r = 4$
Hence the common ratio of the G. P is 4.

3. If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$ $(x \neq 0)$ then show that a, b, c and d are in G. P.

Sol.: Given that $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx}$

$\Rightarrow (a + bx)(b - cx) = (a - bx)(b + cx)$

$\Rightarrow ab - acx + b^2x - bcx^2 = ab + acx - b^2X - bcx^2$

$\Rightarrow 2b^2x = 2acx$

$\Rightarrow \frac{b}{c} = \frac{a}{b}$ …………..(1)

Now , $\frac{b+cx}{b-cx} = \frac{c+dx}{b-dx}$

$\Rightarrow$ $(b + cx)(c- dx) = (b- cx)(c + dx)$

$\Rightarrow bc- bdx + c^2x - cdx^2 = bc + bdx - c^2 x - cdx^2$

$\Rightarrow$ $2c^2x - 2bdx$

$\Rightarrow \frac{c}{d} = \frac{b}{c}$ ……………(2)

From (1) and (2) we have $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$

$\Rightarrow$ *a, b, c* and *d* are in G. P

4. Let S be the sum, P the product and R the sum of reciprocals of n terms in a G. P . Prove that $P^2 R^n = S^n$

Sol.: Let a the first term and r be the common ratio. Given that $S = \frac{a[r^n-1]}{r-1}$,

$P = a.ar.ar^2, ar^3 \ldots ar^{n-1} = a^n r^{1+2+3+\ldots+(n-1)} = a^n r^{\frac{[n-1]n}{2}}$

and $R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} + \ldots \frac{1}{ar^{n-1}} = \frac{r^{n-1} + r^{n-2} + r^{n-3} + r^{n-4} + \ldots\ldots + 1}{ar^{n-1}} = \frac{1}{ar^{n-1}}\left[\frac{r^n-1}{r-1}\right]$

LHS $= P^2 R^n = \left[a^n r^{\frac{(n-1)n}{2}}\right]^2 \left[\frac{1}{ar^{n-1}}\left(\frac{r^n-1}{r-1}\right)\right]^n = [a^{2n} r^{n(n-1)}]\left(\frac{1}{a^n r^{n(n-1)}}\right)\left(\frac{r^n-1}{r-1}\right)^n$

$= \left[a^n\right]\left[\left(\frac{r^n-1}{r-1}\right)^n\right] = \left[\frac{a(r^n-1)}{r-1}\right]^n = S^n =$ RHS

5. If a, b, c, d are in G. P prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G. P.
Sol.: Let r be the common ratio of GP hence $b = ar$, $c = ar^2$ and $d = ar^3$

Now, $\frac{b^n + c^n}{a^n + b^n} = \frac{(ar)^n + (ar^2)^n}{a^n + (ar)^n} = \frac{a^n r^n}{a^n}\left[\frac{1+r^n}{1+r^n}\right] = r^n$……….(1)

Now, $\frac{c^n + d^n}{b^n + c^n} = \frac{(ar^2)^n + (ar^3)^n}{ar^n + (ar^2)^n} = \frac{a^n r^{2n}}{a^n r^n}\left[\frac{1+r^n}{1+r^n}\right] = r^n$……….(2)

From (1) and (2), we get $\frac{c^n + d^n}{b^n + c^n} = \frac{b^n + c^n}{a^n + b^n} \Rightarrow (a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G. P.

6. If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$ where a, b, c, d form a GP. Prove that $(q + p) : (q - p) = 17 : 15$.

Sol.: Given that ***a***, b, c , d form a G. P. Let the common ratio be *r.*
Therefore $b = ar$, $c = ar^2$ and $d = ar^3$
Now, a and b are the roots of $x^2 - 3x + p = 0$
$\Rightarrow$ sum of roots $= a + b = 3$ ………………….(1)
$\Rightarrow a(1+r) = 3$ and product of roots $= ab = p \Rightarrow a^2r = p$………..(2)
And c, d are roots of $x^2 - 12x + q = 0$

$\Rightarrow$ sum of roots = c+d=12, $\Rightarrow ar^2(1+r)=12$ ………………….(3)
And product of roots = cd = q $\Rightarrow$ $a^2 r^5 = q$ ……………….(4)
From (3) and (1) we get $\Rightarrow r^2 = 4, \Rightarrow r = \pm 2$
Case-I : If r = 2, from (1), a (1+2) = 3 $\Rightarrow a = 1$
From (2), p = a^2r = 2 and from (4), q = $a^2 r^5 = 32$

Now, LHS = $\dfrac{q+p}{q-p}=\dfrac{32+2}{32-2}=\dfrac{34}{30}=\dfrac{17}{15}=RHS$

Case-II: If r = 2, from (1), a (1 - 2) = 3 $\Rightarrow$ a = -3
From (2), p = a^2r = -18 and from (4), q = $a^2 r^5$ = 288

Now, LHS $\dfrac{q+p}{q-p}=\dfrac{-288+(-18)}{-288-(-18)}=\dfrac{-306}{-270}=\dfrac{17}{15}=RHS$

7. The ratio of the A.M and G. M of two positive numbers *a* and *b* is *m*:*n* . Show that a : b = $\left(m+\sqrt{m^2-n^2}\right);\left(m-\sqrt{m^2-n^2}\right)$

Sol.: Let A and G are A. M and G. M between two positive numbers *a and b.*

$AM = A = \frac{a+b}{2}$ ………..(1) and $GM = G = \sqrt{ab}$ ……..(2)

Given that $\frac{A}{G}=\frac{m}{n} \Rightarrow \dfrac{a+b}{2\sqrt{ab}}=\dfrac{m}{n}$

Applying componendo and Dividendo rule both the sides

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}=\frac{m+n}{m-n} \quad \Rightarrow \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2}=\frac{m+n}{m-n}$$

$$\Rightarrow \frac{\left(\sqrt{a}+\sqrt{b}\right)}{\left(\sqrt{a}-\sqrt{b}\right)}=\frac{\sqrt{m+n}}{\sqrt{m-n}}$$

Again applying componendo and Dividendo rule both the sides

$$\Rightarrow \frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\left(\sqrt{a}-\sqrt{b}\right)}=\frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

$$\Rightarrow \frac{\cancel{2}\sqrt{a}}{\cancel{2}\sqrt{b}}=\frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

Squaring both sides, we get $\dfrac{a}{b}=\dfrac{\sqrt{m+n}+\sqrt{m-n}+2\sqrt{m+n}\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}-2\sqrt{m+n}\sqrt{m-n}}$

$$\Rightarrow \frac{a}{b}=\frac{2m+2\sqrt{m^2-n^2}}{2m-2\sqrt{m^2-n^2}}=\frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$$

$\therefore a:b=(m+\sqrt{m^2-n^2}):(m-\sqrt{m^2-n^2})$

III.

1. Find the sum of the following series up to n terms

(i) **5 + 55 + 555 + …..**
(ii) **.6 + .66 +.666 + …..**

Sol.: (i) The given sequence is $5 + 55 + 555 + \ldots$..

This sequence is not a G.P. However, it can be changed to G. P by writing the terms as

$$S_n = 5 + 55 + 555 + \ldots \text{ to } n \text{ terms} = \frac{5}{9}[9 + 99 + 999 + \ldots \text{ to } n \text{ terms}]$$

$$= \frac{5}{9}[(10-1) + (100-1) + (1000-1) + \ldots \text{ to } n \text{ terms}]$$

$$= \frac{5}{9}[(10^1 + 10^2 + 10^3 + \ldots + 10^n) - (1 + 1 + 1 + \ldots \text{ to } n \text{ terms})$$

$$= \frac{5}{9}[(10^1 + 10^2 + 10^3 + \ldots + 10^n) - n]$$

$$= \frac{5}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right] = \frac{5}{9}\left[\frac{10(10^n - 1)}{9}\right] - \frac{5}{9}n = \frac{50}{81}(10^n - 1) - \frac{5}{9}n$$

(ii)The given sequence is $.6 + .66 + .666 + \ldots$..

This sequence is not a G. P. However, it can be changed to G. P by writing the terms as

$S_n = [.6 + .66 + .666 + \ldots \text{ to n terms}]$.

$$= \frac{6}{9}[.9 + .99 + .999 + \ldots \text{ to n terms}]$$

$$= \frac{2}{3}[(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \ldots \text{ to n terms}]$$

$$= \frac{2}{3}[(1 + 1 + 1 + \ldots \text{ to n terms}) - (0.1 + 0.01 + 0.001 + \ldots \text{n terms})]$$

$$= \frac{2}{3}[(n - \{(0.1)^1 + (0.1)^2 + (0.1)^3 + \ldots + (0.1)^n\}]$$

$$= \frac{2}{3}\left[n - \frac{(0.1)\{1 - (0.1)^n\}}{1 - 0.1}\right] = \frac{2}{3}n - \frac{2}{3}\left[\frac{0.1\{1 - (0.1)^n}{0.9}\right] = \frac{2}{3}n - \frac{2}{27}\{1 - (0.1)^n\}$$

2. If f is a function satisfying $f(x + y) = f(x) f(y)$ for all $x, y \in N$ such that $f(1) = 3$ and $\sum_{x=1}^{n} f(x) = 120$. Find the value of n.

Sol.: Given that $f(x + y) = f(x) f(y)$ for all $x, y \in N$ ………(1)

And $f(1) = 3$, $\sum_{x=1}^{n} f(x) = 120$

Taking $x = y = 1$ in (1) we get $f(1 + 1) = f(2) = f(1) f(1) = 3 \times 3 = 9 = 3^2$

Similarly, $f(3) = f(1 + 2) = f(1) f(2) = 3 \times 9 = 27 = 3^3$

$f(4) = f(1 + 3) = f(1) f(3) = 3 \times 27 = 81 = 3^4$

$\therefore$ $f(1), f(2), f(3)$….. i.e. 3, 9, 27, ….are in G.P. with common ratio 3.

Now $\sum_{x=1}^{n} f(x) = 120 \Rightarrow f(1) + f(2) + f(3) + \ldots + f(n) = 120$

$\Rightarrow 3 + 3^2 + 3^3 + \ldots + 3^n = 120$

$\Rightarrow \frac{3(3^n - 1)}{3 - 1} = 120 \Rightarrow 3(3^n - 1) = 240 \Rightarrow 3^n - 1 = 80 \Rightarrow 3^n = 3^4 \Rightarrow n = 4$

Hence the value of n is 4.

3. A farmer buys a used tractor for Rs. 12000. He pays Rs. 6000 cash and agrees to pay the balance in annual installments of Rs. 500 plus 12 % interest on the unpaid amount. How much will the tractor cost him ?

Sol.: It is given that the farmer pays Rs.6000 in cash.
Therefore , unpaid amount = Rs.12000 – Rs.6000 = Rs. 6000
Given that, the interest paid annually is 12% of 6000, 12% of 5500,…..
Thus, total interest to be paid = 12% of 6000 + 12 % of 5500 + ….. + 12% of 500
=12% (6000 + 5500 + 5000 + ….. + 500)
= 12% (500 + 1000 + 1500 +….. + 6000)
Now the series 500, 1000, 1500…. 6000 is an AP with the common difference of 500.
Let the number of terms of the A. P be n.
$\Rightarrow$ 6000 = 500 + (n -1) 500 $\Rightarrow$ 1 + (n -1) = 12 $\Rightarrow$ n = 12
Therefore, the sum of AP = $\frac{12}{2}$ [2 x 500 + (12 -1) x 500] =6 [100 + 5500] = 39000.
So, total interest to be paid = 12% of (500 + 1000 + 1500 + ……6000)
= 12% of 39000 = Rs. 4680
Therefore, the cost of tractor = Rs. 12000 + Rs. 4680 = Rs.16680.

4. Shamshad Ali buys a scooter for Rs.22000. He pays Rs.4000 cash and agrees to pay the balance in annual installment of Rs.1000 plus 10% interest on the unpaid amount. How much will the scooter cost him ?

Sol.: Given that Shamshad Ali buys a scooter for Rs.22000 and pays Rs.4000 in cash.
The Unpaid amount = Rs.22000 - Rs.4000 = Rs.18000.
Given that condition, the interest paid annually is 10% of 18000, 10% of 17000 ….. 10% of 1000
Thus, the total interest to be paid = 10% of 18000 +10% of 17000+ ……... + 10 % of 1000
= 10 % of (1000 + 2000 + 3000 + …… + 18000)
Here 1000, 2000, 3000….. 18000 are in A. P with common difference of 1000.
Let the number of terms be n $\Rightarrow$ 18000 = 1000 + (n -1) (1000) $\Rightarrow$ n = 18

S_n =1000 + 2000 + …. + 18000 = $\frac{18}{2}$ [2 x 1000 + (18-1)1000]

= 9 [2000 + 17000] = 171000
Total interest paid = 10% of (18000 + 17000 + 16000+ …. 1000)
= 10 % of Rs.171000 = Rs.17100.
Therefore, the cost of scooter = Rs.22000 + Rs.17100 = Rs.39100.

5. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spend on the postage when 8th set of letter is mailed.

Sol. : Let the numbers of letters mailed be in G. P . i.e., 4, 4^2….. 4^8.
Here a = 4, r = 4 and n = 8.

We know that the sum of n terms of a G.P.is $S_n = \frac{a(r^n - 1)}{r - 1}$

$$\Rightarrow S_8 = \frac{4(4^8 - 1)}{4 - 1} = \frac{4(65536 - 1)}{3} = \frac{4(65535)}{3} = 4(21845) = 87380$$

Given that the cost to mail one letter is 50 paise

Cost of mailing 87380 letters = 87380 x 50 paise = Rs. 43690

Therefore the amount spent when 8th set of letter is mailed is Rs. 43690

6. A man deposited Rs. 10000 in a bank at the rate of 5 % simple interest annually. Find the amount in 15th year since he deposited the amount and also calculate the total amount after 20 years.

Sol.: Given that a man deposited Rs. 10000 at the rate of 5% simple interest annually.

The Interest in first year = $\frac{5}{100}$ x Rs.10000 = Rs.500

The amount in 15th year = Rs.10000 + [Rs.500 + Rs.500 + Rs.500 + ….. 14 times]

= Rs.10000 + 14 x Rs.500 = Rs.17000

Total Amount after 20 years = Rs.10000 + [Rs.500 + Rs.500 + Rs.500 + ….. 20 times]

= Rs.10000 + 20 x Rs.500 = Rs.20000

7. A manufacturer reckons that the value of a machine, which costs him Rs.15625, will depreciate each year by 20% . Find the estimated value at the end of 5 years.

Sol.: The value of machine = Rs.15625

Given that machine depreciates by 20% every year

$\Rightarrow$ The value of machine after every year is 80% of the original cost

$\Rightarrow \frac{4}{5}$ of the original cost.

Therefore the estimated value at the end of 5 years =Rs.15625 x $\frac{4}{5}$ x $\frac{4}{5}$ x $\frac{4}{5}$ x $\frac{4}{5}$ x $\frac{4}{5}$

= 5 x 1024 =Rs. 5120.

8. 150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on third day and so on. It took 8 more days to finish the work Find the number of days in which the work was completed.

Sol.: Let x be the number of days in which 150 workers finish the work.

Given condition is 150x = 150 + 146 + 142 + …….(x + 8) terms

The series 150, 146, 142, …….(x+8) terms are in A. P. with difference -4

a = 150, d = - 4, n = x + 8, We know that $S_n = \frac{n}{2}\left(2a+\left(n-1\right)d\right)$

$\Rightarrow 150x = \frac{x+8}{2}$ [2 x 150 + (x + 8 -1) x (-4)]

$\Rightarrow$ 300x = (x + 8) (300- 4x – 28)

$\Rightarrow$ 300x = 272x – 4x^2 + 2176 – 32x

$\Rightarrow$ 4x^2 + 60x – 2176 = 0

$\Rightarrow$ x^2 + 15x -544 = 0

$\Rightarrow$ (x + 32) (x – 17) = 0

$\Rightarrow$ x = -32 or 17

Here we consider x = 17 (because the no. of days can not be negative)

Therefore the no. of days in which the work was complete = (17 + 8) = 25.

Exercise – 8(d)

I. Find the sum to infinity in each of the following Geometric Progression.

1) 1, $\frac{1}{3}, \frac{1}{9}$,....

Sol. : a = 1, r = $\frac{1}{3}$

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2} = 1.5$$

2) 6, 1.2, .24,…..

Sol.: a = 6, r = $\frac{1}{5}$

$$S_\infty = \frac{a}{1-r} = \frac{6}{1-\frac{1}{5}} = \frac{30}{4} = 7.5$$

3) 5, $\frac{20}{7}, \frac{80}{49}, \ldots.$

Sol.: a = 5, r = $\frac{4}{7}$

$$S_\infty = \frac{a}{1-r} = \frac{5}{1-\frac{4}{7}} = \frac{35}{3}$$

4) $\frac{-3}{4}, \frac{3}{16}, \frac{-3}{64}, \ldots\ldots$

Sol.: a = $\frac{-3}{4}$, r = $\frac{-1}{4}$

$$S_\infty = \frac{a}{1-r} = \frac{\frac{-3}{4}}{1-\left(-\frac{1}{4}\right)} = \frac{-3}{5}$$

5. Prove that $3^{\frac{1}{2}} X 3^{\frac{1}{4}} X 3^{\frac{1}{8}} \ldots\ldots = 3$.

Sol.: $3^{\frac{1}{2}} x\ 3^{\frac{1}{4}} x\ 3^{\frac{1}{8}} \ldots\ldots = 3^{\frac{1}{2}} x\ 3^{\left(\frac{1}{2}\right)^2} x\ 3^{\left(\frac{1}{2}\right)^3} \ldots\ldots.$

$$= 3^{\frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\ldots\ldots\ldots\ldots\ldots\ldots}$$

$$= 3^{\frac{\frac{1}{2}}{1-\frac{1}{2}}} = 3^1 = 3$$

$$\text{Here } \frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\ldots\ldots\ldots\ldots = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1 \quad (\because a = \frac{1}{2}, r = \frac{1}{2})$$

6. Let x = 1 + a + a^2 +…… and y = 1 + b + b^2 +…..where $|a| < 1$ and $|b| < 1$.

Prove that 1 + ab + a^2b^2+…… = $\frac{xy}{x+y-1}$.

Sol.: Given that x = $1+a+a^2+$……. and y = 1 + b + b^2 + ……

we know that $S_\infty = \frac{a}{1-r}$

$x = \frac{1}{1-a} \Rightarrow 1-a = \frac{1}{x} \Rightarrow a = 1-\frac{1}{x}$ and $y = \frac{1}{1-b} \Rightarrow b = 1-\frac{1}{y}$

Hence 1+ab+a^{2}b^2+…..= $\frac{1}{1-ab} = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)} = \frac{xy}{xy-(xy-x-y+1)} = \frac{xy}{x+y-1}$

<u>Multiple Choice Questions (Keys)</u>

1. Here 2, 4, 6, 8, 10 is finite sequence.
2. $a_n = a_{n-1} + a_{n-2}, n > 2$
3. Fibonacci sequence is 1, 1, 2, 3, 5, 8,……
4. Here a_{15} = 4(15) + 6 = 60 + 6 = 66
5. Compact form of sequence is denoted by $\sum a_n$
6. Here 2 + 4 + 6 + 8 + 10 = 30
7. Here $\sum_{n=1}^{4} 2n+3 = 2$ $\sum$n + 3 $\sum$= $\cancel{2}.\frac{n(n+1)}{\cancel{2}} + 3.n$ = 4(4+1)+3(4)=20+12=32.
8. In G.P common ratio r = $\frac{a_{n+1}}{a_n} \Rightarrow a_{n+1} = r.a_n$
9. In G.P. nth term $a_n = a.r^{n-1}$
10. G.P. $\rightarrow$ $a + ar + ar^2 + ... + ar^{n-1}.(n\,terms)$

r = 1 $\Rightarrow$ a + a+ a + ……+ a (n terms) = na.

11. The sum upto 5th term, i.e. 4 + 8 + 16 + 32 + 64 = 128.
12. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32} = \frac{32+16+8+4+2+1}{32} = \frac{63}{32}$
13. G.M = $\sqrt{3x12} = \sqrt{36} = 6$
14. A.M. = $\frac{12+3}{2} = \frac{15}{3}$

G.M. = $\sqrt{12\ x\ 3} = \sqrt{36} = 6$

15. $A \geq G$
16. Here a + b = 10 x, ab = 9x^2, a - b = 8x

Then 2a=18x, 2b = 2x $\Rightarrow$ a= 9x, b = x
a : b =9:1

1)	**3**	2)	**1**	3)	**2**	4)	**4**	5)	**2**	6)	**4**	7)	**4**	8)	**2**	9)	**3**	10)	**1**
11)	**4**	12)	**1**	13)	**2**	14)	**2**	15)	**2**	16)	**1**								

9. STRAIGHT LINES

Exercise – 9(a)

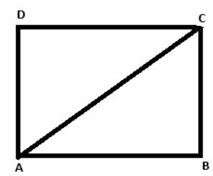
1. Draw a quadrilateral in the Cartesian plane whose vertices are (-4,5) , (0, 7), (5, -5) and (-4, -2) also find its area.

Sol: Let A (-4,5), B (0, 7) , C (5, -5) D(-4, -2) be given vertices of the quadrilateral.

Area of quadrilateral ABCD = Area of Δ ABC + Area of Δ ADC

Area of Δ ABC = $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

= $\frac{1}{2}|-4(7-(-5)) + 0(-5-5) + 5(5-7)|$ = 29



A (-4, 5) D (-4, -2) C (5, -5)

Area of Δ ADC =

$\frac{1}{2}|-4(-2-(-5)) + (-4)(-5-5) + 5(5-(-2))|$

= $\frac{1}{2}|-4(3) - 4(-10) + 5(7)|$ = $\frac{1}{2}|-12 + 40 + 35|$ = $\frac{63}{2}$

∴ Area of Quadrilateral ABCD = 29 + $\frac{63}{2} = \frac{58+63}{2} = \frac{121}{2}$ sq unit.

2. The base of an equilateral triangle with side 2a lies along the *y*-axis such that the mid point of the base is at the origin. Find the vertices of the triangle.

Sol: Let BC be the base of the triangle which lies on *y* axis and let the third vertex be A (b, 0)

Given that Δ ABC is an equilateral.

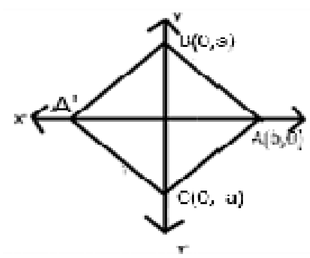
⇒ AB = BC = CA

$AB^2 = BC^2$ ⇒ $(b - 0)^2 + (0 - a)^2 = (0 - 0)^2 + (a - (-a))^2$

⇒ $b^2 + a^2 = (2a)^2 = 4a^2$

⇒ $b^2 = 4a^2 - a^2 = 3a^2$

⇒ $b = \sqrt{3a^2} = \pm\sqrt{3}a$



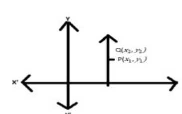
The vertices of the triangle are $(0,a),(\sqrt{3}a,0),(0,-a)$ Or (0, a) (-$\sqrt{3}a$, 0) (0, -a)

3. Find the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ when

(i) PQ is parallel to the y-axis

(ii) PQ is parallel to the x-axis

Ans.: $P(x_1, y_1)$ and $Q(x_2, y_2)$



(i)PQ is parallel to y-axis

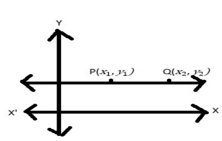
=> $x_1 = x_2$.

∴ PQ =

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{0 + (y_2 - y_1)^2}$ =

$\sqrt{(y_2 - y_1)^2} = |y_2 - y_1|$



(ii)PQ is parallel to the x-axis $\Rightarrow$ $y_1 = y_2$

PQ =

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ =

$\sqrt{(x_2-x_1)^2+0^2}$ =

$\sqrt{(x_2-x_1)^2}$ = $|x_2-x_1|$

4. Find a point on the x-axis which is equidistant from the points (7,6) and (3,4).

Sol: Let a point on x-axis be A (a,0) , B (7,6) and C (3, 4)

Given that AB = AC $\Rightarrow$ $AB^2 = AC^2$

Distance between points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$(7-a)^2+(6-0)^2 = (3-a)^2 + (4-0)^2$

60= 8a $\Rightarrow$ a = $\frac{60}{8}=\frac{15}{2}$

$\therefore$ Required point A ($\frac{15}{2}$,0)

5. Find the slope of a line, which passes through the origin and mid-point of the line segment joining the points P (0, -4) and B (8,0).

Sol: Mid point of the line segment

PB = $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) = \left(\frac{0+8}{2},\frac{-4+0}{2}\right)$ = (4, -2)

$\therefore$ Slope of the line passing through origin (0,0) and point (4, -2) is

$= \frac{y_2-y_1}{x_2-x_1} = \frac{-2-0}{4-0} = \frac{-2}{4} = \frac{-1}{2}$

6. Without using the Pythagoras theorem, show that the point (4, 4) (3, 5) and (-1, -1) are the vertices of a right angled triangle.

Sol: Slope of line joining AB = $\frac{y_2-y_1}{x_2-x_1} = \frac{5-4}{3-4} = -1$

Slope of line joining BC = $\frac{-1-5}{-1-3} = \frac{-6}{-4} = \frac{3}{2}$

Slope of line joining CA = $\frac{4-(-1)}{4-(-1)} = \frac{5}{5} = 1$

Slope of AB x slope of CA = -1 x 1 = -1

$\Rightarrow$ AB is perpendicular to CA

$\therefore \Delta$ ABC is a right angle triangle.

7. Find the slope of the line which makes an angle of 30^0 with positive direction of *y*-axis measured anti clock wise.

Y

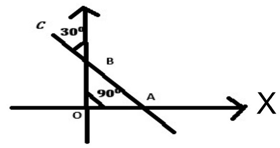
Sol: Given that $\angle CBY = 30^0$

$\angle CBY = \angle OBA$

$\angle OBA + \angle BAO + \angle AOB = 180^0$

$30^0 + \angle BAO + 90^0 = 180^0$

$\Rightarrow \angle BAO = 180 - 120 = 60^0$

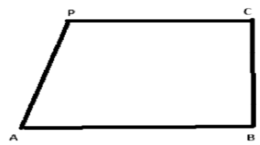


$\angle BAX = 180 - 60 = 120.$

Slope of the line AB = Tan 120 = Tan (180-60) = - Tan 60 = $-\sqrt{3}$.

8. Without using distance formula, show that the points (-2,-1) (4,0), (3,3) and (-3,2) are the vertices of a parallelogram.

Sol: Let A (-2, -1) B (4, 0) C (3,3) D (-3,2)



Slope of AB = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-1)}{4-(-2)} = \frac{1}{6}$

Slope of BC = $\frac{3-0}{3-4} = \frac{3}{-1} = -3$

Slope of CD = $\frac{2-3}{-3-3} = \frac{-1}{-6} = \frac{1}{6}$

Slope of DA = $\frac{2-(-1)}{-3-(-2)} = \frac{3}{-1} = 3$

Slope of AB = Slope of CD, Slope of BC = Slope of DA

Slope of AB x Slope of BC = $\frac{1}{6}(-3) = -\frac{1}{2}(\neq -1)$

$\therefore$ AB is not perpendicular to BC

Mid point of AC = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2+3}{2}, \frac{-1+3}{2}\right) = \left(\frac{1}{2}, 1\right)$

Mid Point of BD = $\left(\frac{4+(-3)}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right)$

Here adjacent sides are non perpendicular and opposite sides are parallel, diagonals bisect each other.

$\therefore$ Given points are vertices of a parallelogram.

9. Find the angle between the *x*- axis and the line joining the points (3, -1) and (4, -2)

Sol: Slope of the line joining (3, -1) and (4, -2) is $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-(-1)}{4-3} = \frac{-2+1}{1} = -1$

Let θ be the angle made by the line with x- axis

Then tan θ= -1 $\Rightarrow$ Tan θ = Tan $(180^0 - 45^0)$ = Tan 135^0, $\therefore$ θ = 135^0.

10. The slope of a line is double the slope of another line. If tangent of the angle between them is $\frac{1}{3}$. Find the slopes of the lines.

Sol: Let slope of first line = m

Slope of second line = 2m

If angle between the two lines is θ then

$$\text{Tan } \theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{1}{3} = \left|\frac{m - 2m}{1 + m(2m)}\right| = \left|\frac{-m}{1+2m^2}\right|$$

$$\Rightarrow \frac{-m}{1+2m^2} = \pm\frac{1}{3}$$

$$\Rightarrow -3\text{m} = \pm\left(1 + 2\text{m}^2\right)$$

$$\therefore 1 + 2\text{m}^2 = -3\text{m}$$

$$\Rightarrow 2\text{m}^2 + 3\text{m} + 1 = 0 \quad \Rightarrow m = \frac{-1}{2} \text{ or } -1$$

$$\text{(or)} \quad -3\text{m} = -(1 + 2\text{m}^2) = -1 - 2\text{m}^2$$

$$\Rightarrow 2\text{m}^2 - 3\text{m} + 1 = 0 \Rightarrow \text{m} = \frac{1}{2} \text{ or } 1$$

$$\therefore m = \pm\frac{1}{2}, \pm 1.$$

11. A line passes through (x_1, y_1) and (h, k). If slope of the line is m, show that $k - y_1 = m(h - x_1)$.

Sol: Let A (x_1, y_1) , B (h,k)

Slope of the line AB = $\frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{k - y_1}{h - x_1} \Rightarrow m(h - x_1) = k - y_1$$

Exercise – 9(b)

I. In exercises 1 to 7, find the equation of the line which satisfy the given conditions.

1. Write the equations for the X-axis, Y-axis

Sol: Any point on X - axis is $(x, 0)$ as Y – coordinate on **X** - axis is zero, also slope of **X** - axis is m=0

$$y - y_1 = \text{m}(x - x_1) \Rightarrow y - 0 = 0(x - x_1) \Rightarrow y = 0$$

Hence equation of **X** – axis is $y = 0$

Similarly we can determine the equation of Y – axis is $x = 0$.

2. Passing through the point (-4, 3) with slope $\frac{1}{2}$.

Sol: Equation of line in one point form is $y - y_1 = m(x - x_1)$

Here $(x_1, y_1) = (-4, 3)$ slope m = $\frac{1}{2}$

$y - 3 = \frac{1}{2}(x + 4) \Rightarrow 2y - 6 = x + 4 \Rightarrow x - 2y + 10 = 0$

3. Passing through (0, 0) with slope m.

Sol: Equation of the line in one point form is $y - y_1 = m(x - x_1)$

Here $(x_1, y_1) = (0, 0)$ slope = m

$y - 0 = m(x - 0) \Rightarrow y = mx$.

4. Passing through $\left(2, 2\sqrt{3}\right)$ and inclined with the **X** – axis at an angle of 75^0.

Sol: Given $(x_1, y_1) = \left(2, 2\sqrt{3}\right)$

Slope m = $\tan 75^0 = 2 + \sqrt{3}$

Equation of a line in one point form is $y - y_1 = m(x - x_1)$

$\Rightarrow y - 2\sqrt{3} = \left(2 + \sqrt{3}\right)(x - 2) \Rightarrow y - 2\sqrt{3} = 2x - 4 + \sqrt{3}x - 2\sqrt{3}$ (or)

$\Rightarrow \left(2 + \sqrt{3}\right)x - y - 4 = 0$

$\Rightarrow \left(\sqrt{3} + 1\right)x - \left(\sqrt{3} - 1\right)y = 4\left(\sqrt{3} - 1\right)$

5. Intersecting the X-axis at a distance of 3 units to the left of the origin with slope-2

Sol: If the line meets **X** – axis (negative side) at distance of 3 units then

(x_1, y_1) = (-3, 0) slope m = -2

Equation of line in one point form is

$y - y_1 = m(x - x_1) \Rightarrow y - 0 = -2(x + 3)$

$y = -2x - 6 \Rightarrow 2x + y + 6 = 0$.

6. Intersecting the Y-axis at a distance of 2 units above the origin and making an angle of 30^0 with positive direction of the X-axis..

Sol: If the line intersect the Y-axis, at a distance of 2 units above the origin then

$(x_1, y_1) = (0, 2)$ and slope m = $Tan\,30^0 = \frac{1}{\sqrt{3}}$

Equation of a line in one point form

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = \frac{1}{\sqrt{3}}(x - 0)$

$$\Rightarrow \sqrt{3}y - 2\sqrt{3} = x$$
$$x - \sqrt{3}y + 2\sqrt{3} = 0$$

7. Passing through the points (-1, 1) and (2, -4).

Sol: Let the given points be

$(x_1, y_1) = A(-1,1)$ and $(x_2, y_2) = B(2,-4)$

Required equation of the straight line is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$
$$y - 1 = \left(\frac{-4-1}{2+1}\right)(x+1)$$

$3(y - 1) = -5(x + 1)$

$3y - 3 = -5x - 5$

$5x + 3y + 2 = 0.$

8. The vertices of ΔPQR are P(2, 1), Q(-2, 3) and R(4, 5). Find equation of the median through the vertex R.

Sol: Given P=(2, 1), Q = (-2, 3) and R = (4, 5) are three vertices of a ΔPQR.

Let S be the mid point of $\overline{PQ}$.

$$\therefore S = \left(\frac{2-2}{2}, \frac{1+3}{2}\right) = (0,2)$$

Now RS is the median, the equation of the median through R is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$
$$y - 5 = \left(\frac{2-5}{0-4}\right)(x-4) \quad \Rightarrow y - 5 = \frac{3}{4}(x-4)$$

$4y - 20 = 3x - 12, \quad 3x - 4y + 8 = 0.$

9. Find the equation of the line passing through (-3, 5) and perpendicular to the line joining the points (2, 5) (-3, 6).

Sol: Let P = (2, 5) and Q = (-3, 6). Now slope of $\overline{PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-5}{-3-2} = \frac{+1}{-5} = \frac{-1}{5}$

Slope of line perpendicular to $\overline{PQ} = \frac{-1}{slope\ PQ} = = \frac{-1}{\left(\frac{-1}{5}\right)} = 5$

Let $(x_1, y_1) = (-3,5)$ and ,m = 5

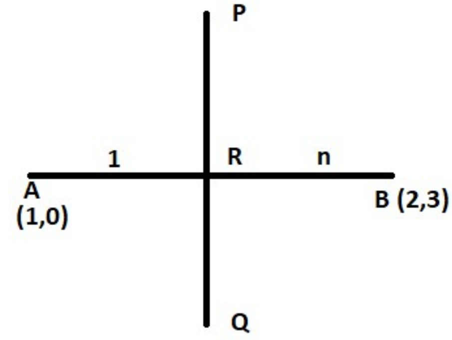
Equation of the line is $y - y_1 = m(x - x_1)$

$\Rightarrow y - 5 = 5(x+3) \Rightarrow y - 5 = 5x + 15 \Rightarrow 5x - y + 20 = 0$

10. A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1 : n. Find the equation of the line.

Sol: Let the given points are A(1,0), B(2,3) let PQ divide AB in the ration 1 : n at R using internally ratio we get

$$R=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)=\left(\frac{2+n}{1+n},\frac{3+0}{1+n}\right)$$



Slope of $\overline{AB}=\frac{3-0}{2-1}=3$

Slope of $\overline{PQ}=\frac{-1}{slope\,of\,\overline{AB}}=-\frac{1}{3}$ $\quad(m_1m_2=-1)$

Now equation of $\overline{PQ}$ is $y-y_1=m(x-x_1)$

$$\Rightarrow y-\frac{3}{1+n}=-\frac{1}{3}\left(x-\frac{n+2}{n+1}\right)$$

$$\Rightarrow \frac{(1+n)y-3}{(n+1)}=-\frac{1}{3}\frac{((n+1)x-(n+2))}{(n+1)}$$

$\Rightarrow 3y$ (n+1) -9 = -(n+1)x +(n+2)

$\Rightarrow$(n+1)x +3(n+1) y = n+11

11. Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3).

Sol: Let a and b be the equal intercepts. Now equation of the straight line in the intercepts form

$\frac{x}{a}+\frac{y}{b}=1$ becomes

$\frac{x}{a}+\frac{y}{b}=1\Rightarrow x+y=a$........................(1)

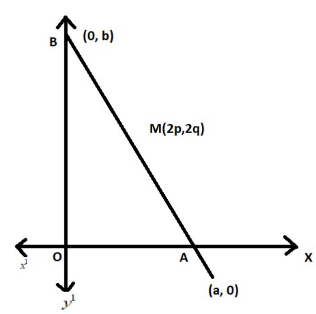
If (1) passing through the point (2, 3) we get 2 + 3 = a

$\therefore$ a = 5 put in equation (1) we get $x+y=5\Rightarrow x+y-5=0$.

12. If the portion of the straight line intercepted between the axes of co-ordinates is bisected at (2p, 2q) write the equation of the straight line.

Sol: Let the straight line meets the co-ordinate axes at A(a, 0) and B(0, b) respectively.

Let M(2p, 2q) be the mid point of $\overline{AB}$.



$$\therefore\left(\frac{a+0}{2},\frac{0+b}{2}\right)=(2p,2q)$$

a =4p, b = 4q

Now equation of the straight line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \quad \Rightarrow\frac{x}{p}+\frac{y}{q}=4$$

13. Find the angle made by the line $y=-\sqrt{3}x+3$ with positive direction of the X–axis measured in the counter – clockwise direction.

Sol: **Given line $y=-\sqrt{3}x+3$**

Compare with y = mx + c, m = - $\sqrt{3}$

Now tan $\theta = -\sqrt{3}$ then $\theta = \frac{2\pi}{3}$.

14. The intercepts of a straight line on the axes of co-ordinates are a and b. If p is the length of the perpendicular from the origin to this line, write the value of p in terms of a and b.

Sol: Let the equation of the straight line with a and b are intercepts on the co-ordinate axes is

$\frac{x}{a}+\frac{y}{b}=1$

$\Rightarrow bx+ay=ab$

divides with $\sqrt{a^2+b^2}$ on both sides

$$\frac{bx}{\sqrt{a^2+b^2}}+\frac{ay}{\sqrt{a^2+b^2}}=\frac{ab}{\sqrt{a^2+b^2}}$$

Now perpendicular distance from origin is P

$$\therefore P=\frac{|ab|}{\sqrt{a^2+b^2}}$$

II. 1) Find the equation of the line passing through the point (2, 2) and cut off intercepts on the axes whose sum is 9.

Sol: Let the equation in the intercept form is $\frac{x}{a}+\frac{y}{b}=1$..................(1)

Given that a + b = 9, $\Rightarrow$ b = 9 – a

Now equation (1) becomes $\frac{x}{a}+\frac{y}{9-a}=1$

$\Rightarrow \frac{2}{a}+\frac{2}{9-a}=1$, $\Rightarrow$ 2(9-a) + 2(a) = a(9-a)

$\Rightarrow$ 18 – 2a+2a = 9a –a^2

$a^2-9a+18=0$

$a^2-6a-3a+18=0$

a(a – 6) – 3(a – 6) = 0 $\Rightarrow$ (a – 6) (a – 3) = 0

a =3 (or) a = 6, Now b = 6 (or) b = 3

$\therefore$ Required equations are $\frac{x}{3}+\frac{y}{6}=1$ (or) $\frac{x}{6}+\frac{y}{3}=1$

$\Rightarrow 2x + y = 6$ (or) $x + 2y = 6$

2. Find the equation of the straight line through the point (0, 2) making an angle $\frac{2\pi}{3}$ with the positive X-axis. Also find the equation of line parallel to it and crossing the Y – axis at a distance of 2 units below the origin.

Sol: Given inclination $\theta = \frac{2\pi}{3}$, so slope of the line is m = $Tan\frac{2\pi}{3} = -\sqrt{3}$

Given point (0, 2)

$\therefore\ y - y_1 = m(x - x_1)$

$\Rightarrow\ y - 2 = -\sqrt{3}(x - 0)\ \Rightarrow \sqrt{3}\,x + y - 2 = 0$……………….(1)

The line $\overleftrightarrow{BD}$ parallel to $\overleftrightarrow{AC}$

$\therefore$ slope of $\overleftrightarrow{BD}$ is also m = $-\sqrt{3}$

It meets the Co-ordinate axis of Y at (0, -2)

$\therefore$ B (0, -2)

Hence required equation is

$y + 2 = -\sqrt{3}(x - 0)\ \Rightarrow \qquad \sqrt{3}\,x + y + 2 = 0$

3. The perpendicular from the origin to a line meets it at the point (-2, 9).

Find the equation of the line.

Sol: Let $\overline{OR}$ be the perpendicular drawn from origin to the line $\overline{PQ}$.

Slope of $\overline{OR} = \frac{-9}{2}$

Slope of $\overline{PQ} = \frac{-1}{slope\ of\ \overline{OR}} = \frac{-1}{\left(\frac{-9}{2}\right)} = \frac{2}{9}$

$\therefore m = \frac{2}{9}, (x_1, y_1) = (-2, 9)$

Now equation of the straight line is $y - y_1 = m(x - x_1)$

$\Rightarrow y - 9 = \frac{2}{9}(x + 2) \quad \Rightarrow 9y - 81 = 2x + 4 \Rightarrow 2x - 9y + 85 = 0$

4. The length L (in centimetres) of a copper rod is a linear function of its Celsius temperature. In an experiment, if L = 124. 942 when C = 20 and L = 125.134 when C = 110, express L in terms of C.

Sol: Given L = 124.942 when C = 20 and L = 125.134 when C = 110 now take in the form of ordered pair (20,124.942), (110,125.134) are two points

$$\therefore y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \text{L} - 124.942 = \frac{125.134 - 124.942}{110 - 20}(C - 20)$$

$$\text{L} - 124.942 = \frac{0.192}{90}(C - 20)$$

$$\text{L} = \frac{0.192}{90}(C - 20) + 124.942$$

5. The owner of a milk store finds that he sell 980L of milk each week at Rs. 14/L and 1220L of milk each week at Rs. 16/L assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs.17 / litres?

Sol: Let price and litre be denoted in the ordered pair (x, y). Where x denotes rupees / L and y denotes the quantity of milk in litre.

Given (14, 980) and (16, 1220) are two points the linear equation is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 980 = \frac{1220 - 980}{16 - 14}(x - 14) \Rightarrow \quad y - 980 = \frac{240}{2}(x - 14) \Rightarrow 120x - y = 700$$

When price x = 17 Rs. Then (120)(17) – y = 700

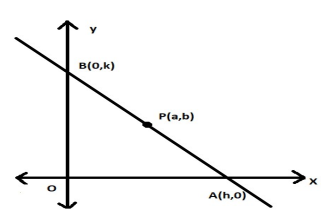
y = 1340

He will sell weekly 1340L milk at the rate of Rs. 17/L

6. P(a, b) is the mid point of a line segment between the axes. Show that the equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$.

Sol: Let the line meets the axes at A(h, 0) and B(0,k) and given P(a, b)

be the mid point of $\overline{AB}$.



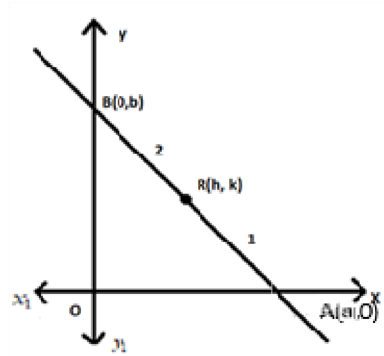
$$\therefore \left(\frac{h+0}{2}, \frac{0+k}{2}\right) = (a, b) \qquad h = 2a \text{ and } k = 2b$$

Now intercept form of the straight line is $\frac{x}{h} + \frac{y}{k} = 1$ becomes

$$\frac{x}{2a} + \frac{y}{2b} = 1, \; \frac{x}{a} + \frac{y}{b} = 2$$

7. The point R(h, k) divides a line segment between the axes in the ratio 1: 2. Find the equation of the straight line.

Sol: Let the straight line meets the Co-ordinate axes at A=(a, 0) and B=(0, b), then equation of a line is $\frac{x}{a}+\frac{y}{b}=1 \rightarrow (1)$



Given R divides $\overline{AB}$ in the ratio 1 : 2 then

$$R=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$$

$$=\left(\frac{0+2a}{1+2},\frac{b+0}{1+2}\right)=(h,k)$$

$$\therefore \frac{2a}{3}=h,\frac{b}{3}=k$$

$$a=\frac{3h}{2},b=3k$$

Put in equation (1) we get

$$\frac{x}{\left(\frac{3h}{2}\right)}+\frac{y}{(3k)}=1 \Rightarrow \frac{2x}{h}+\frac{y}{k}=3$$

2kx + hy=3hk

8. By using the concept of equation of a line, prove that the three points are collinear

i) (3, 0), (-2, -2), (8, 2)

ii) (-5, 1), (5, 5), (10, 7)

iii) (a, b+c), (b, c+a). (c, a+b)

Sol: **(i) Let the given points are A(3, 0), B(-2, -2) and C(8, 2)**

Equation of line $\overline{AB}$ is $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$$\Rightarrow y-0=\frac{-2-0}{-2-3}(x-3) \Rightarrow y=\frac{2}{5}(x-3)$$

$5y = 2x - 6$

$2x - 5y - 6 = 0$…….(1)

Put C(8, 2) in equation (1)

2(8) – 5(2) -6 =0 $\Rightarrow 0 = 0$

Hence the points A, B and C are collinear

(ii) Let the given points are A(-5, 1), B(5, 5) and C(10, 7).

Equation of a line $\overleftrightarrow{AB}$ is $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$\Rightarrow y-1=\frac{5-1}{5+5}(x+5)\Rightarrow y-1=\frac{2}{5}(x+5)$

$5y-5=2x+10 \Rightarrow \quad 2x-5y+15=0$........(1)

On putting the point C(10,7) in equation (1)

$2(10)-5(7)+15=0 \quad \Rightarrow 20-35+15=0 \Rightarrow 0=0$

Hence A, B, C are collinear.

(iii) **Let the given points are A = (a, b+c), B=(b, c+a) and C = (c, a+b).**

Equation of the line $\overleftrightarrow{AB}$ is $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$\Rightarrow (y-b-c)=\frac{c+a-b-c}{b-a}(x-a)$

$y-b-c=-1(x-a)$

$x+y-(a+b+c)=0 \rightarrow (1)$

on putting the point C = (c, a+b) we get

$c+a+b-(a+b+c)=0$

$(a+b+c)-(a+b+c)=0 \Rightarrow 0=0$

Hence A, B, C are collinear.

EXERCISE – 9(C)

I. 1) Reduce the following equations into slope intercept form and find their slopes and y – intercepts.

(i) $x+7y=0$ (ii) $6x+3y-5=0$ (iii) $y=0$

Sol: (i) Given $x+7y=0 \quad \Rightarrow 7y=-x,\ y=\left(\frac{-1}{7}\right)x+0$

On comparing with $y=mx+c$ we get, Slope m = $\frac{-1}{7}$, C = 0 is y – intercept

(ii) Given $6x+3y-5=0 \Rightarrow 3y=-6x+5 \quad y=-2x+\frac{5}{3}$

On comparing with $y=mx+c$ we get , Slope m = -2 and y – intercept c = $\frac{5}{3}$

(iii) Given $y = 0$, $y = 0x + 0$ On comparing with $y = mx + c$ we get

Slope m = 0, y – intercept C = 0

2) Reduce the following equations into intercept form and find their intercepts on the axes (i) $3x + 2y - 12=0$, (ii) $4x - 3y = 6$, (iii) $3y + 2 = 0$

Sol: Given equation $3x + 2y - 12 = 0$

$\Rightarrow 3x + 2y = 12 \quad \Rightarrow \frac{3x}{12}+\frac{2y}{12}=1 \quad \Rightarrow \frac{x}{4}+\frac{y}{6}=1$

x – Intercept =4, y – Intercept = 6

(ii) Given $4x - 3y = 6$

$\frac{4x}{6}-\frac{3y}{6}=1. \qquad \frac{x}{\left(\frac{3}{2}\right)}+\frac{y}{(-2)}=1$, x – Intercept = $\frac{3}{2}$, y – Intercept = -2

(iii) Given $3y + 2 = 0$ $\qquad 3y = -2$

$\Rightarrow 0.x+\frac{y}{\left(\frac{-2}{3}\right)}=1$, No intercept on x – axis, y – intercept = $\frac{-2}{3}$

3. Find the distance of the point (-1, 1) from the line $12(x+6) = 5(y-2)$

Sol: Given equation is $12(x + 6) = 5(y - 2)$

$$12x + 72 = 5y - 10$$

$$12x - 5y + 82 = 0 \ldots\ldots\ldots(1)$$

Given point $(x_1, y_1) = (-1,1)$

Perpendicular distance = $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$$=\frac{|-12-5+82|}{\sqrt{144+25}}=\frac{65}{13}=5.$$

4. Find the point on the X – axis whose distance from the line $\frac{x}{3}+\frac{y}{4}=1$ are 4 units.

Sol: Let (h, 0) be any point the **X**– axis.

Given line $\frac{x}{3}+\frac{y}{4}=1$

$\Rightarrow 4x + 3y - 12 = 0$ .... (1)

Given perpendicular distance from (h, 0) to line (1) is 4

$\frac{|4h+0-12|}{\sqrt{16+9}} = 4$, $|4h-12| = 20$, $\therefore 4h - 12 = \pm 20$

4h – 12= 20 (or) 4h – 12 = -20, 4h = 32 (or) 4h=-8, h = 8 (or) h = - 2

Now the points on the **X** – axis are (8, 0) and (-2, 0)

5. Find the distance between the parallel lines (i) 15x+8y–34 =0, and 15x+8y+31=0

(ii) $\ell(x+y)$+p = 0 and $\ell(x+y)$ – r = 0

Sol: (i) Given $15x + 8y - 34 = 0$

$$15x+8y+31=0$$

The distance between parallel lines ax+by+c=0 and ax+by+d=0 is $\frac{|c-d|}{\sqrt{a^2+b^2}}$

$$\therefore \frac{|-34-31|}{\sqrt{225+64}} = \frac{65}{17}$$

(ii) Given lines are $\ell x + \ell y + p = 0$ and $\ell x + \ell y - r = 0$

Now distance between parallel lines = $\frac{|c-d|}{\sqrt{a^2+b^2}} = \frac{|p+r|}{\sqrt{l^2+l^2}} = \frac{|p+r|}{\sqrt{2}l}$

6. Find the equation of the line parallel to the line 3x – 4y +2 = 0 and passing through the point (-2, 3).

Sol: Equation of a line passing through (x_1, y_1) and parallel to ax + by + c = 0 is

$a(x - x_1) + b(y - y_1) = 0.$

$\therefore$ Given line $3x - 4y + 2 = 0$, point $(x_1, y_1) = (-2, 3)$

$3(x+2) - 4(y-3) = 0 \Rightarrow 3x - 4y + 18 = 0$

7. Find the equation of the line perpendicular to the line x – 7y + 5 = 0 and having x – intercept 3.

Sol: Equation of line passing through (x_1, y_1) and perpendicular to the line ax+by+c=0 is

$b(x-x_1) - a(y-y_1) = 0$.

Given line $x - 7y + 5 = 0$ and point is (3, 0)

$-7(x-3) - 1(y-0) = 0 \Rightarrow -7x - y + 21 = 0 \Rightarrow 7x + y - 21 = 0$

8. Find the angle b/w the lines $\sqrt{3}x+y=1$ and $x + \sqrt{3}\,y = 1$.

Sol: Given lines are $\sqrt{3}\,x + y = 1 \Rightarrow y = -\sqrt{3}\,x + 1$

$m_1 = -\sqrt{3}$, $c_1 = 1 \Rightarrow$ and $x + \sqrt{3}\,y = 1 \Rightarrow y = \frac{-1x}{\sqrt{3}} + \frac{1}{\sqrt{3}}$, $m_2 = \frac{-1}{\sqrt{3}}$, $C_2 = \frac{1}{\sqrt{3}}$

$$\mathrm{Tan}\,\theta = \left|\frac{m_1 - m_2}{1+m_1m_2}\right| = \left|\frac{-\sqrt{3}+\frac{1}{\sqrt{3}}}{1+\left(-\sqrt{3}\right)\left(-\frac{1}{\sqrt{3}}\right)}\right| = \left|\frac{\frac{-3+1}{\sqrt{3}}}{\left(\frac{2}{1}\right)}\right| = \pm\frac{1}{\sqrt{3}} \qquad \therefore \theta = 30^0 \text{ (or) } \theta = 150^0$$

9. The line through the points (h, 3) and (4, 1) intersects the line $7x - 9y - 19 = 0$ at right angle then find the value of h.

Sol: Let A = (h, 3) and B = (4, 1)

Slope of AB $= \frac{1-3}{4-h} = \frac{-2}{4-h}$

Given line $7x - 9y - 19 = 0$ …..(1)

Slope of (1) is $= \frac{-7}{-9} = \frac{7}{9}$ $\qquad \therefore m_1m_2 = -1 \Rightarrow \left(\frac{2}{h-4}\right)\left(\frac{7}{9}\right) = -1$

$\Rightarrow 14 = -9h + 36 \qquad 9h = 22 \qquad h = \frac{22}{9}$.

10. Find the ratio in which the straight line $3x + 4y = 6$ divides the line segment joining the points (2, -1) and (1, 1).

Sol: The ratio of the line $ax + by + c = 0$ divides the line segment joining the points (x_1, y_1) and (x_2, y_2) is $= \frac{-(ax_1+by_1+c)}{(ax_2+by_2+c)}$.

Given line 3x + 4y – 6 = 0 and $(x_1, y_1) = (2, -1)$, $(x_2, y_2) = (1, 1)$

Ratio $= \frac{-(6-4-6)}{(3+4-6)} = \frac{4}{1}$; Ratio is 4 : 1, opposite signs $L_{11} < 0, L_{22} > 0$

11.Find the point of intersection of the lines $7x + y + 3 = 0$ and $x + y = 0$

Sol: Given lines $7x + y + 3 = 0$ and $x + y = 0$

Let p (x_1, y_1) be the point of intersection

$\therefore\ 7x_1 + y_1 + 3 = 0$...........(1)

$x_1 + y_1 + 0 = 0$...........(2)

Solve (1) and (2) we get $x_1 = \frac{-3}{6} = \frac{-1}{2}$, $y_1 = \frac{3}{6} = \frac{1}{2}$

point of inter section = $\left(\frac{-1}{2}, \frac{1}{2}\right)$

12. Show that the line (a – b) x +(b-c) y = c – a, (b – c) x + (c – a) y = a – b

and (c – a) x + (a – b) y = b – c are concurrent.

Sol: Note : If $L_1 = a_1x + b_1y + c_1 = 0$, $L_2 = a_2x + b_2y + c_2 = 0$, $L_3 = a_3x + b_3y + c_3 = 0$, no two lines are parallel and if non – zero numbers λ_1, λ_2 and λ_3 exists such that $\lambda_1 L_1 + \lambda_2 L_2 + \lambda_3 L_3 = 0$ then $L_1 = 0$, $L_2 = 0$ and $L_3 = 0$ are concurrent.

Let $\lambda_1 = \lambda_2 = \lambda_3 = 1$.

Given lines $L_1 + L_2 + L_3 =$ (a – b + b – c + c – a) x +(b - c + c – a + a – b) y

$$0.x + 0.y = 0, \Rightarrow 0 = 0$$

$\therefore$ The lines are concurrent.

13. Find the value of P, if the lines x + p = 0, y + 2 = 0 and 3x + 2y + 5 = 0 are concurrent.

Sol: Given lines are x + p = 0…….. (1), y + 2 = 0 ……. (2), and 3x + 2y + 5 = 0 ……(3)

Solve (1) and (2) we get P = (– P, - 2) put in (3)

-3p – 4 + 5 = 0, -3p + 1 = 0, P = $\frac{1}{3}$.

14. Find the area of the triangle formed by the following straight lines and the co-ordinate axes (i) x – 4y + 2 = 0, (ii) 3x – 4y + 12 = 0

Sol: Note :- The area of the triangle formed by the straight line ax+by+c=0 with coordinate axes is $\frac{c^2}{2|ab|}$ sq.u.

(i) Given line x – 4y + 2 = 0, Hence area = $\frac{c^2}{2|ab|} = = \frac{4}{2|(1)(-4)|} = \frac{1}{2} sq.u.$

(ii) Given line 3x – 4y + 12 = 0, Hence area $= \frac{c^2}{2|ab|} = \frac{144}{2|(3)(-4)|} = \frac{144}{24} = 6$ Sq.u

II) 1. Prove that the line through the point (x_1, y_1) and parallel to the line Axx+By+C= 0 is A(x – x_1)+B(y – y_1) = 0.

Sol: Given line Ax + By + C = 0 …….(1)

Equation of the line parallel to line (1) is Ax + By + K = 0 ….(2)

If (2) passing through (x_1, y_1) we get $Ax_1 + By_1 + K = 0$

K = -A x_1 - B y_1, put in (2) A x + B y – A x_1 – B y_1 = 0 $\Rightarrow A(x-x_1)+B(y-y_1)=0$

2. Two lines passing through the point (2, 3) intersect each other at an angle of 60^0. If slope of one line is 2, find the equation of the other line.

Sol: Let m be the slope of the required line

Let m_1 = m and m_2 = 2 given line slope

Now $Tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$Tan60^0 = \dfrac{m-2}{1+2m} \Rightarrow \pm\sqrt{3}(1+2m) = m-2 \Rightarrow \sqrt{3}+2\sqrt{3}m = m-2$ (or) - $\sqrt{3}-2\sqrt{3}m = m-2$

$m(2\sqrt{3}-1) = -2-\sqrt{3}$ | $(2\sqrt{3}+1)m = 2-\sqrt{3}$

$\therefore m = \dfrac{-(2+\sqrt{3})}{(2\sqrt{3}-1)}$ | $m = \dfrac{2-\sqrt{3}}{2\sqrt{3}+1}$

Required equations are

$y - 3 = \dfrac{-(2+\sqrt{3})}{(2\sqrt{3}-1)}(x-2)$ and $y-3 = \dfrac{(2-\sqrt{3})}{(2\sqrt{3}+1)}(x-2)$

$\therefore (2+\sqrt{3})x + (2\sqrt{3}-1)y = +8\sqrt{3}+1$ and $(2-\sqrt{3})x - (2\sqrt{3}+1)y + 8\sqrt{3} - 1 = 0$

3. Find the equation of the right bisector of the line segment joining the points

(3, 4) and (-1, 2).

Sol: Let the given points are P(3, 4) and Q(-1, 2).

Let M be the midpoint PQ : M = $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right) = \left(\dfrac{3-1}{2}, \dfrac{4+2}{2}\right) = (1,3)$

Slope of $\overline{PQ} = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{2-4}{-1-3} = \dfrac{-2}{-4} = \dfrac{1}{2}$

Slope of perpendicular bisector is = $\dfrac{-1}{slope\, of\, \overline{PQ}} = -2$

Equation of right bisector is $y - y_1$ = m(x - x_1)

$\Rightarrow y - 3 = -2(x-1) \Rightarrow y - 3 = -2x + 2 \Rightarrow 2x + y - 5 = 0$

4. Find the co-ordinate of the foot of the perpendicular from the point (-1, 3) to the line $3x - 4y - 16 = 0$.

Sol: If Q(h, k) is the foot of the perpendicular from P(x_1, y_1) on the straight line

ax + by +c = 0 then (h – x_1) : a = (k – y_1) : b = -(ax_1 + by_1 + c) : a^2+b^2.

Given P = (x_1, y_1) = (-1, 3), Q = (h, k) be the foot of the perpendicular.

Given line $3x - 4y - 16 = 0$

$$\therefore \frac{h+1}{3} = \frac{k-3}{-4} = \frac{(-3-12-16)}{9+16} = \frac{31}{25} \Rightarrow h + 1 = \frac{93}{25},\ K - 3 = \frac{124}{25}$$

$$h = \frac{68}{25},\ k = -\frac{49}{25} \Rightarrow (h, k) = \left(\frac{68}{25}, \frac{-49}{25}\right)$$

5. The perpendicular from the origin to the line y = mx +c meets it at the point (-1, 2). Find the values of m and c.

Sol: Let 0 = (0, 0) and given point P = (-1, 2)

Given line is y = mx + c ....(1)

(slope of $\overline{OP}$) x (slope of line(1)) = - 1

$$\left(\frac{2}{-1}\right)(m) = -1,\ m = \frac{1}{2}$$

The point (-1, 2) lies on the line (1) $2 = -m + c \Rightarrow 2 = -\frac{1}{2} + c \Rightarrow c = \frac{5}{2}$

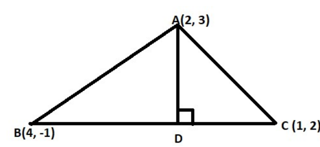
$$\therefore\ m = \frac{1}{2} \text{ and } c = \frac{5}{2}$$

6. If p and q are the lengths of perpendiculars from origin to the lines x cosθ - y sinθ = kcos2θ and x secθ + y cosecθ = k respectively then prove that $p^2 + 4q^2 = k^2$.

Sol: Given lines are x cosθ - y sinθ = kcos2θ............(1)

and x secθ + y cosecθ = k

$$\frac{x}{\cos\theta} + \frac{y}{\sin\theta} = k \Rightarrow x\sin\theta + y\cos\theta = \frac{k}{2}\sin 2\theta \ \text{.......... (2)}$$



Given p = perpendicular distance from (0, 0) to line ....(1)

$$= \frac{|0 - 0 - K\cos 2\theta|}{\sqrt{\cos^2\theta + \sin^2\theta}} = |k\cos 2\theta|$$

q = perpendicular distance from (0, 0) to line ......(2)

$$= \frac{\left|0 - 0 - \frac{K}{2}\sin 2\theta\right|}{\sqrt{\sin^2\theta + \cos^2\theta}} = \left|\frac{k}{2}\sin 2\theta\right|$$

Now L.H.S $= p^2 + 4q^2$

$= k^2\cos^2 2\theta + 4\left(\frac{k^2}{4}\sin^2 2\theta\right) = k^2\left(\cos^2 2\theta + \sin^2 2\theta\right) = k^2 = \text{R.H.S}$

7. In the ΔABC with vertices A(2, 3), B(4, -1) and C(1, 2) find the equation and length of the altitude from the vertex A.

Sol: Given vertices of a ΔABC are A=(2, 3), B = (4, -1), C = (1, 2)

Let AD be the altitude.

(slope of BC) (slope of AD) = -1

$\left(\frac{2+1}{1-4}\right)$ (slope of AD) = - 1 $\Rightarrow$ Slope of AD = 1

Equation of $\overline{AD}$ is $y - y_1 = m(x - x_1) \Rightarrow y - 3 = 1(x - 2) \Rightarrow x - y + 1 = 0$

Equation of $\overline{BC}$ is $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) \Rightarrow y + 1 = \left(\frac{2+1}{1-4}\right)(x-4)$

$\Rightarrow y + 1 = -x + 4 \Rightarrow x + y - 3 = 0$

Length of altitude $\overline{AD} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}} = \frac{|2+3-3|}{\sqrt{1+1}} - \frac{2}{\sqrt{2}} = \sqrt{2}$

8. If P is the length of the perpendicular from origin to the line whose intercepts on the axes are a and b then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

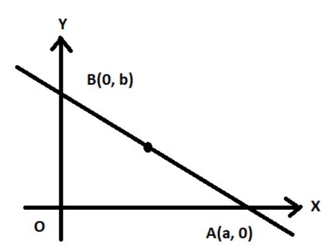
Sol: Equation of the line in the intercept form is $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow bx + ay - ab = 0$ ……(1)

Given p = perpendicular distance from (0, 0) to the line(1)

$p = \frac{|-ab|}{\sqrt{b^2+a^2}} \Rightarrow \frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2} = \frac{1}{a^2} + \frac{1}{b^2}$

9. A straight line meets the co-ordinate axes in A and B. Find the equation of the straight line when (i) $\overline{AB}$ is divided in the ratio 2 : 3 at (-5, 2) (ii) $\overline{AB}$ is divided in the ratio 1 : 2 at (-5, 4) (iii) (p, q) bisects $\overline{AB}$.

Sol: Let the line meets the co-ordinate axes at A(a, 0) and B(0, b). Such that OA = a, OB = b.



(i) Let P(-5, 2) divides $\overline{AB}$ in the ratio 2 : 3 then $\left(\frac{0+3a}{5}, \frac{0+2b}{5}\right) = (-5, 2)$

$\frac{3a}{5} = -5, \frac{2b}{5} = 2 \Rightarrow a = \frac{-25}{3}, b = 5$

Now equation of the straight line is $\frac{-3x}{25}+\frac{y}{5}=1 \Rightarrow 3x - 5y + 25 = 0$

(ii) Let P(-5, 4) divides $\overline{AB}$ in the ratio 1 : 2

$\therefore \left(\frac{2a}{3},\frac{b}{3}\right)=(-5,4) \Rightarrow \frac{2a}{3}=-5, \frac{b}{3}=4 \Rightarrow a=\frac{-15}{2}, b=12$

Now equation $\overline{AB}$ is $\frac{-2x}{15}+\frac{y}{12}=1 \Rightarrow -8x + 5y = 60 \Rightarrow 8x - 5y + 60 = 0$

(iii) Let P(p, q) bisects the line segment $\overline{AB}$

$\therefore \left(\frac{a+0}{2},\frac{0+b}{2}\right)=(p,q) \Rightarrow$ a = 2p, b = 2q

Now equation of the straight line in $\frac{x}{2p}+\frac{y}{2q}=1 \Rightarrow \frac{x}{p}+\frac{y}{q}=2$

10. A triangle of area 24 sq. units is formed by a straight line and the co-ordinate axes in the first quadrant. Find the equation of a straight line, if it passes through (3, 4).

Sol: Let the straight line meets **X** -axis at A(a, 0) and **Y**-axis at B(o, b)

Area of ΔOAB = 24 $\Rightarrow \frac{1}{2}ab = 24 \Rightarrow$ ab = 48… (1)

Intercept form of the straight line is $\frac{x}{a}+\frac{y}{b}=1$, this passing through (3, 4)

We get $\frac{3}{a}+\frac{4}{b}=1 \Rightarrow$ 3b + 4a = ab $\Rightarrow$ 4a + 3b= 48…(2)

4a – 3b = $\sqrt{(4a+3b)^2-4(4a)(3b)} = \sqrt{(48)^2-(48)^2}$ =0 $\Rightarrow$ 4a – 3b = 0 …(3)

Solve (2) and (3)

$$\begin{array}{lr} 4a + 3b = 48 & \\ 4a - 3b = 0 & \text{(add)} \\ \hline 8a \quad = 48 \Rightarrow a = 6 & \end{array}$$

Put a = 6 in equation (3) 24 – 3b = 0

3b = 24 $\Rightarrow$ b = 8

Now equation is $\frac{x}{6}+\frac{y}{8}=1 \Rightarrow 4x+3y-24=0$

11. Find the set of values of 'a' if the prints (1, 2) and (3, 4) lie to the same side of the straight line 3x – 5y + a = 0.

Sol: Find $L_{11}=ax_1+by_1+C$, $L_{22}=ax_2+by_2+C$

case(i) $L_{11}<0$ and $L_{22}<0$ then points lies on the same side.

$3 - 10 + a < 0$ and $9 - 20 + a < 0$

$a < 7$ and $a < 11$

$a < 7$-------(1)

case(ii) $L_{11} > 0$ and $L_{22} > 0$ then also points lies on the same side of the line now

3-10+a>0 and 9-20+a>0, a>7 and a>11 $\Rightarrow$ a>11--------(2)

$\therefore$ The set of values of a is $(-\infty, 7) \cup (11, \infty)$

12. Show that the lines $2x + y - 3 = 0$, $3x + 2y - 2 = 0$ and $2x - 3y - 23 = 0$ are con current. Find the point of concurrency.

Sol: Given lines are $2x + y - 3 = 0$……….(1), 3x $x + 2y - 2 = 0$……..(2)

$2x - 3y - 23 = 0$…….. (3)

Point of intersection of (1) and (2) is $\frac{x}{-2+6} = \frac{y}{-9+4} = \frac{1}{4-3} \Rightarrow \frac{x}{4} = \frac{y}{-5} = \frac{1}{1}$; x = 4, y = 5

Point of intersection is P(4, -5) put in equation …(3)

2(4)-3(-5)-23 = 0 $\Rightarrow$ 8 + 15 – 23 = 0 $\Rightarrow$ 23 – 23 = 0 ; 0 = 0

Hence the lines are concurrent and point of concurrency is (4, -5).

13. Find the value of p if the following lines are concurrent (i) $3x+4y=5$, $2x+3y=4$, $px+4y = 6$.

Sol: Given lines are $3x+4y - 5 = 0 \rightarrow (1)$, $2x+3y - 4 = 0 \rightarrow (2)$, $px+4y - 6 = 0 \rightarrow (3)$

Solve (1) and (2) $\frac{x}{-16+15} = \frac{y}{-10+12} = \frac{1}{9-8} \Rightarrow \frac{x}{-1} = \frac{y}{2} = \frac{1}{1} \Rightarrow (x, y) = (-1, 2)$

Put (-1, 2) in equation (3) we get p(-1) + 4(2) – 6 = 0 $\Rightarrow$ -p + 2 = 0; p = 2

14. If 3a + 2b + 4c = 0 then show that the equation $ax+by+c=0$ represents a family of concurrent straight lines and find the point of concurrency.

Sol: Given line $ax+by+c=0$ …………….(1)

Given condition is 3a+2b+4c=0 $\Rightarrow$ 4c = -3a-2b $\Rightarrow C = \frac{-(3a+2b)}{4}$ put in equation (1)

we get $ax + by - \frac{(3a+2b)}{4} = 0 \Rightarrow$ $4ax + 4by - 3a - 2b = 0$

$a(4x-3)+b(4y-2) = 0$ is in the form of $\lambda_1 L_1 + \lambda_2 L_2 = 0$ represents family of lines $\therefore L_1 = 4x - 3 = 0$, $L_2 = 4y - 2 = 0$

The point of intersection of L_1 and L_2 is point of concurrency

$\therefore$ $4x = 3$ | $4y = 2$

$x = \frac{3}{4}$ $\quad y = \frac{1}{2}$

Point = $\left(\frac{3}{4},\frac{1}{2}\right)$.

III) 1) Find the point on the straight line $3x + y + 4 = 0$ which is equidistance from the points (-5, 6) and (3, 2).

Sol: Given equation of a line is $3x + y + 4 = 0$ ………………(1)

Let A(-5, 6) and B(3, 2) are two given points

Let P(α,β) be any point on the line and equidistance to A and B.

P(α,β) lies on (1) then $3\alpha+\beta+4=0$……..(2)

and PA = PB, $PA^2 = PB^2$ $\quad \Rightarrow (\alpha+5)^2+(\beta-6)^2=(\alpha-3)^2+(\beta-2)^2$

$\Rightarrow \alpha^2+25+10\alpha+\beta^2+36-12\beta=\alpha^2+9-6\alpha+\beta^2+4-4\beta \Rightarrow 16\alpha-8\beta-48=0$

$\Rightarrow 2\alpha-\beta+6=0$………..(3)

Solve (2) and (3) $\dfrac{\alpha}{\begin{matrix}1 & 4\\ -1 & 6\end{matrix}} \dfrac{\beta}{\begin{matrix}3 & 1\\ 2 & -1\end{matrix}}\dfrac{1}{}$ $(\alpha,\beta)=\left(\frac{6+4}{-3-2},\frac{8-18}{-3-2}\right)=\left(\frac{10}{-5},\frac{-10}{-5}\right)$ = (-2, 2)

2. A straight line through P(3, 4) makes an angle of 60^0 with the positive direction of the X – axis. Find the co-ordinates of the points on the line which are 5 units away from P.

Sol: Given point P = (3, 4) = (x_1, y_1)

Given angle $\theta=60^0$

Given distance r = 5

The points on the line are $(x_1 \pm r\cos\theta, y_1 \pm r\sin\theta)$

$= (3+5\cos 60^0, 4+5\sin 60^0) = \left(3+\frac{5}{2},4+\frac{5\sqrt{3}}{2}\right) = \left(\frac{6+5}{2},\frac{8+5\sqrt{3}}{2}\right)=\left(\frac{11}{2},\frac{8+5\sqrt{3}}{2}\right)$

and also $(x_1 - r\cos\theta, y_1 - r\sin\theta)$

$= (3-5\cos 60^0, 4-5\sin 60^0)=\left(3-\frac{5}{2},4-\frac{5\sqrt{3}}{2}\right)=\left(\frac{1}{2},\frac{8-5\sqrt{3}}{2}\right)$.

3. A straight line through Q($\sqrt{3}$, 2) makes an angle $\frac{\pi}{6}$ with the positive direction of X–axis. If the straight line intersects the line $\sqrt{3}x - 4y + 8=0$ at P. Find the distance PQ.

Sol: Given line $\sqrt{3}x - 4y + 8 = 0$ …….(1)

Given point Q(x_1, y_1) = ($\sqrt{3}$, 2), Let PQ = r and angle $\theta = \frac{\pi}{6}$.

$\therefore$ P $= \left(x_1 + r\cos\theta, y_1 + r\sin\theta\right) = \left(\sqrt{3} + r\cos\frac{\pi}{6}, 2 + r\sin\frac{\pi}{6}\right)$

P $= \left(\sqrt{3} + \frac{r\sqrt{3}}{2}, 2 + \frac{r}{2}\right)$ lies on (1)

$\therefore \sqrt{3}\left(\sqrt{3} + \frac{r\sqrt{3}}{2}\right) - 4\left(2 + \frac{r}{2}\right) + 8 = 0 \Rightarrow 3 + \frac{3r}{2} - 8 - 2r + 8 = 0$

$\Rightarrow 3 - \frac{r}{2} = 0$; $\Rightarrow r = 6$; $\therefore$ the distance PQ = r = 6.

4. A straight line through Q(2, 3) makes an angle $\frac{3\pi}{4}$ with the negative direction of X-axis. If the straight line intersects the line $x + y - 7 = 0$ at P. find the distance PQ.

Sol: Given line $x + y - 7 = 0$ ………………(1)

Given point Q = (x_1, y_1) = (2, 3) and angle $\frac{3\pi}{4}$ with negative direction means $\frac{\pi}{4}$ in the positive direction of X – axis

$\therefore \theta = \frac{\pi}{4}$, Let PQ = r

Now P = $\left(x_1 + r\cos\theta, y_1 + r\sin\theta\right) = \left(2 + r\cos\frac{\pi}{4}, 3 + r\sin\frac{\pi}{4}\right) = \left(2 + \frac{r}{\sqrt{2}}, 3 + \frac{r}{\sqrt{2}}\right)$

P lies on line (1) then we get, $2 + \frac{r}{\sqrt{2}} + 3 + \frac{r}{\sqrt{2}} - 7 = 0 \Rightarrow \frac{2r}{\sqrt{2}} = 2$; $r = \sqrt{2}$

$\therefore$ The distance PQ = $r = \sqrt{2}$

5. Show that the lines $x + y = 0$, $3x + y = 0$ and $3x + y - 4 = 0$ forms an isosceles triangle.

Sol: Given straight lines are $x + y = 0$ …..(1), $3x + y - 4 = 0$ …….(2) , $x + 3y - 4 = 0$ ….(3)

Let A$\left(x_1, y_1\right)$ be the point of intersection of (1) and (2)

A=$\left(x_1, y_1\right) = \left(\frac{-4-0}{1-3}, \frac{0+4}{1-3}\right) = \left(2, -2\right)$

Let B = (x_2, y_2) be the point of intersection of (2) and (3)

B = $\left(x_2, y_2\right) = \left(\frac{-4+12}{9-1}, \frac{-4+12}{9-1}\right) = \left(1, 1\right)$

Let C = $\left(x_3, y_3\right)$ be the point of intersection of (3) and (1)

C = $\left(x_3, y_3\right) = \left(\frac{0+4}{1-3}, \frac{-4-0}{1-3}\right)$ = (-2, 2)

Vertices of triangle are A = (2, -2), B = (1, 1), C = (-2, 2)

AB = $\sqrt{1+9} = \sqrt{10}, BC = \sqrt{9+1} = \sqrt{10}$

CA = $\sqrt{16+16} = \sqrt{32}$

$\therefore AB = BC = \sqrt{10}$

$\therefore \Delta$ ABC is isosceles triangle.

EXERCISE : 9(d)

I) 1. Find the value of K for which the line (k -3)x – (4-k^2) y +k^2-7k+6=0 is
(a) parallel to the x-axis (b) parallel to the y-axis (c) passing through origin.

Sol: Equation of given line is (k – 3) x -(4-k^2) y +(k^2 – 7k + 6) = 0….. (1)

(a) If line (1) parallel to x – axis then slope = 0 $\Rightarrow \frac{-\left(k-3\right)}{-\left(4-k^2\right)} = 0 \Rightarrow k = 3$

(b) If line (1) parallel to y-axis then slope = $\infty = \frac{1}{0}$

$\therefore \frac{k-3}{4-k^2} = \infty \Rightarrow 4-k^2 = 0;\ k = \pm 2$

(c) If a line (1) passing through O(0, 0) then constant is zero

$\therefore k^2 - 7k + 6 = 0 \Rightarrow k^2 - 6k - 1k + 6 = 0 \Rightarrow \left(k-1\right)\left(k-6\right) = 0$

$\therefore k = 1 \ or\ k = 6$

2. Find the equations of the lines which cut off intercepts on the axes whose sum and product are 1 and – 6 respectively.

Sol: Let the intercept form of the straight line is $\frac{x}{a} + \frac{y}{b} = 1$........(1)

Given a + b = 1 ………..(2), a b = - 6 ……..(3)

Solve (2) and (3) we get a – a^2 = -6 $\Rightarrow$ a = - 2 (or) a = 3

If a = -2 we get b = 3 then equation is $\frac{-x}{2} + \frac{y}{3} = 1 \Rightarrow$ 3x – 2y + 6 = 0

If a = 3 we get b = -2 then equation is $\frac{x}{3} - \frac{y}{2} = 1 \Rightarrow$ 2x – 3y – 6 = 0

3. What are the point on the y – axis whose distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ is 4 units.

Sol: Let (0, h) be any point on the y – axis given line is 4x + 3y -12 = 0 …..(1)

Given perpendicular distance from (0, h) to line (1) is 4

$\therefore \dfrac{|0+3h-12|}{\sqrt{16+9}}=4 \Rightarrow |3h-12|=20 \Rightarrow 3h - 12 = \pm 20 \Rightarrow h = \dfrac{32}{3}$ (or) $h = -\dfrac{8}{3}$

$\therefore$ The points on y-axis are $\left(0,\dfrac{32}{3}\right)$ (or) $\left(0,\dfrac{-8}{3}\right)$

4. Find the perpendicular distance from the origin to the line joining the points ($\cos\theta$, $\sin\theta$) and ($\cos\phi$, $\sin\phi$).

Sol: Equation of the line joining the points

Let A = ($\cos\theta$, $\sin\theta$) and B = ($\cos\phi$, $\sin\phi$)

Slope of $\overline{AB} = \dfrac{\sin\phi-\sin\theta}{\cos\phi-\cos\theta} = \dfrac{2\cos\left(\dfrac{\phi+\theta}{2}\right)\sin\left(\dfrac{\phi-\theta}{2}\right)}{-2\sin\left(\dfrac{\phi+\theta}{2}\right)\sin\left(\dfrac{\phi-\theta}{2}\right)} = -\dfrac{\cos\left(\dfrac{\phi+\theta}{2}\right)}{\sin\left(\dfrac{\phi+\theta}{2}\right)}$

Equation of the line $\overline{AB}$ is y - $\sin\theta$ = $\dfrac{-\cos\left(\dfrac{\phi+\theta}{2}\right)}{\sin\left(\dfrac{\phi+\theta}{2}\right)}(x-\cos\theta)$

$y\sin\left(\dfrac{\theta+\phi}{2}\right) - \sin\left(\dfrac{\theta+\phi}{2}\right)\sin\theta = -x\cos\left(\dfrac{\theta+\phi}{2}\right)+\cos\theta\cos\left(\dfrac{\theta+\phi}{2}\right)$

$x\cos\left(\dfrac{\theta+\phi}{2}\right)+y\sin\left(\dfrac{\theta+\phi}{2}\right) = \cos\theta\cos\left(\dfrac{\phi+\theta}{2}\right)+\sin\theta\sin\left(\dfrac{\phi+\theta}{2}\right)$

$x\cos\left(\dfrac{\phi+\theta}{2}\right)+y\sin\left(\dfrac{\phi+\theta}{2}\right) = \cos\left(\dfrac{\phi-\theta}{2}\right)$.........(1)

perpendicular distance from 0(0, 0) to line (1) is $P = \dfrac{\left|0+0-\cos\left(\dfrac{\theta-\phi}{2}\right)\right|}{\sqrt{\cos^2\left(\dfrac{\phi+\theta}{2}\right)+\sin^2\left(\dfrac{\phi+\theta}{2}\right)}}$

$$P=\left|\cos\left(\dfrac{\theta-\phi}{2}\right)\right|$$

5. Find the equation of the line parallel to y-axis and drawn through the point of intersection of the lines $x - 7y + 5 = 0$ and $3x + y = 0$.

Sol: Given lines are $x - 7y + 5 = 0$.......(1), $3x + y = 0$..........(2)

Put $y = -3x$ in equation (1) $\Rightarrow x + 21x + 5 = 0 \Rightarrow x = \dfrac{-5}{22}$

Put $x = \frac{-5}{22}$ we get $y = \frac{15}{22}$

$\therefore P = \left(\frac{-5}{22}, \frac{15}{12}\right)$

Any line parallel to y-axis is in the form $x = \text{h} \Rightarrow x = \frac{-5}{22} \Rightarrow 22x + 5 = 0$

6. Find the equation of a line drawn perpendicular to the line $\frac{x}{4} + \frac{y}{6} = 1$ through the point where it meets the y-axis.

Sol: Give line is $\frac{x}{4} + \frac{y}{6} = 1 \Rightarrow 6x + 4y - 24 = 0 \Rightarrow 3x + 2y - 12 = 0$……..(1)

Line (1) meets the y-axis at P = (0, 6)

Equation of perpendicular line is $b(x - x_1) - a(y - y_1) = 0$

$\Rightarrow 2(x - 0) - 3(y - 6) = 0 \Rightarrow 2x - 3y + 18 = 0$

7. Find the length of the perpendicular from the point (3, 4) to the straight line $3x - 4y + 10 = 0$.

Sol: Perpendicular distance from (3, 4) to the line

$3x - 4y + 10 = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|9 - 16 + 10|}{\sqrt{9 + 16}} = \frac{3}{5}$

8. Find the distance between the following parallel lines (i) $3x - 4y - 12 = 0$ and $3x - 4y - 7 = 0$ (ii) $5x - 3y - 4 = 0$ and $10x - 6y - 9 = 0$.

Sol: The distance between the parallel lines

$ax + by + c = 0$ and $ax + by + d = 0$ is $\frac{|c - d|}{\sqrt{a^2 + b^2}}$

(i) Given lines are $3x - 4y - 12 = 0$… (1)and $3x - 4y - 7 = 0$ … (2)

Distance between parallel lines (1) and (2) is $\frac{|c - d|}{\sqrt{a^2 + b^2}} = \frac{|-12 + 7|}{\sqrt{9 + 16}} = \frac{5}{5} = 1$

(ii) Given lines are $5x - 3y - 4 = 0$ and $10x - 6y - 9 = 0$
First equation can be taken as $10x - 6y - 8 = 0$ ………(1) x y
and second equation $10x - 6y - 9 = 0$ …….(2)
distance between parallel lines (2) and (2) is $= \frac{|-8 + 9|}{\sqrt{100 + 36}} = \frac{1}{\sqrt{136}} = \frac{1}{2\sqrt{34}}$

9. Find the value of K if the straight lines $y - 3kx + 4 = 0$ and

(2k -1) x -(8k-1) y – 6 = 0 are perpendicular.

Sol: Given line are $y - 3kx + 4 = 0$ ……(1)

Slope of (1) is m1 = 3k And (2k – 1) x – (8k – 1)y -6 = 0 …………(2)

Slope of (2) is $m_2 = \frac{2k-1}{8k-1}$

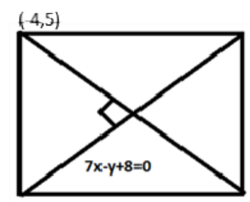
$$\therefore m_1 m_2 = -1 \Rightarrow (3k)\frac{(2k-1)}{(8k-1)} = -1 \Rightarrow 6k^2 - 3k + 8k - 1 = 0 \Rightarrow 6k^2 + 6k - k - 1 = 0$$

$\Rightarrow$ 6k(k+1) – 1(k+1) = 0

$\therefore$ k = $+\frac{1}{6}$ (or) k = -1

10. (-4, 5) is a vertex of a square and one diagonal is $7x - y + 8 = 0$. Find the equation of the other diagonal.

Sol: Given point (-4,5) does not lies on the given diagonal.



The other diagonal is perpendicular line

$\therefore b(x - x_1) - a(y - y_1) = 0$

$\Rightarrow -(x + 4) - 7(y - 5) = 0 \Rightarrow -x - 7y + 31 = 0$

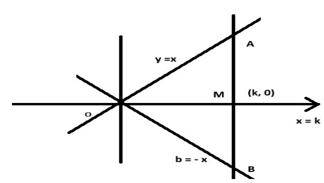
$\therefore x + 7y - 31 = 0$

II. 1) Find the area of the triangle formed by the lines $y - x = 0$ and $x + y = 0$ and x -k=0.

Sol: Given lines are y = x … (1) , $y = -x$ … (2) and x = k ………..(3) x y

(1) and (2) are perpendicular lines O(0, 0), A=(k, k),

B= (k, -k)



Area of ΔOAB = $\frac{1}{2}\left|(OM)(AB)\right|$

$$= \frac{1}{2}\left|(k)(2k)\right| = \frac{2k^2}{2} = k^2 \text{ units}$$

2. Find the value of P so that the three lines $3x + y$ -2 = 0, px +2y -3=0 and 2x - y -3 = 0 may intersect at one point .

Sol: Given that lines are con current and

$3x + y - 2 = 0$….(1), Px +2y - 3= 0 … (2), $2x - y - 3 = 0$ ……(3)

Solve (1) and (3) we get (x, y) = $\left(\frac{-3-2}{-3-2}, \frac{-4+9}{-3-2}\right) = (1, -1)$

Put (1, -1) in equation (2)

P(1) +2(-1) - 3 = 0 ; P = 5

3) If three lines whose equations are y = $m_1 x + c_1$, $y = m_2 x + c_2$ and $y = m_3 x + c_3$ are concurrent then show that $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$.

Sol: Given lines are $m_1 x - y + c_1 = 0 .... \rightarrow (1)$, $m_2 x - y + c_2 = 0 .... \rightarrow (2)$

$m_3 x - y + c_3 = 0 .... \rightarrow (3)$ are concurrent

solving (1) and (2) we get $(x, y) = \left(\frac{c_1 - c_2}{m_2 - m_1}, \frac{c_1 m_2 - c_2 m_1}{m_2 - m_1}\right)$

Substituting $(x, y) = \left(\frac{c_1 - c_2}{m_2 - m_1}, \frac{c_1 m_2 - c_2 m_1}{m_2 - m_1}\right)$ in equation (1)

We get $\frac{m_3(c_1 - c_2)}{(m_2 - m_1)} - \frac{(c_1 m_2 - c_2 m_1)}{(m_2 - m_1)} + c_3 = 0 \Rightarrow m_3(c_1 - c_2) - (c_1 m_2 - c_2 m_1) + c_3(m_2 - m_1) = 0$

$\Rightarrow m_3(c_1 - c_2) - c_1 m_2 + c_2 m_1 + c_3 m_2 - c_3 m_1 = 0 \Rightarrow m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$

4. Find the equation of the line through the point (3, 2) which makes an angle of 45^0 with line $x - 2y = 3$.

Sol: Let m be the slope of the line which makes an angle 45^0 with given line $x - 2y = 3$ and passing through (3, 2).

Let $m_1 = m$, m_2 = slope of line $x - 2y - 3 = 0 = \frac{1}{2}$ and $\theta = 45^0$

$$\therefore Tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow Tan45^0 = \left|\frac{m_1 - \frac{1}{2}}{1 + \frac{m}{2}}\right| = \left|\frac{2m - 1}{2 + m}\right| = 1$$

$2m - 1 = \pm(2 + m) \Rightarrow 2m - 1 = 2 + m$ (or) $2m - 1 = -2 - m$

$m = 3 \Rightarrow 3m = -1 \Rightarrow m = \frac{-1}{3}$

case (i) $m = 3$, $(x_1, y_1) = (3, 2)$

$y - 2 = 3(x - 3) \Rightarrow 3x - y - 7 = 0$

Case (ii) $m = \frac{-1}{3}$, $(x_1, y_1) = (3, 2)$

$y - 2 = \frac{-1}{3}(x - 3) \Rightarrow 3y - 6 = -x + 3 \Rightarrow x + 3y - 9 = 0$

Required equations are $3x - y - 7 = 0$, $x + 3y - 9 = 0$.

5. Find the equation of the line passing through the point of intersection of the lines $4x + 7y - 3 = 0$ and $2x - 3y + 1 = 0$ that has equal intercepts on the axes.

Sol: Given lines are $4x + 7y - 3 = 0$ …(1), $2x - 3y + 1 = 0$ …(2)

solve (1) and (2) $(x, y) = \left(\frac{7-9}{-12-14}, \frac{-6-4}{-12-14}\right) = \left(\frac{1}{13}, \frac{5}{13}\right)$

Let a, a are intercepts on the co-ordinate axes equation of line in the intercept form is

$\frac{x}{a} + \frac{y}{a} = 1 \rightarrow (3)$

$x + y =$ a passing through $\left(\frac{1}{13}, \frac{5}{13}\right)$ we get $\frac{1}{13} + \frac{5}{13} = a \Rightarrow a = \frac{6}{13}$

put a = $\frac{6}{13}$ in equation (3) we get $x + y = \frac{6}{13} \Rightarrow 13x + 13y - 6 = 0.$

6. Show that the equations of the lines passing through origin and making an angle θ with the line $y = mx + c$ is $\frac{y}{x} = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}$.

Sol: Let m be the slope of the line passing through origin and making an angle θ with the line $y = mx + c$

m_1=M and m_2 = m, let θ be the angle

$$Tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow Tan\theta = \left|\frac{M - m}{1 + Mm}\right| \Rightarrow \frac{M - m}{1 + Mm} = \pm Tan\theta$$

$M - m = \pm Tan\theta(1 + Mm) \Rightarrow M - m = Tan\theta(1 + Mm)$ (or) $M - m = -Tan\theta(1 + Mm)$

M-Mm Tanθ= m+Tanθ (or) M+ MmTanθ= m - Tanθ

$M = \frac{m + Tan\theta}{1 - mTan\theta}$ (or) $\frac{m - Tan\theta}{1 + mTan\theta}$

$\therefore$ slopes are $\frac{m \pm Tan\theta}{1 \mp mTan\theta}$

The equations passing through origin $y = \left(\frac{m \pm Tan\theta}{1 \mp mTan\theta}\right)x$ (or) $\frac{y}{x} = \frac{m \pm Tan\theta}{1 \mp mTan\theta}$

7. In what ratio, the line joining (-1, 1) and (5, 7) is divided by the line $x + y = 4$?

Sol: The ratio is $-\frac{(ax_1 + by_1 + c)}{(ax_2 + by_2 + c)} = -\frac{(-1 + 1 - 4)}{(5 + 7 - 4)} = \frac{4}{8} = \frac{1}{2}$

$\therefore$ Ratio is 1 : 2

8. Find the equation of the straight line passing through (1, 3) and (i) parallel to (ii) perpendicular to the line passing through the points (3, -5) and (-6, 1).

Sol: Let A = (3, -5) and B=(-6, 1)

i) Slope of parallel line is also $m = -\frac{2}{3}$, let $(x_1, y_1) = (1,3)$

Now equation of the line is $y-3 = -\frac{2}{3}(x-1) \Rightarrow 3y - 9 = -2x + 2$

$\therefore 2x + 3y - 11 = 0$

ii) Slope of line perpendicular to $\overline{AB}$ is $m = \frac{3}{2}$ and passing through (1, 3) is

$y - 3 = \frac{3}{2}(x-1) \Rightarrow 2y - 6 = 3x - 3$

$\therefore 3x - 2y + 3 = 0$

9. Find the equation of the line perpendicular to $3x + 4y + 6 = 0$ and making an intercept – 4 on X – axis.

Sol: Equation given line is $3x + 4y + 6 = 0$ ……………..(1)

The point is P = (-4, 0)

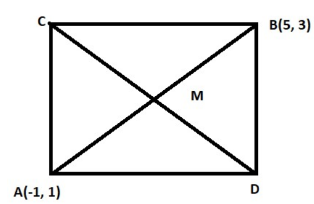
Now perpendicular line is $b(x - x_1) - a(y - y_1) = 0$

$\Rightarrow 4(x+4) - 3(y-0) = 0 \Rightarrow 4x - 3y + 16 = 0$

10. A(-1, 1), B(5, 3) are opposite vertices of a square in XY – plane. Find the equation of a diagonal (not passing through A, B) of the square.

Sol: Given A = (-1, 1), B = (5, 3) are two opposite vertices of a square

Let M be the mid point of $\overline{AB}$ and $\overline{CD}$



$\therefore M = \left(\frac{-1+5}{2}, \frac{1+3}{3}\right) = (2,2)$

Slope of AB $= \frac{3-1}{5+1} = \frac{2}{6} = \frac{1}{3}$

Slope of the diagonal $\overline{CD} = \frac{-1}{slope\ of\ \overline{AB}} = -3$

Equation on of the diagonal is $y - 2 = -3(x - 2) \Rightarrow y - 2 = -3x + 6 \Rightarrow 3x + y - 8 = 0$

11. Find the foot of the perpendicular (3, 0) upon the straight line $5x + 12y - 41 = 0$.

Sol: If (h, k) is foot of the perpendicular from (x_1, y_1) to the line ax + by + c = 0 then

$$\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2} \Rightarrow \frac{h-3}{5}=\frac{k-0}{12}=\frac{-(15+0-41)}{25+144}$$

$$\Rightarrow \frac{h-3}{5}=\frac{k}{12}=\frac{+26}{169}=\frac{2}{13}$$

$h-3=\frac{+10}{13}$

$h=3+\frac{10}{13}=\frac{49}{13}$

$\frac{k}{12}=\frac{2}{13}$

$k=\frac{24}{13}$

$\therefore$ foot of the perpendicular $\left(\frac{49}{13},\frac{24}{13}\right)$

12. Show that the distance of the point (6, -2) from the line 4x + 3y = 12 is half the distance of the point (3,4) from the line 4x -3y =12.

Sol: Perpendicular distance from (x_1, y_1) to the line $ax+by+c=0$ is $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

Let d_1 = perpendicular distance from (6, -2) to the line 4x + 3y – 12 = 0

$$=\frac{|24-6-12|}{\sqrt{16+9}}=\frac{6}{5}$$

Let d_2 = perpendicular distance from (3, 4) from the line 4x–3y=12 $\Rightarrow$ 4x-3y-12=0

$$=\frac{|12-12-12|}{\sqrt{16+9}}=\frac{12}{5}=2.\ \frac{6}{5}=2d_1$$

$$\therefore d_1=\frac{1}{2}d_2$$

III. 1) Find the distance of the line 4x+7y+5=0 from the point (1, 2) along the line 2x-y =0.

Sol: Let the equation of $\overline{AB}$ is 4x +7y+5 = 0 … (1) $\overline{PQ}$ is 2x – y = 0 ……. (2)

Solve (1) and (2) we get Q

4x + 7(2x) + 5 = 0 $\Rightarrow$ 18x = -5

$x=\frac{-5}{18}$ and $y=\frac{-5}{9}$

$Q=\left(\frac{-5}{18},\frac{-5}{9}\right)$, P = (1,2)

Now PQ length is = $\sqrt{\left(1+\frac{5}{18}\right)^2+\left(2+\frac{5}{9}\right)^2} \quad = \sqrt{\left(\frac{23}{18}\right)^2+\left(\frac{23}{9}\right)^2}$

$$= \frac{23}{9}\sqrt{\frac{1}{4}+1}=\frac{23}{18}\sqrt{5} \ .$$

2. Find the direction in which a straight line must be drawn through the point (-1, 2). So that its points of intersection with the line $x+y$=4 may be at a distance of 3 units from the point.

Sol: Let m be the slope of the line passing through P(-1, 2)

y -2 = m(x+1) $\Rightarrow$ mx - y + 2 + m = 0 $\rightarrow$ ……(1)

and given line is $x+y=4 \Rightarrow x+y-4=0 \rightarrow$ ….(2)

Solve (1) and (2) put $y=4-x$ in equation (1)

mx – 4 + x + 2 + m = 0 $\Rightarrow$ x (1 + m) = 2 – m

$x=\frac{2-m}{1+m}$ put in equation ………..(2)

y = 4 - $\frac{2-m}{1+m} = \frac{4+4m-2+m}{1+m}=\frac{2+5m}{1+m}$

$\therefore$ point of intersection is Q $\left(\frac{2-m}{1+m},\frac{2+5m}{1+m}\right)$

Given PQ = 3; PQ^2 = 9

$$\left(\frac{2-m}{1+m}+1\right)^2+\left(\frac{2+5m}{1+m}-2\right)^2=9 \Rightarrow (2-m+1+m)^2+(2+5m-2-2m)^2=9(1+m)^2$$

$$\Rightarrow (3)^2+(3m)^2=9(1+m)^2 \Rightarrow 1+m^2=1+m^2+2m$$

$\Rightarrow m=0$ (or) m is not defined.

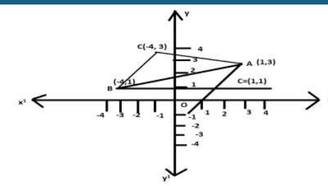
The line is parallel to x – axis (or) parallel to y-axis

3. The hypotenuse of a right angled triangle has its ends at the points (1, 3) and (-4, 1). Find the equations of the legs (perpendicular sides)of the triangle.

Sol: Let A = (1, 3) and B = (-4, 1)

Let $\lfloor C = 90^0$ then $\overline{BC}$ is a line parallel to x – axis it equation is $y=1$.

And $\overline{AC}$ is parallel to y –axis it equation is $x=1$



$\therefore$ C = (1, 1)

Equations of legs are $x = 1$, $y = 1$ (or) $x = -4$, $y = 3$

4. Find the image of the given point w.r.t to the given line

(i) point (3, 8) w.r.t line $x + 3y = 7$

(ii) point (1, 2) w.r.t line $3x + 4y - 1=0$

Sol:(i) Let (h, k) be the image of the point (x_1, y_1) w.r.t the line $ax + by + c = 0$ then

$$\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}.$$

(x_1, y_1) = (3, 8) and line $x + 3y - 7 = 0$

$$\therefore \frac{h-3}{1}=\frac{k-8}{3}=\frac{-2(3+24-7)}{1+9} = -\frac{40}{10}=-4$$

$$\frac{h-3}{1}=-4,\ \frac{k-8}{3}=-4 \Rightarrow \text{h = -1, k = -4}$$

Image (-1, -4)

(ii) (x_1, y_1) = (1, 2) and line is $3x + 4y - 1=0$

$$\frac{h-1}{3}=\frac{k-2}{4}=\frac{-2(3+8-1)}{9+16} = -\frac{20}{25}=-\frac{4}{5}$$

$$\frac{h-1}{3}=\frac{-4}{5},\frac{k-2}{4}=\frac{-4}{5} \Rightarrow \text{h} = -\frac{12}{5}+1, k=\frac{-16}{5}+2 \Rightarrow \text{h} = -\frac{7}{5}, k=\frac{-6}{5}$$

image = $\left(\frac{-7}{5},\frac{6}{5}\right)$

5. If the lines y = 3x + 1 and 2y = x + 3 are equally inclined to the line y = mx + 4, find the value of m.

Sol: Let the angle between the lines $y = 3x + 1$ and $y = mx + 4$ is θ,

$m_1 = 3$, $m_2 = m$ $\therefore Tan\theta=\left|\frac{3-m}{1+3m}\right| \to (1)$

And also take angle between y = $\frac{x}{2}+\frac{3}{2}$ and y = mx+4 is also θ.

$m_1 = \frac{1}{2}$, m_2=m. Hence Tanθ= $\left|\frac{\frac{1}{2}-m}{1+\frac{m}{2}}\right|=\left|\frac{1-2m}{2+m}\right| \to (2)$

from (1) and (2) we get $\frac{3-m}{1+3m}=\pm\left(\frac{1-2m}{2+m}\right)$

By taking positive sign (3 – m) (2+m) = (1+3m) (1-2m)

$\Rightarrow$ 6 + 3m – 2m –m^2 = 1 – 2m +3m – 6m^2 $\Rightarrow$ 5m^2 = - 5 $\Rightarrow$m^2 = -1 not possible

Again taking negative sign we get $\frac{3-m}{1+3m}=\frac{-(1-2m)}{2+m}$

$\Rightarrow$(3 – m)(2+m) = -(1-2m)(1+3m) $\Rightarrow$6 + 3m - 2m – m^2 = -(1+3m – 2m – 6m^2)

$$\Rightarrow 7m^2 - 2m - 7 = 0$$

$$m = \frac{2\pm\sqrt{4+4(49)}}{14} = \frac{2\pm2\sqrt{50}}{14} = \frac{20\left(1\pm5\sqrt{2}\right)}{14} = \frac{1\pm5\sqrt{2}}{7}$$

6. If sum of the perpendicular distance of a variable point P(x y) from the lines $x+y$-5 = 0 and 3x-2y+7=0 is always 10. Show that P must move on a line.

Sol: Given equations of lines are $x+y-5=0$……..(1)

$$3x - 2y + 7 = 0 \; \text{…..(2)}$$

Given sum of the perpendicular distances from P(x y) to line (1) and (2) is 10.

$$\Rightarrow \frac{|x+y-5|}{\sqrt{1+1}}+\left|\frac{3x-2y+7}{\sqrt{9+4}}\right|=10 \Rightarrow \frac{|x+y-5|}{\sqrt{2}}=10-\left|\frac{3x-2y+7}{\sqrt{13}}\right|$$

$$\Rightarrow \sqrt{13}\,(x + y - 5) = 10\sqrt{2}\,.\sqrt{13}-\sqrt{2}\left(13x-2y+7\right)$$

$\left(\sqrt{13}+2\sqrt{13}\right)x+\left(\sqrt{13}-2\sqrt{2}\right)y+7\sqrt{2}-5\sqrt{13}-10\sqrt{26}=0$ is a straight line

7. Find the equation of the line which is equidistant from parallel lines 9x + 6y – 7 = 0 and 3x + 2y + 6 = 0.

Sol: Mid way line between parallel lines ax + by + c_1 = 0 and ax + by + c_2 =0,

$$ax+by+\left(\frac{c_1+c_2}{2}\right)=0$$

Let the given lines are 9x+6y–7 = 0 ..(1) and 3x + 2y +6 =0 $\Rightarrow$9x + 6y+18 = 0 ..(2)

$\therefore$ Now required line is $9x+6y+\left(\frac{-7+18}{2}\right)=0 \Rightarrow 18x+12y+11=0$.

8. A ray of light passing through the point (1, 2) reflects on the x-axis at point A and the reflected ray passes through the point (5, 3) find co-ordinate of A.

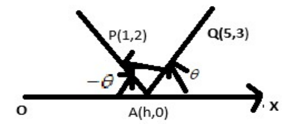
Sol: Let P(1, 2) and Q(5, 3)

Let A(h, 0) be the ray through P(1, 2) is reflected

Slope of AQ in Tanθ= $\frac{3-0}{5-h}=\frac{3}{5-h}$ and slope of AP is - tanθ = $\frac{-(2-0)}{1-h}=\frac{2}{h-1}$

$$\frac{3}{5-h}=\frac{2}{h-1} \quad \Rightarrow \text{h} = \frac{13}{5}$$

$$\therefore A=(h,0)=\left(\frac{13}{5},0\right)$$



9. Prove that the product of the lengths of the perpendiculars drawn from the points $\left(\sqrt{a^2-b^2},0\right)$ and $\left(-\sqrt{a^2-b^2},0\right)$ to the line $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ is b^2.

Sol: Given line (b cosθ)x + (a sinθ) y – ab = 0 ….(1)

Let d_1 = perpendicular distance from $\left(\sqrt{a^2-b^2},0\right)$ the line (1) = $\frac{\left|b\cos\theta\sqrt{a^2-b^2}-ab\right|}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}}$

d_2 = perpendicular distance from $\left(-\sqrt{a^2-b^2},0\right)$ to line (2) = $\frac{\left|-b\cos\theta\sqrt{a^2-b^2}-ab\right|}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}}$

$$\therefore \text{ product } d_1d_2 = \frac{\left|b\sqrt{a^2-b^2}\cos\theta-ab\right|}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}}\cdot\frac{\left|-b\sqrt{a^2-b^2}\cos\theta-ab\right|}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}} = \frac{\left|\left(a^2-b^2\right)\cos^2\theta-a^2b^2\right|}{b^2\cos^2\theta+a^2\sin^2\theta}$$

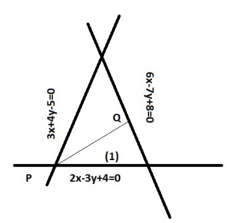
$$= b^2\frac{\left|\left[a^2\cos^2\theta-b^2\cos\theta-a^2\right]\right|}{\left(b^2\cos^2\theta+a^2\sin^2\theta\right)} = b^2\frac{\left|\left[a^2\left(1-\sin^2\theta\right)-b^2\cos\theta-a^2\right]\right|}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$= b^2\frac{\left|-a^2\sin^2\theta-b^2\cos\theta\right|}{\left|b^2\cos^2\theta+a^2\sin^2\theta\right|}=b^2$$

10. A person studying at the junction (crossing) of two straight paths represented by the equations 2x – 3y + 4 = 0 and 3x + 4y – 5 = 0 wants to reach the path whose equations is 6x -7y + 8 = 0 in the least two. Find the equation of the path that he should follow.

Sol: Let the given lines are $2x-3y+4=0$….(1), $3x+4y-5=0$….(2)

and other line $6x-7y+8=0$……….(3)



Let point of intersection of (1) and (2) is P

$$P=(x,y)=\left(\frac{15-16}{8+9},\frac{12+10}{8+9}\right)=\left(\frac{-1}{17},\frac{22}{17}\right)$$

Equation of the line perpendicular to (3) and passing this $P=\left(\frac{-1}{17},\frac{22}{17}\right)$ is the least path

$$\text{Hence } 7\left(x+\frac{1}{17}\right)-6\left(y-\frac{22}{17}\right)=0 \Rightarrow -7(17x+1)-6(17y-22)=0$$

$$\therefore\ 119x+102y-125=0.$$

11. Show that the lines x -7y – 22 = 0, 3x + 4y + 9 = 0 and 7x + y – 54 = 0 form a right angled isosceles triangle.

Sol: Let the equations of the given lines are

$x - 7y - 22 = 0 \rightarrow (1), \quad 3x + 4y + 9 = 0 \rightarrow (2), 7x + y - 54 = 0 \rightarrow (3)$

Let α be the angle between (1) and (2)

Slope of (1) is $m_1 = \frac{1}{7}$, slope of (2) $m_2 = -\frac{3}{4}$

$$\therefore Tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{3}{28}}\right| = \left|\frac{\frac{4+21}{28}}{\frac{28-3}{28}}\right| = \left|\frac{25}{25}\right| = 1$$

$$\therefore \alpha = 45^0$$

Let β be the angle between (2) and (3) slope of (2) is $m_1 = \frac{-3}{4}$, slope of (3) is m_2 = -7

$$Tan\beta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-\frac{3}{4} + \frac{7}{1}}{1 + \frac{21}{4}}\right| = \left|\frac{-\frac{3+28}{4}}{\frac{4+21}{4}}\right| = \left|\frac{25}{25}\right| = 1$$

$$\therefore \beta = 45^0$$

Let γ be the angle between (1) and (3)

$\alpha + \beta + \gamma = 180^0 \Rightarrow 45^0 + 45^0 + \gamma = 180^0 \Rightarrow \quad \gamma = 90^0$

$\therefore \alpha = 45^0, \beta = 45^0, \gamma = 90^0$

$\therefore \Delta$ right angled Isosceles triangle.

12. Find the equation of the straight line passing through the point (-3, 2) and making an angle of 45^0 with the straight line 3x – y + 4 = 0.

Sol: Let m be the slope of line passing through the point (-3, 2) and making an angle 45^0 with line 3x – y + 4 = 0.

Let m_1 = m, θ=45^0 and m_2 = slope of line 3x – y + 4 = 3

Now $\text{Tan}\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow Tan45^0 = \left|\frac{m-3}{1+3m}\right|$

$\Rightarrow$ m – 3 = 1 + 3m (or) m – 3 = -(1+3m)

$\Rightarrow$ m = -2 (or) 4m = 2

case (i) If m = - 2 and (x_1, y_1) = (-3, 2) then equation of the line is

$y - 2 = -2(x + 3) \Rightarrow 2x + y + 4 = 0$

case (ii) If m = $\frac{1}{2}$ and $(x_1, y_1) = (-3, 2)$ then equation of the line is

$$y - 2 = \frac{1}{2}(x + 3) \Rightarrow x - 2y + 7 = 0$$

$\therefore$ Required equations are $x - 2y + 7 = 0$ and $2x + y + 4 = 0$

MULTIPLE CHOICE QUESTIONS

1) $\theta = 45^0$ (inclined) m = Tan45^0 = 1; C = 3

$$y = x + 3$$

2) Let O = (0, 0), P(5, 2), Q(6, -15)

m_1 = slope of OP = $\frac{2}{5}$, m_2 = slope of OQ = $\frac{-15}{6} = \frac{-5}{2}$, $m_1 m_2 = -1$, $\theta = \frac{\pi}{2}$

3) A = (3, 0), B = (4, 0), C = (5, 0)

By verification (2, 2) and (5, 0) satisfiers (verification (2, 2) and (5, 0))

4) $\frac{C^2}{2|ab|} = \frac{144}{24} = 6$

5) $\frac{C^2}{2|ab|} = \frac{4}{2|\cos\alpha.\sec\alpha|} = 2$

6) $\frac{a^2}{24} = 1 \Rightarrow a = \pm\sqrt{24} = 2\sqrt{6}$

7) A = (3, 0), B(4, 0), C = (5, 0)

By verification (2, 2) and (5, 0) satisfies $2x + 3y = 10$

8) $\frac{x}{x_1} + \frac{y}{y_1} = 2; \frac{x}{3} + \frac{y}{4} = 2; 4x + 3y - 24 = 0$

9) $m_1 = -3$, $m_2 = \frac{1}{a}$, $m_3 = -\frac{b}{2}$

$m_1 m_2 = -1$

$\frac{-3}{a} = -1$

a = 3

$m_2 \times m_3 = -1$

$\left(\frac{1}{3}\right)\left(\frac{-b}{2}\right) = -1$

b = 6

$\therefore$ ab = (3) (6) = 18

10) $\frac{x^2}{l^1}+\frac{y^2}{m^2}=\frac{k^2}{(l+m)^2}$

l= 1, m = 2, k = 1

$\frac{x^2}{1}+\frac{y^2}{4}=\frac{l^2}{9}=\frac{4x^2+y^2}{4}=\frac{l^2}{9}\Rightarrow 36x^2-9y^2=4l^2$

11) By verification (1, -10) satisfies $3x + y + 7 = 0$

12) $\frac{|12+10-7|}{13}=\frac{15}{13}$

13) Result only $4p^2 + q^2 = a^2$

14) By verification (1) and (4) are parallel lines now

(1) distance $\frac{|5+0+34|}{\sqrt{25+144}}=\frac{39}{13}=3$

(2) distance = $\frac{|5+0+44|}{13}=\frac{49}{13}\Rightarrow$ 5x + 12y+34=0 is parallel line which is at a distance of 3 units.

15. $r=\frac{1}{2}\left(\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}\right)=\frac{1}{2}\left(\frac{|-15+5|}{5}\right)=\frac{1}{2}(2)=1$

Area = $\pi r^2=\pi(1)^2=\pi$

16) Slope of AB = $\frac{6}{4}=\frac{3}{2}$ and Slope of the line $\perp^r$ $to\,\overline{AB}=-\frac{2}{3}$

Equation is y – 1 $=-\frac{2}{3}(x-4)\Rightarrow\quad 2x+3y-11=0$

Ratio = $\frac{-(4-3-11)}{(12+15-11)}=\frac{10}{16}=\frac{5}{8}=5:8$

17) Let $\frac{x}{a}+\frac{y}{a}=1$ be the line $x + y = $ a perpendicular distance from origin is a = $\sqrt{2}$

Now equation is $x + y = \sqrt{2}$ ……(1) and Given line y = $2x+3+\sqrt{2}$ ….. (2)

From (1) and (2) we get x = -1 and $y = \sqrt{2}+1$;

$x_0 = -1$, $y_0 = \sqrt{2}+1$

$2x_0 + y_0 = -2+\sqrt{2}+1 = \sqrt{2}-1$

18) If a, b, c are in AP then $\dfrac{a+c}{2} = b$

a – 2b + c = 0 $\rightarrow ax+by+c=0$; $(x, y) = (1, -2)$

19) From (18) the line passing through fixed point

20) By verification option (2) and (3) satisfies the $2x - 3y = 5$ and distance from option is $\sqrt{9+1} = \sqrt{10}$ and $\sqrt{4+25} = \sqrt{29}$

Option (3) not satisfies.

MULTIPLE CHOICE QUESTIONS KEYS

1) 2	**2) 3**	**3) 1**	**4) 3**	**5) 3**	**6) 2**	**7) 1**	**8) 1**	**9) 1**	**10) 1**
11) 1	**12) 1**	**13) 1**	**14) 1**	**15) 4**	**16) 2**	**17) 2**	**18) 3**	**19) 1**	**20) 2**

10. CONIC SECTIONS

CIRCLE

Exercise -10(a)

I. In each of the following exercise 1 to 5

1. Find the equation of the circle with Centre(0,2) and radius 2.

Sol: The equation of a circle with centre C(h,k) and radius 'r' is $(x-h)^2 + (y-k)^2 = r^2$

Here C(h, k) = (0, 2) and r = 2

$(x-0)^2+(y-2)^2=2^2$ $\quad\therefore x^2+y^2-4y=0$

2.Find the equation of the circle with centre (-2, 3) and radius 4.

Sol: C(h, k) = (-2, 3) and $r = 4$

$(x+2)^2+(y-3)^2=4^2$

$\therefore x^2+y^2+4x-6y-3=0$

3. Find the equation of the circle with Centre $\left(\frac{1}{2},\frac{1}{4}\right)$ and radius $\frac{1}{12}$

Sol: C(h,k) = $\left(\frac{1}{2},\frac{1}{4}\right)$ and r = $\frac{1}{12}$

$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{4}\right)^2=\left(\frac{1}{12}\right)^2 \Rightarrow x^2+y^2-x-\frac{y}{2}+\frac{1}{4}+\frac{1}{16}-\frac{1}{144}=0$

$\therefore 36x^2+36y^2-36x-18y+11=0$

4. Find the equation of the circle with Centre (1,1) and radius $\sqrt{2}$

Sol: C (1,1) and r = $\sqrt{2}$

$(x-1)^2+(y-1)^2=\left(\sqrt{2}\right)^2$

$\therefore x^2+y^2-2x-2y=0$

5. Find the equation of the circle with Centre (-a,-b) and radius $\sqrt{a^2-b^2}$.

Sol: C(h,k) = (-a,-b) and r = $\sqrt{a^2-b^2}$

$(x+a)^2+(y+b)^2=(\sqrt{a^2-b^2})^2$

$\Rightarrow x^2+y^2+2ax+2by+a^2+b^2-a^2+b^2=0$

$x^2+y^2+2ax+2by+2b^2=0$

In each of the following Exercises 6 to 10

6. Find the centre and radius of the circle $(x+5)^2+(y-3)^2=36$

Sol: $(x+5)^2+(y-3)^2=36 \Rightarrow (x-(-5)^2+(y-3)^2=\left(\sqrt{36}\right)^2$

$\therefore$Centre C(h,k) =(-5,3) and radius r = $\sqrt{36}$

7. Find the centre and radius of the circle $x^2+y^2-4x-8y-45=0$

Sol: $x^2+y^2+2gx+2fy+c=0$, $\quad 2g=-4, 2f=-8, c=-45$

$\Rightarrow g=-2, \Rightarrow f=-4$

$\therefore C(-g,-f)=(2,4)$ and $r=\sqrt{g^2+f^2+c}=\sqrt{4+16+45}=\sqrt{65}$

8. Find the centre and radius of the circle $x^2+y^2-8x+10y-12=0$

Sol: $C(-g,-f)=(4,-5)$, $r=\sqrt{g^2+f^2+c}=\sqrt{16+25+12}=\sqrt{53}$

9. Find the centre and radius of the circle $2x^2+2y^2-x=0$

Sol: $x^2+y^2-\left(\frac{1}{2}\right)x=0$

$$2g=-\frac{1}{2},2f=0,c=0$$

$$g=\frac{-1}{4},f=0$$

$$C(-g,-f)=\left(\frac{1}{4},0\right)\ and\ r\ =\ \frac{1}{4}$$

10. Find the centre and radius of the circle $\sqrt{1+m^2}(x^2+y^2)-2cx-2mcy=0$

Sol: $x^2+y^2-\frac{2c}{\sqrt{1+m^2}}x-\frac{2mc}{\sqrt{1+m^2}}y=0$

$$C(-g,-f)=\left(\frac{c}{\sqrt{1+m^2}},-\frac{mc}{\sqrt{1+m^2}}\right)$$

$$r=\sqrt{g^2+f^2-c}=C$$

11. Find the equation of the circle passing through origin and having the centre at (-4,-3).

Sol: Centre C(-4,-3) and point P(0,0)

Radius, r =CP =$\sqrt{16+9}=5$

The equation of circle is $(x-h)^2+(y-k)^2=r^2$

$$(x+4)^2+(y+3)^2=5^2\Rightarrow x^2+y^2+8x+6y=0$$

12. Find the equation of the circle passing through (2,-1) having the centre at (2,3).

Sol: $P(2-1)\ and\ C(2,3)=(h,k)$

$$r=CP=\sqrt{0+4^2}=4$$

The equation of circle is $(x-h)^2+(y-k)^2=r^2$

$$(x-2)^2+(y-3)^2=4^2\Rightarrow x^2+y^2-4x-6y-3=0$$

13. Find the equation of the circle passing through (-2,3) having the centre at (0,0).

Sol: $P(-2,3)\ and\ C(0,0)=(h,k)$

$$r=CP=\sqrt{4+9}=\sqrt{13}$$

The equation of circle is $x^2+y^2=13$

14. Find the equation of the circle passing through (3,4) having the centre at (-3,4).

Sol: $P(3,4)\ and\ C(h,k)=(-3,4)$

$$r=CP=\sqrt{(-3-3)^2+(4-4)^2}=6$$

The equation of a circle $(x+3)^2+(y-4)^2=6^2$

$$\Rightarrow x^2+y^2+6x-8y-11=0$$

15. Find the value of a if $2x^2+ay^2-3x+2y-1=0$ represents a circle and also find its radius.

Sol: $2x^2+ay^2-3x+2y-6=0 \Rightarrow a=2$

$$x^2+y^2-\frac{3}{2}x+y-\frac{1}{2}=0$$

$$r=\sqrt{\frac{9}{16}+\frac{1}{4}+\frac{1}{2}}=\frac{\sqrt{21}}{4}$$

16. Find the values of a, b if $ax^2+bxy+3y^2-5x+2y-3=0$ represents a circle. Also find the radius and centre of the circle.

Sol: If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a circle then a = b and h = 0,

$\therefore a=3$ and b = 0.

The given equation becomes $3x^2+3y^2-5x+2y-3=0$

$$\left.\begin{array}{l} 2g=\frac{-5}{3} \\ g=\frac{-5}{6} \end{array}\right| \left.\begin{array}{l} 2f=\frac{2}{3} \\ f=\frac{1}{3} \end{array}\right| c=-1 \Rightarrow \text{Centre } C(-g,-f)=\left(\frac{5}{6},\frac{-1}{3}\right),$$

$$r=\sqrt{\frac{25}{36}+\frac{1}{9}+1}$$

$$=\sqrt{\frac{25+4+36}{36}}=\sqrt{\frac{65}{36}}=\frac{\sqrt{65}}{6}$$

17. If $x^2+y^2+2gx+2fy-12=0$ represents a circle with centre (2,3) find g, f and its radius.

Sol: Given $x^2+y^2+2gx+2fy-12=0$, Centre $C(-g,-f)=(2,3)$ & $c=-12$, $g=-2$ *and* $f=-3$

Radius r= $\sqrt{g^2+f^2-c}$ $=\sqrt{4+9+12}=5$

18. If $x^2+y^2+rgx+2fy=0$ represents a circle with centre (-4,-3) then find g, f and the radius of the circle.

Sol: Given $x^2+y^2+rgx+2fy=0$, Centre $C(-g,-f)=(-4,-3)$ *and* $c=0$

$g=4, f=3$ & $C=0$

$r=\sqrt{g^2+f^2-c}$ $=\sqrt{16+9}=\sqrt{25}$ $=5$

19. If $x^2+y^2-4x+6y+c=0$ represents a circle with radius 6 then find the value of c.

Sol: Let $x^2+y^2-4x+6y+c=0$

$$\left.\begin{array}{l} 2g=-4 \\ g=-2 \end{array}\right| \begin{array}{l} 2f=6 \\ f=3 \end{array}$$

Given that radius r=6 $=\sqrt{g^2+f^2-c}$ $\Rightarrow c = -23$

20. Find the equation of a circle which is concentric with $x^2+y^2-6x-4y+12=0$ and passing through (-2, 14).

Sol: Let the equation of circle which is concentric is $x^2+y^2-6x-4y+k=0$

Given that it passes through the point (-2,14)

$\Rightarrow$ 4+196+12-56+k =0 $\Rightarrow$ k = -156

$\therefore$ Required equation of circle is $x^2+y^2-6x-4y-156=0$

21. Find the equation of a circle with centre (2,2) and passes through the point (4,5).

Sol: Given that centre C(h,k) = (2,2) and P(x,y) = (4,5)

Radius $r = CP = \sqrt{(4-2)^2+(5-2)^2} = \sqrt{13}$

$$(x-1)^2+(y-k)^2=r^2 \Rightarrow (x-2)^2+(y-2)^2=\left(\sqrt{13}\right)^2$$

$\therefore$ Required equation of circle is $x^2+y^2-4x-4y-5=0$

22. Does the point (-2.5, 3.5) lie inside, outside or on the circle $x^2+y^2=25$?

Sol: $S_{11}=x_1^2+y_1^2-25 = (-2.5)^2+(3.5)^2-25 = 6.25+12.25-25=-6.50<0$

$\therefore S_{11}<0.$ The Given point lies inside the circles $x^2+y^2=25$

(II)1. If the abscissae of points A,B are the roots of the equation $x^2+2ax-b^2=0$ and ordinates of A,B are roots of $y^2+2py-q^2=0$ then find the equation of a circle for which $\overline{AB}$ is a diameter.

Sol: Let $x^2+2ax-b^2=0 \to (1)$ and $y^2+2py-q^2=0 \to (2)$

If x_1,x_2 are the roots of equation (1) and y_1y_2 are the roots of equation (2) then

$x_1+x_2=-2a, x_1x_2=-b^2$ and $y_1+y_2=-2py_1y_2=-q^2$

If A (x_1,y_1) & $B(x_2,y_2)$ are end points diameter $\overline{AB}$ then

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$\Rightarrow x^2-(x_1+x_2)x+x_1x_2+y^2-(y_1+y_2)y+y_1y_2=0$

$\Rightarrow x^2-(-2a)x-b^2+y^2-(-2p)y-q^2=0$

$\Rightarrow x^2+2ax-b^2+y^2+2py-q^2=0$

$\therefore$ Required equation of circle is $x^2+y^2+2ax+2py-(b^2+q^2)=0$

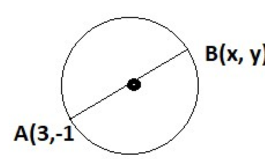
2. Show that A(3, -1) lies on the circle $x^2+y^2-2x+4y=0$. Also find the other end of the diameter through A.

Sol: Let $x^2+y^2-2x+4y=0 \to (1)$ and $A(3,-1)=(x,y)$

$(3)^2+(-1)^2-2(3)+4(-1)=0$

$9+1-6-4=0$

$\therefore A(3,-1) lies$ on the circle.



Center C$(-g,-f)=(1,-2)$

Mid point of $\overline{AB}=C(1,-2) \Rightarrow \left(\left(\frac{x+3}{2}\right),\left(\frac{-1+y}{2}\right)\right)=(1,-2)$

$\Rightarrow \frac{x+3}{2}=1, \frac{y-1}{2}=-2 \Rightarrow x+3=2, \quad y-1=-4 \quad \therefore x=-1 \qquad y=-3$

$\therefore$ The other end point of diameter through A is B(x, y) = (-1,-3)

3. Show that A(-3,0) lies on $x^2+y^2+8x+12y+15=0$ and find the other end of diameter through A.

Sol: Let S = $x^2+y^2+8x+12y+15=0$ ….(1) and A(-3, 0) = $(x,0)$

CentreC(-g, -f) = (-4, -6)

$(-3)^2 + 0 + 8(-3) + 12(0) + 15 = 0$

$9 - 24 + 0 + 15 = 0$

$\therefore$ A(-3, 0) lies on the circle S = 0

If B (x,y) is an another end point of diameter $\overline{AB}$ then

centre of circle = mid point of $\overline{AB}$

(-4, -6) = $\left(\dfrac{x-3}{2},\dfrac{y+0}{2}\right) \Rightarrow \dfrac{x-3}{2}=-4, \dfrac{y}{2}=-6 \Rightarrow x=-5, y=-12$

$\therefore$ The other end point of diameter $\overline{AB}$ is B(x,y)= (-5, -12).

4. Find the equation of the circle whose center lies on the x - axis and passingthrough (-2,3) and (4,5)

Ans.: Let equation of the circle be $x^2+y^2+2gx+2fy+c=0 \to (1)$

Given that centre $C(-g,-f)$ *lies* on x –axis i. e $-f=0 \Rightarrow f=0$

If equation (1) passes through (-2,3) then

$(-2)^2+(3)^2+2g(-2)+2f(3)+c=0$

$4+9-4g+6f+c=0 \Rightarrow -4g+c=-13 \to (2)(\because f=0)$

If equation $\to$ (1) passes through (4,5) then

$(4)^2+5^2+2g(4)+2f(5)+c=0 \Rightarrow 16+25+8g+10f+c=0$

8g+c = -41 $\to$ (3) ($\because f=0$)

Solving(2) and (3) we get $g=-\dfrac{7}{3}$ and $c=\dfrac{-67}{3}$

$\therefore$ Required equation of circle is $x^2+y^2+2\left(\dfrac{-7}{3}\right)x+\left(\dfrac{-67}{3}\right)=0 \Rightarrow 3x^2+3y^2-14x-67=0$

III.1. **Find the equation of the circle passing through the points (4,1) and (6,5) and Whosecentre is on the line** $4x+y=16$

Sol: Let equation of the circle be $x^2+y^2+2gx+2fy+c=0 \to (1)$

If equation (1) passes through (4,1) then

$(4)^2+(1)^2+2g(4)+2f(1)+c=0 \Rightarrow 16+1+8g+2f+c=0$

$8g+2f+c=-17 \to (2)$

If equation (1) passes through (6, 5) then

$(6)^2+5^2+2g(6)+2f(5)+c=0 \Rightarrow 36+25+12g+10f+C=0$

$\Rightarrow 12g+10f+c=-61 \to (3)$

Given that centreC(-g, - f) lies on 4x + y = 16 then

4(-g) + (- f) = 16$\Rightarrow$-4g – f = - 16………….(4)

Solving (2) and (3) we get $-4g-8f \quad =44$..............(5)

Solving (4) and(5) becomes $7f=-28 \Rightarrow f=-4$, We get $g=-3$,We get C =15

$\therefore$ Required equation of circle is $x^2+y^2-6x-8y+15=0$

2. **Find the equation of a circle which passes through (2,3) and (-1,1) Whose centre is on the line $x-3y-11=0$.**

Sol: Let equation of circle be $x^2+y^2+2gx+2fy+c=0 \to (1)$

If equation (1) pass through (2,3) then

$2^2+3^2+2g(2)+2f(3)+c=0$

$4g+6f+c=-13 \qquad \to (2)$

If equation (1) passes through (-1,1) then

$(-1)^2+(1)^2+2g(-1)+2f(1)+c=0$

$-2g+2f+c=-2 \to (3)$

If c(-g,-f) lies on $x-3y-11=0$

$-g+3f-11=0 \to (4)$

Solving (2), (3)& (4) we get $f = \frac{5}{2}, \quad g = \frac{-7}{2} \ \& C = -14$

$\therefore$ Required equation of circle is $x^2+y^2-7x+5y-14=0$

3. **Find the equation of the circle which passes through(2,-3) and (-4,5) and having the centre on $4x+3y+1=0$**

Sol:let the equation of circle be $x^2+y^2+2gx+2fy+c=0 \to (1)$

Equation (1) passes through (2,-3), then $4g-6f+c-13 \to (2)$

Equation (1) passes through (-4,5), then $-8g+10f+c=41 \to (3)$

If centre $(g,-f)$ lies on $4x+3y+1=0$

Solving (2), (3) & (4), we get $g = 1, \ f = -1, \ \& C = -23$

$\therefore$ The equation of required circle is $x^2+y^2+2x-2y-23=0$

4. **Find the equation of a circle which passes through (4,1), (6,5) and having the centre on 4x+3y -24 = 0.**

Sol: Let the equation of the circle be $x^2+y^2+2gx+2fy+c=0 \to (1)$

$(4,1)=(x,y)$

$4^2+1^2+2g(4)+2f(1)+c=0 \Rightarrow 8g+2f+c=-17 \to (2)$

$(6,5)=(x,y)$

$6^2+5^2+2g(6)+2f(5)+c=0 \Rightarrow 12g+10f+c=-61 \to (3)$

If centre $c(-g,-f)$ *lieson* $4x+3y-24=0$ *then*

$4(-g)+3(-f)=24 \Rightarrow -4g-3f=24 \to (4)$

Solving (2), (3)& (4), we get $f = -4, \ g = \frac{-5}{2}, C = 11$

$\therefore$ The equation of required circle is $x^2+y^2-5x-8y+11=0$

5. **Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2,3).**

Sol: Let the equation of the circle with centreC(h, k) and radius 'r' be $(x-h)^2+(y-k)^2=r^2 \ldots(1)$

Given that r = 5 and (x , y) = (2, 3) i.e. $(2 - h)^2+(3-k)^2 = 5^2$

If centreC(h, k) lies on x -axis then k = 0

$(2 - h)^2 + 3^2 = 5^2 \Rightarrow 2^2 + h^2 - 4h = 25 - 9 \Rightarrow (h+2)\ (h-6) = 0$, h = -2 or 6.

From equation (1)

Case -1	ase -2
When h =6, k =0,r =5	when h =-2, k =0,r =5
$(x-6)^2+(y-0)^2=5^2$	$x-2)^2+(y-0)^2=5^2$
$x^2+y^2-12x+36-25=0$	$^2+y^2-4x+4-25=0$
$x^2+y^2-12x+11=0$	$^2+y^2-4x-21=0$

$\therefore$ The required equations of the circles are $x^2+y^2-12x+11=0, x^2+y^2+4x-21=0$

6. Find the equation of the circle passing through (0,0) and making intercepts a change the font on the coordinate axes.

Sol: let the equation of the circle be $x^2+y^2+2gx+2fy+c=0 \to (1)$

$if\,(x,y)=(0,0)\ then\ c=0$

$(x,y)=(a,o)\ then\ \ a^2+2ga+c=0 \Rightarrow g\ =\ \frac{-a}{2}$

$and\ (x,y)=\ (0,b)\ \ then\ b^2+2fb+c=0 \Rightarrow\ \ \ f\ =\frac{-b}{2}$

Substitute g,f,c values in Eq (1)

$$x^2+y^2+2\left(\frac{-a}{2}\right)x+\ 2\left(\frac{-b}{2}\right)y+0=0$$

$\therefore$ The required equation of the circle is $x^2+y^2-ax-by=0$

7. Find the equation of circle passing through each of the following three points

(i) (0,0) (2 ,0) (0,2)

Sol: Let equation of the circle be $x^2+y^2+2gx+2fy+c=0 \to (1)$

Since eqn (1) passes through (0,0), then $c=0$,

Since eqn (1) passes through (2,0), then $4+0+4g+c\ =0$, $\Rightarrow g=-1$

Since eqn (1) passes through (0,2), then $4+4f+c=0$, $\Rightarrow 4f=-4\ (\ c=0)$, $\Rightarrow f\ =-1$

Substituting g,f,c values in eqn(1) $x^2+y^2+2(-1)x+2(-1)y+c=0$

$\therefore$ The required equation of the circle $x^2+y^2-2x-2y=0$

(ii) (3,4),(3,2),(1,4)

Sol: Let equation of the circle be $x^2+y^2+2gx+2fy+c=0 \to (1)$

Since eqn (1) passes through (3,4), then

$9+16+6g+8f+c=0 \Rightarrow 6g+8f+c=-25 \rightarrow (2)$

Since eqn (1) passes through (3,2), then

$9+4+6g+4f+c=-13 \Rightarrow 6g+4f+c=-13 \rightarrow (3)$

Since eqn (1) passes through (1,4), then

$1+16+2g+8f+c=0 \Rightarrow 2g+8f+c=-17 \rightarrow (4)$

Solving (2),(3) & (4) we get $\Rightarrow g=-2, f=-3 \,\&\, c=11$

Substituting g, f ,c values in eq(1) we get $x^2+y^2-4x-3y+11=0$

$\therefore$ The required equation of the circle $x^2+y^2-4x-3y+11=0$

(iii) (2,1), (5,5) (-6,7)

Sol: Let equation of the circle be $x^2+y^2+2gx+2fy+c=0 \rightarrow (1)$

Since eqn (1) passes through (2,1), then $4g+2f+c=-5 \rightarrow (2)$

Since eqn (1) passes through (5,5) then $10g+10f+c=-50 \rightarrow (3)$

Since eqn (1) passes through (-6,7)then $-12g+14f+c=-85 \rightarrow (4)$

Solving (2),(3) & (4) we get $\Rightarrow g=\frac{1}{2}, f=-6 \,\&\, c=5$

Substituting g, f ,c values in eq(1) we get $x^2+y^2+x-12y+5=0$

$\therefore$ The required equation of the circle $x^2+y^2+x-12y+5=0$

EXERCISE – 10 (b)

I. In each of the following exercise 1 to 6, find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latusrectum

1) $y^2=12x$

Sol: Given equation of the parabola is

$\Rightarrow y^2=4(3)x \Rightarrow$ a=3

Focus s= (a,0) = (3,0)

Axis is x -axis$\Rightarrow$ y=0

Equation of the directrix $x=-a \Rightarrow x=-3$

and length of the latusrectum = 4a =12

2) $x^2=6y$

Sol: Given equation of the parabola is $x^2=6y$

$\Rightarrow x^2 = 4\left(\frac{3}{2}\right)y \Rightarrow a = \frac{3}{2}$

Focus S= (0,a) = $\left(0, \frac{3}{2}\right)$

Axis is y-axis$\Rightarrow$ x=0

Equation of directrix $y = -a$ $\Rightarrow$y= $\frac{-3}{2}$

Length of latusrectum = 4a = $4\left(\frac{3}{2}\right)$=6

3) $y^2 = -8x$

Sol: Given equation of the parabola is $y^2 = -8x$

$\Rightarrow y^2 = -4(2)x$

$\Rightarrow a = 2$

Focus s= $(-a,0) = (-2,0)$

Axis is x-axis$\Rightarrow$ y=0

Equation of directrix $x = a \Rightarrow x = 2$

Length of latusrectum = 4a = 4(2) =8

4) $x^2 = -16y$

Sol: Given equation of the parabola is $x^2 = -16y \Rightarrow x^2 = -4(4y) \Rightarrow a = 4$

Focus S=(0,-a) = (0,-4)

Axis is y-axis$\Rightarrow$ x=0

Equation of directrix is $y = a \Rightarrow y = 4$

Length of latusrectum = 4a=16

5) $y^2 = 10x$

Sol: Given equation of the parabola is $y^2 = 10x \Rightarrow y^2 = 4\left(\frac{5}{2}\right)x \Rightarrow a = \frac{5}{2}$

Focus s= (a,0) = $\left(\frac{5}{2}, 0\right)$

Axis is x-axis$\Rightarrow$ $y = 0$

Equation of directrix is $x = -a \Rightarrow x = \frac{-5}{2}$

Length of latusrectum = $4a = 10$

6) $x^2 = -9y$

Sol: Given equation of parabola is $x^2 = -9y \Rightarrow x^2 = -4\left(\frac{9}{4}\right)y \Rightarrow a = \frac{9}{4}$

Focus s=(0,-a) = $\left(0, \frac{-9}{4}\right)$

Axis is y-axis$\Rightarrow$ x=0

Equation of directrix is y=a$\Rightarrow$ $y=\frac{9}{4}$

Length of latusrectum = 4a=9

In each of the exercise 7 to 12, find the equation of the parabola that satisfy the given conditions.

7) Focus (6,0), directrix x=-6

Sol: Given, focus s= (6,0) $\Rightarrow a=6$

And directrix x=-6

The equation of the parabola is

$y^2=4ax$

$\Rightarrow y^2=4(6)x$

$\therefore y^2=24x$

8) Focus (0,-3) and directrix y=3

Sol: Given Focus s= (0,-3) $\Rightarrow a=+3$ and its lies on y-axis And directrix y=3,

The equation of the parabola is $x^2=-4ay$

$\therefore x^2=-12y$

9) Vertex (0,0) Focus (3,0)

Sol: Given Focus S =(3,0) $\Rightarrow a=3$ and it lies on x-axis in Q_1

The equation of the parabola is $y^2=4ax$ $\Rightarrow y^2=4(3)x$

$\therefore y^2=12x$

10) Vertex (0,0), Focus (-2,0)

Sol: Given focus S= (-2,0) $\Rightarrow a=2$ and it lies on x-axis in Q_2

The equation of the parabola is $y^2=-4ax$

$\Rightarrow y^2=-4(2)x$

$\therefore y^2=-8x$

11) Vertex (0,0) passing through (2,3) and axis is along x-axis.

Sol: Given axis is x-axis and vertex = (0,0), so equation of the parabola is of the form $y^2=4ax$ or $y^2=-4ax$. But it passes through (2,3). That isit lies in Q_1.

So required equation of the parabola is $y^2=4ax$

$\Rightarrow 3^2=4a(2)$

$\Rightarrow 9=8a$

$\Rightarrow a=\frac{9}{8}$

$\therefore$Equation of the parabola is $\therefore 2y^2=9x$

12) Vertex (0,0) passing through (5,2) and symmetric with respect to Y-axis

Sol:Given vertex = (0,0) and parabola symmetric about Y-axis. So, the form of the parabola is $x^2=4ay$ it passes through the point (5,2)

$$\Rightarrow 5^2 = 4a(2)$$

$$\Rightarrow a = \frac{25}{8}$$

$\therefore$ Equation of the parabola is $x^2 = 4ay$

$$\Rightarrow x^2 = 4\left(\frac{25}{8}\right)y$$

$$\therefore 2x^2 = 25y$$

II. 1) Find the equation of the parabola whose focus is S(3,5) and vertex is A (1,3)

Sol:Given focus S= (3,5) and vertex A = (1,3)

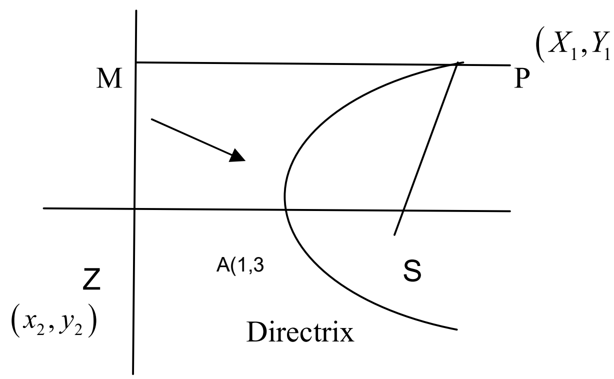
Let SA meets the directrix at $Z(x_2, y_2)$

$\Rightarrow$ A is mid point of SZ

$$\Rightarrow \left(\frac{x_2+3}{2}, \frac{y_2+5}{2}\right) = (1,3)$$

$$\Rightarrow x_2 = -1, y_2 = 1$$

$$\therefore Z = (-1,1)$$



Let slope of SZ is $m_1 = \frac{5-1}{3+1} = 1$

Let slope of directrix MZ be m_2

Since $SZ \perp MZ$

$$\Rightarrow m_1 \times m_2 = -1$$

$$\Rightarrow m_2 = -1$$

$\therefore$ Equation of the directrix is y-1=-1(x+1)$\Rightarrow x + y = 0$

Let $P(x_1, y_1)$ be any point on the parabola. By the definition of parabola.

$$\frac{SP}{PM} = 1 \Rightarrow SP = PM$$

$$\Rightarrow SP^2 = PM^2$$

$$\Rightarrow (x_1-3)^2 + (y_1-5)^2 = \left(\frac{|x_1+y_1|}{\sqrt{1^2+1^2}}\right)^2$$

$$\Rightarrow x^2 - 6x_1 + 9 + y_1^2 - 10y_1 + 25 = \frac{(x_1+y_1)^2}{2}$$

$$\Rightarrow 2x_1^2 - 12x_1 + 18 + 2y_1^2 - 20y_1 + 50 = x_1^2 + y^2_1 + 2x_1y_1$$

$$\Rightarrow 2x_1^2 - 12x_1 + 18 + 2y_1^2 - 20y_1 + 50 - x_1^2 - y^2_1 - 2x_1y_1 = 0$$

$$\Rightarrow x^2_1 - 2x_1y_1 + y^2_1 - 12x_1 - 20y_1 + 68 = 0$$

$\therefore$ Equation of the locus of P is the parabola

$$x^2 - 2xy + y^2 - 12x - 20y + 68 = 0$$

2) Find the equation of the parabola whose focus is (-2,3) and direction is the line $2x + 3y - 4 = 0$ **Also find the length of the latus roctum and the equation of the axis of the parabola.**

Sol: Given focus S(-2,3)& Equation of directirix is $2x+3y-4=0$

Let $p(x_1 y_1)$ be any point on the parabola by the definition of the parabola we have

$$\frac{SP}{PM}=1$$

$$\Rightarrow (x_1+2)^2+(y_1-3)^2=\left(\frac{2x_1+3y_1-4}{\sqrt{2^2+3^2}}\right)^2$$

$$\Rightarrow x_1^2+4x_1+4+y_1^2-6y_1+9=\left(\frac{2x_1+3y_1-4}{\sqrt{2^2+3^2}}\right)^2$$

$$\Rightarrow 13(x_1^2+4x_1+4+y_1^2-6y_1+9)=4x_1^2+9y_1^2+16+12x_1y_1-24y_1-16x_1$$

$$\Rightarrow 9x_1^2-12x_1y_1+4y_1^2+68x_1-54y_1+153=0$$

$\therefore$ Equation of the locus of P is the parabola $9x^2-12xy+4y^2+68x-54y+153=0$

Length of the latus rectum=4a $=2(2a)=2d$

$=2(\textit{The distance from the focus } (-2,3) \textit{ to the directrix } (2x+3y-4=0)$

$$=2\frac{|2(-2)+3(3)-4|}{\sqrt{2^2+3^2}}=2\frac{1}{\sqrt{13}}$$

$$\therefore \textit{Length of the latus rectum}=4a=\frac{2}{\sqrt{13}}$$

Since axis is perpendicular to the directrix ,slope of the directrix $2x+3y-4=0$ *is* $\frac{-2}{3}$

$\Rightarrow$ slope of the axis is $\frac{3}{2}$ and axis passing through the focus (-2,3)

$\therefore$ *Equation of the axis is* $y-3=\frac{3}{2}(x+2)\Rightarrow 2y-6=3x+6\Rightarrow 3x-2y+12=0$

3) **Find the equation of the parabola whose axis is parallel to X-axis and which passes through the points (-2,1), (1,2) and (-1,3)**

Sol: Let A = $(-2,1)B=(1,2)$ *and* $C=(-1,3)$

The equation of the parabola whose axis is parallel to $X-axis\ is\ x=ly^2+my+n\rightarrow(1)$

Given that A(-2,1) $\in(1)\Rightarrow l+m+n=-2\rightarrow$ (2)

Let $B\,(1,2)\in(1)\Rightarrow 4l+2m+n=1\rightarrow(3)$

and $C(-1,3)\in(1)\Rightarrow 9l+3m+n=-1\rightarrow(4)$

By solving (2),(3) and (4) we get

$$l=-\frac{5}{2},m=\frac{21}{2},n=-10$$

Put the values of l,m,n in (1), we get

The required equation of the parabola is

$$x=-\frac{5}{2}y^2+\frac{21}{2}y-10\ or\ 5y^2+2x-21y+20=0$$

4) **Find the equation of the parabola whose axis is parallel to Y-axis and which passes through the points (4,5),(-2,11) and (-4,21)**

Sol: $Let\ A=(4,5)\,,B=(-2,11)\,,C=(-4,21)$

The equation of the parabola whose axis is parallel to Y- axis is $y=lx^2+m\,x+n\rightarrow(1)$

$Let\ A=(4,5)\in(1)\Rightarrow 16l+4m+n=5\rightarrow(2)$

$B=(-2,11)\in(1)\Rightarrow 4l-2m+n=11\rightarrow(3)$

$C=(-4,21)\in(1)\Rightarrow 16l-4m+n=21\rightarrow(4)$

$By\ solving(2)\,,(3)and(4)we\ get$

$l=\frac{1}{2},m=-2and\ n=5$

$\therefore Equation\ of\ the\ parabola\ is\ y=\frac{1}{2}x^2-2x+5\ or\ x^2-4x-2y+10=0$

ELLIPSE

I. 1. **Find the eccentricity of the ellipse** $\frac{x^2}{9}+\frac{y^2}{16}=1$

Sol: Given Ellipse is $\frac{x^2}{9}+\frac{y^2}{16}=1$

$\Rightarrow a^2=9,b^2=16$

Since b>a $\Rightarrow$ eccentricity $e=\sqrt{\frac{b^2-a^2}{b^2}}\Rightarrow e=\sqrt{\frac{16-9}{16}}=\frac{\sqrt{7}}{4}$

2. **Find the length of the latus rectum of the ellipse** $x^2+16y^2=16$

Sol: Given ellipse is $x^2+16y^2=16$

$\frac{x^2}{16}+\frac{y^2}{1}=1\Rightarrow a^2=16,b^2=1,a>$b

Length of latusrectum = $\frac{2b^2}{a}=\frac{2(1)}{4}=\frac{1}{2}$

II.In each of the exercises 1 to 9, find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse

1. $\frac{x^2}{36}+\frac{y^2}{16}=1$

Sol: The given equation is $\frac{x^2}{36}+\frac{y^2}{16}=1$

Here, the denominator of $\frac{x^2}{36}$ is greater than the denominator of $\frac{y^2}{16}$

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\therefore C=\sqrt{a^2-b^2}=\sqrt{36-16}=\sqrt{20}=2\sqrt{5}$

Now we have a=6 and b=4.

Therefore,the coordinates of the foci are $\left(2\sqrt{5},0\right)$ and $\left(-2\sqrt{5},0\right)$

The coordinates of the vertices are (6,0) and (-6,0)
Length of major axis = 2a=12
Length of minor axis = 2b=8

Eccentricity, e=$\frac{c}{a}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$

Length of latusrectum = $\frac{2b^2}{a}=\frac{2\times16}{6}=\frac{16}{3}$

2. $\frac{x^2}{4}+\frac{y^2}{25}=1$

Sol: The given equation is $\frac{x^2}{4}+\frac{y^2}{25}=1$ or $\frac{x^2}{2^2}+\frac{y^2}{5^2}=1$

Here, the denominator of $\frac{y^2}{25}$ is greater than the denominator of $\frac{x^2}{4}$

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

Now, we have b=2 and a=5,

$\therefore c=\sqrt{a^2-b^2}=\sqrt{25-4}=\sqrt{21}$

Therefore, the coordinates of the foci are $\left(0,\sqrt{21}\right)$ and $\left(0,-\sqrt{21}\right)$

The coordinates of the vertices are (0,5) and (0,-5)
Length of major axis = 2a=10
Length of minor axis=2b=4

Eccentricity, e= $\frac{c}{a}=\frac{\sqrt{21}}{5}$

Length of latus rectum $\frac{2b^2}{a}=\frac{2\times4}{5}=\frac{8}{5}$

3. $\frac{x^2}{16}+\frac{y^2}{9}=1$

Sol: The given equation is $\frac{x^2}{16}+\frac{y^2}{9}=1$ or $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$

Hence, the denominator of $\frac{x^2}{16}$ is greater than the denominator of $\frac{y^2}{9}$

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Now, we have a=4 and b=3

$\therefore c=\sqrt{a^2-b^2}=\sqrt{16-9}=\sqrt{7}$

Therefore, the coordinates of the foci are $\left(\pm\sqrt{7},0\right)$

The coordinates of the vertices are $(\pm 4, 0)$

Length of major axis =2a=8

Length of minor axis =2b=6

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{7}}{4}$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{9}{2}$

4. $\frac{x^2}{25} + \frac{y^2}{100} = 1$

Sol: The given equation is $\frac{x^2}{25} + \frac{y^2}{100} = 1$ or $\frac{x^2}{5^2} + \frac{y^2}{10^2} = 1$

Hence, the denominator of $\frac{y^2}{100}$ is greater than the denominator of $\frac{x^2}{25}$

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis

On comparing the given equation with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Now, we have b=5 and a=10

$\therefore c = \sqrt{a^2 - b^2} = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}$

Therefore, the coordinates of the foci are $(0, 5\sqrt{3})$

The coordinates of the vertices are $(0, \pm 10)$

Length of major axis = 2a = 20 length of minor axis = 2b=10

Eccentricity, c= $\frac{c}{a} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$

Length of latusrectum $\frac{2b^2}{a} = \frac{2 \times 25}{10} = 5$

5. $\frac{x^2}{49} + \frac{y^2}{36} = 1$

Sol: The given equation is $\frac{x^2}{49} + \frac{y^2}{36} = 1$ or $\frac{x^2}{7^2} + \frac{y^2}{6^2} = 1$

Here, the denominator of $\frac{x^2}{49}$ is greater than the denominator of $\frac{y^2}{36}$

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis

On comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Now, we have a=7 and b=6

$\therefore c = \sqrt{a^2 - b^2} = \sqrt{49 - 36} = \sqrt{13}$

Therefore, the coordinates of the foci are $(\pm\sqrt{13}, 0)$

The coordinates of the vertices are $(\pm 7, 0)$

Length of major axis = 2a = 14

length of minor axis = 2b= 12

Eccentricity, e= $\frac{c}{a}=\frac{\sqrt{13}}{7}$

Length of latusrectum $\frac{2b^2}{a}=\frac{2X36}{7}=\frac{72}{7}$

6. $\frac{x^2}{100}+\frac{y^2}{400}=1$

Sol: The given equation is $\frac{x^2}{100}+\frac{y^2}{400}=1$ or $\frac{x^2}{10^2}+\frac{y^2}{20^2}=1$

Here, the denominator of $\frac{y^2}{400}$ is greater than the denominator of $\frac{x^2}{100}$

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

we have b=10 and a=20

$\therefore c=\sqrt{a^2-b^2}=\sqrt{400-100}=\sqrt{300}=10\sqrt{3}$

Therefore, the coordinates of the foci are $\left(0,\pm 10\sqrt{3}\right)$

The coordinates of the vertices are $\left(0,\pm 20\right)$

Length of major axis = 2a = 40

length of minor axis = 2b= 20

Eccentricity, $e=\frac{c}{a}=\frac{10\sqrt{3}}{20}=\frac{\sqrt{3}}{2}$

Length of latusrectum $\frac{2b^2}{a}=\frac{2X100}{20}=10$

7. $36x^2+4y^2=144$

Sol: The given equation $36x^2+4y^2=144$

$36x^2+4y^2=144$ can be written as $\frac{x^2}{4}+\frac{y^2}{36}=1 \Rightarrow \frac{x^2}{2^2}+\frac{y^2}{6^2}=1$ ----------------(1)

Here, the denominator of $\frac{y^2}{6^2}$ is greater than the denominator of $\frac{x^2}{2^2}$

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing equation (1) with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

Now, we have b=2 and a= 6.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{36-4}=\sqrt{32}=4\sqrt{2}$

Therefore, the coordinates of the foci are $\left(0,4\sqrt{2}\right)$

The coordinates of the vertices are $\left(0,\pm 6\right)$.

Length of major axis = 2a=12

Length of minor axis = 2b = 4

Eccentricity, $e=\frac{c}{a}=\frac{4\sqrt{2}}{6}=\frac{2\sqrt{2}}{3}$

Length of latusrectum $\frac{2b^2}{a}=\frac{2\times 4}{6}=\frac{4}{3}$

8. $16x^2+y^2=16$.

Sol: The given equation is $16x^2+y^2=16$

It can be written as $\frac{x^2}{1}+\frac{y^2}{16}=1 \Rightarrow \frac{x^2}{1^2}+\frac{y^2}{4^2}=1$ ---------------(1)

Here, the denominator of $\frac{y^2}{4^2}$ is greater than the denominator of $\frac{x^2}{1^2}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing equation (1) with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

Now, we have b=1 and a=4.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{16-1}=\sqrt{15}$

Therefore, the coordinates of the foci are $\left(0,\pm\sqrt{15}\right)$.

The coordinates of the vertices are $\left(0,\pm 4\right)$.

Length of major axis = 2a = 8

Length of minor axis = 2b = 2

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{15}}{4}$

Length of latusrectum $\frac{2b^2}{a}=\frac{2\times 1}{4}=\frac{1}{2}$

9. $4x^2+9y^2=36$.

Sol: The given equation is $4x^2+9y^2=36$

It can be written as $\frac{x^2}{9}+\frac{y^2}{4}=1 \Rightarrow \frac{x^2}{3^2}+\frac{y^2}{2^2}=1$ -------------(1)

Here, the denominator of $\frac{x^2}{3^2}$ is greater than the denominator of $\frac{y^2}{2^2}$.

Therefore, the major axis is along the x-axis, while the major axis is along the y-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

We have a=3 and b=2. Therefore, $c=\sqrt{a^2-b^2}=\sqrt{9-4}=\sqrt{5}$

Therefore, the coordinates of the foci are $\left(\pm\sqrt{5},0\right)$

The coordinates of the vertices are $\left(\pm 3,0\right)$

Length of major axis = 2a=6

Length of minor axis = 2b=4

Eccentricity , $e=\frac{c}{a}=\frac{\sqrt{5}}{3}$

Length of latusrectum $\frac{2b^2}{a}=\frac{2\times 4}{3}=\frac{8}{3}$

II. 10: Find the equation of the ellipse with focus at (1,-1), $e=\frac{2}{3}$ and directrix as $x+y+2=0$

Sol: Let p(x ,y) be any point on the ellipse.

Given equation of the directrix is $x+y+2=0$

By the definition of ellipse SP=e.PM

⇨ $SP^2=e^2.PM^2$

⇨ $\left(x_1-1\right)^2+\left(y_1+1\right)^2=\left(\frac{2}{3}\right)^2\left(\frac{x_1+y_1+2}{\sqrt{1+1}}\right)^2$

=> $\left(x_1-1\right)^2+\left(y_1+1\right)^2=\frac{4}{9}\frac{\left(x_1+y_1+2\right)^2}{2}$

=> $9\left[x_1{}^2-2x_1+1+y_1{}^2+2y_1+1\right]=2\left(x_1{}^2+y_1{}^2+4+2x_1y_1+4y_1+4x_1\right)$

=> $7x_1{}^2-4x_1y_1+7y_1{}^2-26x_1+10y_1+10=0$

Locus of $p\left(x_1,y_1\right)$ is $7x^2-4xy+7y^2-26x+10y+10=0$

This is equation of the required ellipse.

11. Find the equation of the ellipse in the standard form whose distance between the foci is 2 and the length of latusrectum is $\frac{15}{2}$

Sol: Let the equation of the required ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ------------ (1) $\because (a>b>0)$

Given distance between foci = 2

⇨ 2ae =2 => ae=1

And length of latusrectum = $\frac{15}{2}$

$=>\frac{2b^2}{a}=\frac{15}{2}=>b^2=\frac{15}{4}a$

Since $b^2=a^2-(ae)^2$

$=>\frac{15}{4}a=a^2-1\Rightarrow a=4\ or\ a=-\frac{1}{4}$

Since $b^2=a^2-(ae)^2\Rightarrow b^2=16-1=15$ and $a^2=16\because (a>b>0)$

∴The requiredequation of ellipse is $\frac{x^2}{16}+\frac{y^2}{15}=1$.

12.Find the equation of the ellipse in the standard form such that distance between foci is 8 and distance between directrix is 32.

Sol: Let the equation of the required ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,(a>b>0)$ -----------(1)

Given distance between foci= 8

$\Rightarrow 2ae=8\Rightarrow ae=4$ and distance between directrix =32

$\Rightarrow \frac{2a}{e}=32\Rightarrow \frac{a}{e}=16$

Now ae x $\frac{a}{e}$ = 4 x 16$\Rightarrow a^2$ =64

Since $b^2=a^2-(ae)^2\Rightarrow b^2=64-(4)^2=64-16\Rightarrow b^2=48$

∴From (1) equation of ellipse is $\frac{x^2}{64}+\frac{y^2}{48}=1$

In each of the following exercises 13 to 23, find the equation for the ellipse that satisfies the given conditions:

13. vertices (±5,0), foci (±4,0).

Sol: vertices (±5,0), foci (±4,0).Here, the vertices are on the x- axis .

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$,

where a is the semi-major axis. Accordingly, $a=5, c=4$

It is known that $a^2=b^2+c^2 \Rightarrow b^2=25-16 \Rightarrow b=\sqrt{9}=3$

Thus, the equation of the ellipse is $\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$ or $\frac{x^2}{25}+\frac{y^2}{9}=1$.

14: vertices (0,±13), foci (0,±5).

Sol: vertices (0,±13), foci (0,±5).Here, the vertices are on the y –axis.

So, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi- major axis ,Accordingly, $a=13, c=5$

It is known that $a^2=b^2+c^2 \Rightarrow 169=b^2+25$, $\Rightarrow b^2=169-25$, $\Rightarrow b=\sqrt{144}=12$

Thus, the equation of the ellipse is $\frac{x^2}{12^2}+\frac{y^2}{13^2}=1$ or $\frac{x^2}{144}+\frac{y^2}{169}=1$

15. vertices (±6,0), foci (±4,0).

Sol: vertices (±6,0), foci (±4,0). Here, the vertices are on the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$,

where a is the semi - major axis.Accordingly, $a=6, c=4$

. It is known that $a^2=b^2+c^2$, $\Rightarrow 36=b^2+16$, $\Rightarrow b^2=36-16 \Rightarrow b=\sqrt{20}$

Thus, the equation of the ellipse is $\frac{x^2}{6^2}+\frac{y^2}{\left(\sqrt{20}\right)^2}=1\ (or)\ \frac{x^2}{36}+\frac{y^2}{20}=1$

16. Ends of major axis (±3,0), ends or minor axis (0,±2)

Sol: Ends of major axis (±3,0), ends or minor axis (0,±2)

Here, the major axis is along the x- axis .

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$,

where a is the semi-major axisAccordingly, $a=3, b=2$

Thus, the equation of the ellipse is $\frac{x^2}{3^2}+\frac{y^2}{2^2}=1 \Rightarrow \frac{x^2}{9}+\frac{y^2}{4}=1$

17.Ends of major axis (0,±√5), ends or minor axis (±1,0).

Sol: Ends of major axis (0,±$\sqrt{5}$), ends of minor axis (±1,0).
Here, the major axis is along the y- axis.
Therefore the equation of the ellipse will be of the form$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$
where a is the semi-major axis. Accordingly, a = $\sqrt{5}$and b = 1

Thus, the equation of the ellipse is $\frac{x^2}{1^2}+\frac{y^2}{\left(\sqrt{5}\right)^2}=1 \Rightarrow \frac{x^2}{1}+\frac{y^2}{5}=1$.

18.Length of major axis 26, foci ($\pm$5,0).

Sol: Length of major axis 26, foci ($\pm$5,0).
Since the foci are on the x- axis, the major axis is along the x-axis.

Let equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}$=1, where a is the semi-major axis

Accordingly, $2a=26$
$\Rightarrow a=13 \; and \; c=5$
It is known that $a^2=b^2+c^2$
$\therefore 13^2=b^2+5^2$
$\Rightarrow 169=b^2+25$, $\Rightarrow \mathbf{b}^2=169-25$, $\Rightarrow b=\sqrt{144}=12$
Thus, the equation of the ellipse is $\frac{x^2}{13^2}+\frac{y^2}{12^2}=1 \Rightarrow \frac{x^2}{169}+\frac{y^2}{144}=1$

19. Length of minor axis 16, foci (0,$\pm$6).
Sol: Length of minor axis 16, foci (0,$\pm$6).
Since the foci are on the y- axis, the major axis is along the y-axis.
Therefore, equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$,
where a is the semi-major axis

Accordingly, 2b=16
$\Rightarrow$b = 8 and c =6. It is known that $a^2=b^2+c^2$=64+36 = 100
Thus, the equation of the ellipse is $\frac{x^2}{8^2}+\frac{y^2}{10^2}=1 \Rightarrow \frac{x^2}{64}+\frac{y^2}{100}=1$

20. foci ($\pm$3,0), a =4.
Sol: foci ($\pm$3,0), a =4.
Since the foci are on the x- axis, the major axis is along the x-axis.
Therefore, equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}$=1,
where a is the semi-major axis
Accordingly c =3 and a =4. It is known that $a^2=b^2+c^2 \Rightarrow b^2=16-9=7$
Thus, the equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{7}=1$.

21.b=3, c=4, centre at the origin; foci on the x-axis.

Sol: It is given that b = 3, c = 4, centre at the origin; foci on the x axis.

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1,

Where a is the semi – major axis.

Accordingly, b = 3, c = 4. It is known that $a^2 = b^2 + c^2$

$\therefore\ a^2 = 3^2 + 4^2 = 9 + 16 = 25 \quad \Rightarrow a = 5$

Thus, the equation of the ellipse is $\dfrac{x^2}{5^2} + \dfrac{y^2}{3^2} = 1 \Rightarrow \dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

22. Centre at (0, 0), major axis on the y-axis and passes through the points (3,2) and (1,6).

Sol: As the centre is at (0, 0) and the major axis is on the y-axis, the equation of the ellipse will be of the form

$\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1 \quad ...(1)$, Where a is the semi – major axis.

The ellipse passes through points (3, 2) and (1,6). Therefore,

$\dfrac{9}{b^2} + \dfrac{4}{a^2} = 1 \qquad ...(2)$

$\dfrac{1}{b^2} + \dfrac{36}{a^2} = 1 \qquad ...(3)$

On solving equations (2) and (3), we have $b^2 = 10$ *and* $a^2 = 40$

Thus, the equation of the ellipse is $\dfrac{x^2}{10} + \dfrac{y^2}{40} = 1 \qquad \Rightarrow 4x^2 + y^2 = 40$

23. Major axis on the x–axis and passes through the points (4, 3) and (6,2).

Sol: Since the major axis is on the x-axis, the equation of the ellipse will be of the form

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \qquad ...(1)$

Where, a is the semi –major axis. The ellipse passes through points (4, 3) and (6, 2). Therefore,

$\dfrac{16}{a^2} + \dfrac{9}{b^2} = 1 \qquad ...(2)$, $\dfrac{36}{a^2} + \dfrac{4}{b^2} = 1 \qquad ...(3)$

On solving equations (2) and (3), we have $a^2 = 52$ *and* $b^2 = 13$

Thus, the equation of the ellipse is $\dfrac{x^2}{52} + \dfrac{y^2}{13} = 1$

$x^2 + 4y^2 = 52$

HYPERBOLA (EXERCISE-10(d)

I.1) Find the eccentricity of hyperbola $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$

Sol: Given hyperbola $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$

$\Rightarrow \frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is called conjugate hyperbola.

$\therefore$ eccentricity $e=\sqrt{\frac{a^2+b^2}{b^2}}$

2) Find the eccentricity of equilibrium of equilateral hyperbola.

Sol: In an equilateral hyperbola b = a then the hyperbola is called rectangular hyperbola.

$\therefore$ eccentricity $e=\sqrt{\frac{a^2+a^2}{a^2}}=\sqrt{\frac{2a^2}{a^2}}=\sqrt{2}$

3) Find the length of the conjugate axis of the hyperbola $3x^2-4y^2=12$

Sol: Given hyperbola $3x^2-4y^2=12 \Rightarrow \frac{x^2}{4}-\frac{y^2}{3}=1$

Compare with $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$,

we get $a^2=4 \Rightarrow a=2$ and $b^2=3 \Rightarrow b=\sqrt{3}$

Length of the conjugate axis 2b = $2\sqrt{3}$

4) Find length of the transverse axis of a hyperbola $x^2-4y^2=4$

Sol: Given equation of the hyperbola is $x^2-4y^2=4 \Rightarrow \frac{x^2}{4}-\frac{y^2}{1}=1$

Compare with $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ we get $a^2=4 \Rightarrow a=2$ and $b^2=1 \Rightarrow b=1$

Length of the transverse axis is $2a$ = 2(1) =2

Latus rectum = $\frac{2b^2}{a}=2\left(\frac{27}{3}\right)=18$

II. In each of the exercise 1 to 6 find the co-ordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbolas

1) $\frac{x^2}{16}-\frac{y^2}{9}=1$ 2) $\frac{y^2}{9}-\frac{x^2}{27}=1$ 3) $9y^2-4x^2=36$ 4) $16x^2-9y^2=576$

5) $5y^2-9x^2=36$ 6) $49y^2-16x^2=784$

Sol: Given equation $\frac{x^2}{16}-\frac{y^2}{9}=1$

compare with $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ we get $a^2=16 \Rightarrow a=4$ and $b^2=9 \Rightarrow b=3$,

$c^2=a^2+b^2, c^2=25 \Rightarrow c=5$

Here in a hyperbola equation co-efficient of x^2 is positive. So transverse axis is along the x-axis

Foci = $(\pm c,0)=(\pm 5,0)$

Vertices = $(\underline{+}a,0)=(\underline{+}4,0)$

Eccentricity $e=\frac{c}{a}=\frac{5}{4}$

Latus rectum = $\frac{2b^2}{a}=\frac{18}{4}=\frac{9}{2}$

2)**Sol:** Given equation $\frac{y^2}{9}-\frac{x^2}{27}=1$

Compare with $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ we get $a^2=9 \Rightarrow a=3, b^2=27 \Rightarrow b=3\sqrt{3}$,

$c^2=a^2+b^2=9+27=36$

$c=6$ (c must be positive)

Here in the hyperbola equation y^2 is positive co-efficient, so major axis is y-axis.

Foci = $(0,\pm c)=(0,\pm 6)$

Vertices = $(0,\pm a)=(0,\pm 3)$

Eccentricity $e=\frac{c}{a}=\frac{6}{3}=2\,(c>a)$

Length latus rectum $=\frac{2b^2}{a}=\frac{2(27)}{3}=18$

Note:In hyperbola equation, if the co-efficient of x^2 is positive then its transverse axis is x-axis and if the co-efficient of y^2 is positive then its transverse axis y-axis.

3) **Sol:** Given equation $9y^2-4x^2=36$ divided by 36 we get $\frac{9y^2}{36}-\frac{4x^2}{36}=1$

$\frac{y^2}{4}-\frac{x^2}{9}=1$ compare the equation with $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ we get

$$a^2=4\Rightarrow a=2, \qquad b^2=9\Rightarrow b=3$$

$$c^2=a^2+b^2=4+9\Rightarrow c=\sqrt{13} \quad \text{(c is positive only)}$$

Here, is the hyperbola the co-efficient of y^2 is positive so transverse axis is along y-axis

Foci = $(0,\pm c)=(0,\pm\sqrt{13})$

Vertices = $(0,\pm a)=(0,\pm 2)$

Eccentricity $e=\frac{c}{a}=\frac{\sqrt{13}}{2}$

Latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{2}=9$

4) Given equation is $16x^2-9y^2=576$ divide by 576 we get

$$\frac{16x^2}{576}-\frac{9y^2}{576}=\frac{576}{576}$$

$$\frac{x^2}{36}-\frac{y^2}{64}=1$$

Now, comparing with $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ we get

$a^2=36\Rightarrow a=6$ and $b^2=64\Rightarrow b=8$

$c^2=a^2+b^2=36+64=100\Rightarrow c=10 \quad (\therefore c>0)$

Here in hyperbola equation the co-efficient x^2 is positive, so transverseaxis is along x-axis

Foci= $(\pm c,0)=(\pm 10,0)$

Vertices = $(\pm a,0)=(\pm 6,0)$

Eccentricity $e=\frac{c}{a}=\frac{10}{6}=\frac{5}{3}$

Latus rectum = $\frac{2b^2}{a}=\frac{2\times 64}{9}=\frac{64}{3}$

5) Given equation is $5y^2-9x^2=36$ divided with 36 we get

$$\frac{5y^2}{36}-\frac{9x^2}{36}=1$$

$$\Rightarrow \frac{Y^2}{\left(36/5\right)}-\frac{X^2}{4}=1 \text{ compare with } \frac{y^2}{a^2}-\frac{x^2}{b^2}=1$$

Weight $a^2=\frac{36}{5}\Rightarrow a=\frac{6}{\sqrt{5}}, b^2=4\Rightarrow b=2$

$$c^2=a^2+b^2=\frac{36}{5}+4=\frac{56}{5}\Rightarrow c=\sqrt{\frac{56}{5}}=\frac{2\sqrt{14}}{\sqrt{5}}$$

Here in the hyperbola equation the co-efficient of y^2 is a positive so transverse axis is along y-axis

Foci = $(0,\pm C)=(0,\pm\frac{2\sqrt{14}}{\sqrt{5}})$

Vertices $=(0,\neq a)=(0,\neq\frac{6}{\sqrt{5}})$

Eccentincity e $=\frac{c}{a}=\frac{\sqrt{14}}{3}$

Latusrectum=$\frac{2b^2}{a}=\frac{2\times 4}{(\frac{6}{\sqrt{5}})}=\frac{4\sqrt{5}}{3}$

6) Given equation is $49y^2-16x^2=784$ divided it by 784 we get

$$\frac{49y^2}{784}-\frac{16x^2}{784}=\frac{784}{784}$$

$$\frac{y^2}{16}-\frac{x^2}{49}=1$$

compare with $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1 weget$

$a^2=16\Rightarrow a=4$

$b^2=49\Rightarrow b=7$

$c^2=a^2+b^2=16+49=65\Rightarrow c=\sqrt{65}\ (\because c>0)$

Here in the hyperbola equation the co-efficient y^2 is positive so the transverse axis is along y - axis

Foci = $\left(0,\pm\right)=\left(0,\pm\sqrt{65}\right)$

Vertices = $(0,\pm a)=(0,\pm 4)$

Eccenticity $=a=\dfrac{c}{a}=\dfrac{\sqrt{65}}{4}$

Latusrectum $=\dfrac{2b^2}{a}=\dfrac{2\times 49}{4}=\dfrac{49}{2}$

In each of the exercise 7 to 16 find the equations of the hyperbola satisfying the given conditions

7.vertices ($\pm$2, 0), foci ($\pm$3, 0)

Sol: since given vertices ($\pm$2, 0) and foci ($\pm$2, 0)

Lies on the x- axis, and the co-efficient of x^2 is positive, the equation of hyperbola is in the form of $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

Given $a=2, and\ c=3$

$c^2=a^2+b^2$

$a=4+b^2\Rightarrow b^2=5\Rightarrow b=\sqrt{5}$

Now equation of hyperbola is $\dfrac{x^2}{4}-\dfrac{y^2}{5}=1$

8. Vertices $(0,\pm 5)$, foci $(0,\pm 8)$

Sol: Given vertices $(0,\pm 5)$ and $(0,\pm 8)$ lies on the y –axis

Now hyperbola in the form of $\dfrac{y^2}{a^2}-\dfrac{x^2}{b^2}=1$

Given $(0,\pm a)=(0,\pm 5)\ and\ (0,\pm C)=(0,\pm 8)$

$a=5,\quad c=8$

$c^2=a^2+b^2\Rightarrow b^2=64-25=39\Rightarrow b=\sqrt{39}$

Equation of the hyberbola is $\dfrac{y^2}{9}-\dfrac{x^2}{16}=1$

9. Vertices $(0,\pm 3)\ and\ (0,\pm 5)$ foci

Sol: Given vertices $(0,\pm 3)\ and\ foci(0,\pm 5)b^2\Rightarrow\dfrac{x^2}{16}-\dfrac{y^2}{9}=1(0,\pm 13)$ lies on y-axis.

Hence equation of the hyperbola will be the form.

$\dfrac{y^2}{a}-\dfrac{x^2}{b}=1$

Given that vertices $(0,\neq 0)=(0,\neq 2)$

And Foci $(0,\neq c)=(0,\neq 5)$

$a=3\ and\ c=5$

$c^2=a^2+b^2\Rightarrow 25=a+b^2\Rightarrow b^2=16$

$a^2 = 9$ *and* $b^2 = 16$ equation of the hyperbola is $\frac{y^2}{9} - \frac{x^2}{16} = 1$

10. Foci $(\neq 5, 0)$ and the length of transverse axis 8:

Sol: Given foci $(\neq c, 0) = (\neq C, 0) = (\neq 5, 0)$ lies on $x - axis$ them transverse axis is along $x - axis$ equation of the hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Given 2a = 8 $\Rightarrow a = 4, c = 5 \Rightarrow c^2 = a^2 + b^2 \Rightarrow$ 25 = 16+ b^2

$b^2 = 9, a^2 = 16$

Now equation of hyperbola is $\frac{x^2}{a^2} - \frac{b^2}{b^2} = 1$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

11 Foci $(0, \pm 13)$ the conjugate axis is of length 24.

Sol: Given foci $(0, \pm 13)$ lies on the y-axis and x- coordinate zero.

Here the equation of the hyperbola will be the form

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Where it is given that foci (0,$\pm$13) = (0,$\pm$C), and conjugate axis 2b = 24 $\Rightarrow$ b=12,c =13

$c^2 = a^2 + b^2$

169 = a^2 +144

$\Rightarrow a^2$ = 169-144 = 25

a^2 = 25 and b^2 = 144 we get the equation of the hyperbola is

$$\frac{y^2}{25} - \frac{x^2}{144} = 1$$

12. Foci ($\pm 3\sqrt{5}$, 0) the latusrecture is of length 8.

Sol: Given foci = ($\pm 3\sqrt{5}$, 0)lies on x-axis as y – axis co-ordinate is zero.

Hence equation of hyperbola will be the four

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Given foci ($\pm 3\sqrt{5}$, 0)= $(\pm C, 0)$, C = $3\sqrt{5}$

And length of latusrectum is $\frac{2b^2}{a}$ = 8

b^2 = 4a, $c^2 = a^2 + b^2$

$\Rightarrow (3\sqrt{5})^2 = a^2$ +4a

a^2 +4a-45 = 0

$a = 5, a = -9$ not possible

a^2=25 and b^2 = 4x5=20

Now equation of the hyperbola is $\frac{x^2}{25}-\frac{y^2}{20}=1$

13. Foci $(\pm 4,0)$ the latusrectum is of length 12.

Sol: Given focus $(\pm 4,0)$ lies on x-axis and y- co-ordinate is zero. Given length of latusrectum is $\frac{2b^2}{a}$ = 12, b^2 = 6a

Given foci $(\pm C, 0) = (\pm 4,0)$

$\therefore$ C = 4 ($\because$ C > 0)

$c^2 = a^2 + b^2$

16 = a^2+6a, $\Rightarrow a^2+6a-16=0 \Rightarrow (a-2)(a+8)$ = 0

$a=2, a=-8$ not possible

here a =2, $a^2=4$, b^2 = 6(2) =12

Now equation of the hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$$\frac{x^2}{4}-\frac{y^2}{12}=1$$

14) vertices $(\pm 7,0)$ and e = $\frac{4}{3}$.

Sol: Given vertices $(\pm 7,0)$ lies on x-axis as y – co-ordinate is zero and e = $\frac{c}{a}$ = $\frac{4}{3}$

a=7 and we know that

$c^2 = a^2 + b^2$, C = $\frac{28}{3}$

$$\left(\frac{28}{3}\right)^2 = 49+b^2 \Rightarrow b^2 = \frac{784}{9}-\frac{49}{1}=\frac{343}{9}$$

Now required equation of the hyperbola is $\frac{x^2}{49}-\frac{9y^2}{343}=1$

15) Foci (0, $\pm\sqrt{10}$) and passing through (2,3).

Sol: Given foci (0, $\pm\sqrt{10}$) lies on y-axis a x co-ordinate is zero. Now equation of the hyperbola is in the form of $\frac{y^2}{9^2}$ - $\frac{x^2}{b^2}$ =1-----------(1)

Given foci $(0, \pm\sqrt{10}) = (0, \pm C)$ $\therefore$ C = $\sqrt{10}$, $C^2 = 10$

Now $C^2 = a^2 + b^2$

$10 = a^2 + b^2$

$b^2 = 10 - a^2$ -----------(2)

If (1) passing through (2,3) we get

$\frac{9}{9^2} - \frac{4}{b^2} = 1$ from (2)

$\frac{9}{9^2} - \frac{4}{10 - a^2} = 1$

$90 - 9a^2 - 4a^2 = 10a^2 - a^4$

$a^2 = 5 (or) a^2 = 18$

If $a^2 = 5$ from (2) $b^2 = 5$

For $a^2 = 18$, $b^2 =$ is negative, not possible

$\therefore$ The equation of the hyperbola is $\frac{y^2}{5} - \frac{x^2}{5} = 1$.

16) Find the equation of the hyperbola of given length of transverse axis 6 and whose vertex bisects the distance between centre and focus.

Sol: Given length of Transverse axis, $2a = 6$

a=3 $\Rightarrow a^2 = 9$

let (0,0) be the centre and (c,0) be the focus and vertex is (a,0) =(3,0).

Given vertex bisects the centre and focus

$\therefore$ vertex is the mid point

$\therefore \left(\frac{0+c}{2}, \frac{0+0}{2}\right) = (3,0) \Rightarrow$ c=6

We know that $c^2 = a^2 + b^2 \Rightarrow 36 = 9 + b^2 \Rightarrow b^2 = 27$

$\therefore$ The required equation of the hyperbola is $3x^2 - y^2 = 27$

Exercise 10(e)

I.

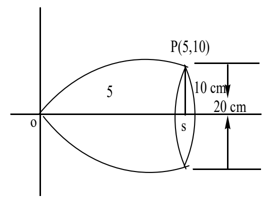
1. If a parabola reflector is 20cm in diameter and 5cm deep, find the focus.

Sol: Taking vertex of the parabola reflector at origin ,x- axis along the axis of parabola. The equation of the parabola is $y^2 = 4ax$ ----------(1)

Given diameter is 20cm and depth is 5cm, P(5,10) be any point on the parabola (1)

$\therefore 100 = 2a \Rightarrow a = 5$

$focus = (a,0) = (5,0)$



2. An arch is in the form of a semi ellipse. It is 8m wide and 2m high at the centre. Find the height of the arch at a point 1.5m from one end,

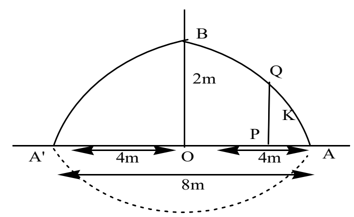
Sol: let the equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ -------(1).

Here $2a = 8 \Rightarrow a = 4$ and $b = 2$

Now equation (1) becomes $\frac{x^2}{16}+\frac{y^2}{4}=1$---------(2)

If AP = 1.5 then P = (2.5,0)

Let Q = (2.5,k) be any point on the parabola (2)



$\frac{(2.5)^2}{16}+\frac{k^2}{4}=1 \Rightarrow \frac{k^2}{4}=1-\frac{6.25}{16}=\frac{16-6.25}{16}=\frac{9.75}{4} \Rightarrow k^2$=2.4375,

$\therefore$ k = 1.56m

II.

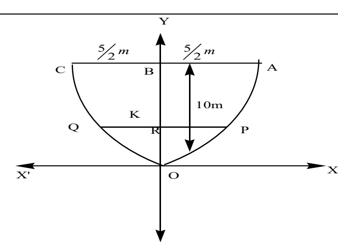
1. An arch is in the form of a parabola with its axis vertical. The arch is 10 m heigh and 5m wide at the base. How wide it is 2m from the vertex of the parabola.

Sol: since axis is vertical let the required equation of the parabola is $x^2 = 4ay$ -------(1)

let OB = 10m, AC =5, AB =CB = $\frac{5}{2}$.

Hence the co-ordinator of A = $(\frac{5}{2},10)$

From (1) $\frac{25}{4} = 40a \Rightarrow a = \frac{5}{32}$

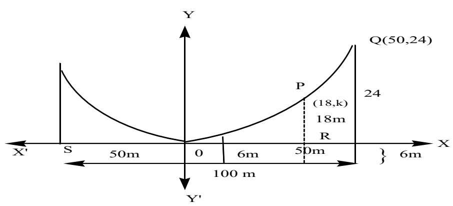


Equation of parabola is $x^2 = 4(\frac{5}{32})y \Rightarrow 8x^2 = 5y$ -----------(2), let PQ = K, RP = $\frac{k}{2}$, OR = 2, let p =($\frac{k}{2}$,2) lies on (2)

$\frac{8k^2}{4}$ =5 (2), $k^2 = 5 \Rightarrow k = \pm\sqrt{5}$ = 2.23 m

2. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The road way which is horizontal and 100 m long is supported a vertical wires attached to the cable. The longest wire being 30m and shortest being 6m. find the length of a supporting wire attached to the roadway 18m from the middle.

Sol: Since wires are vertical. Consider the equation of the parabola is $x^2 = 4ay$ ------(1)



Focus is at the middle of the cable the shortest and longest vertical supports are 6m and 30 m and road way is 100 m long.

$\therefore$ Q(50,24) put in eq (1)

$$(50)^2 = (4a)(24) \Rightarrow \frac{2500}{96} = a$$

Hence equation (1) becomes $x^2 = 4\left(\frac{2500}{96}\right)y$ ----------(2)

Let PR = K , let the point P(18,k) will satisfies equation of the parabola (2)

$$(18)^2 = \frac{2500}{24}k \Rightarrow \frac{2500}{24}k = 324 \Rightarrow k = \frac{324x24}{2500} = \frac{324x6}{625} = \frac{194}{625} \Rightarrow k = 3.11.$$

Required length is 6+k = 6+3.11 = 9.11m.

3. A rod of length 12 cm moves with its ends always touching the co-ordinate axis. Determine the equation of the focus of a point P on the rod which is 3cm from the end in contact with the x-axis.

Sol: let A and B be the ends of the rod moves on the x-axis and y – axis respectively.

Let P(x,y) be any point on the focus. Let P divides $\overline{AB}$ in the ratio 3:9.

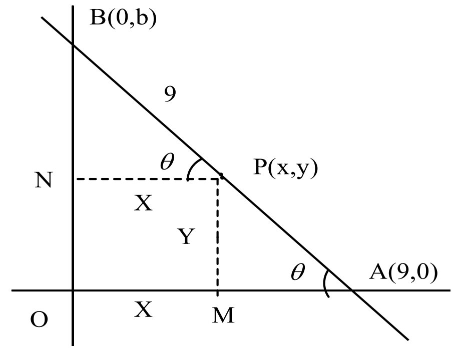
Let M and N are projection from t p to the x-axis and y-axis respectively, let $\underline{|MAP} = \theta$ and $\underline{|NPB} = \theta$

In a right angled triangle MAP sin $\theta = \frac{y}{3}$ -----(1)

and in a right angled triangle NPB

cos $\theta = \frac{x}{9}$

now $\cos^2\theta + \sin^2\theta = 1 \Rightarrow \frac{x^2}{81} + \frac{y^2}{9} = 1$



III.

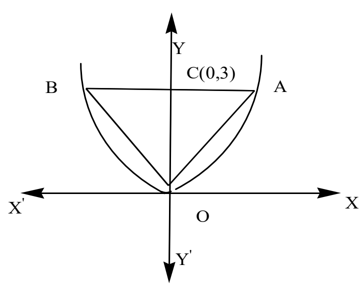
1. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the two ends of itslatusrectum.

Sol: Given equation of the parabola $x^2 = 12y$ ---------(1)

Since $4a = 12 \Rightarrow a = 3$

Focus is C = (0,3)

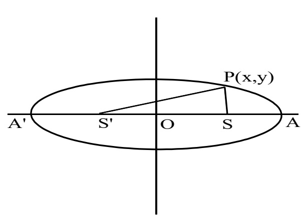
Area of ΔOAB = $\frac{1}{2}(ABxOC) = \frac{1}{2}(12)(3) = 18sq.$units



2. A man running a race course notes that the sum of its distances from two flag posts from him is always 10 m and the distance between the flag posts is 8m. Find the equation of the path traced by the man.

Sol: The path traced by the man is an Ellipse Let S and S^1 be the flag Posts and P (x,y) be any print on the Locus.

$\therefore$ SP + S^1P = $2a = 10 \Rightarrow a = 5$



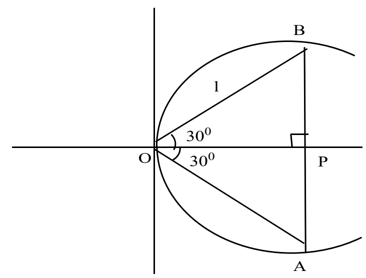
Since the coordinates of S and S^1 are (c,o) and (-c,o),therefore distance between s and s^1 is 2C = 8, C= 4 and $c^2 = a^2 – b^2 \Rightarrow 16=25-b^2 \Rightarrow b^2 = 25-16=9$ $\therefore$ b=3

$\therefore$ The required equation of the Ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$

3. An equilateral Triangle is inscribed in the parabola $y^2 = 4ax$ where one vertex is at the vertex of the parabola. Find the length of the side of the Triangle.

Sol: Given equation of the Parabola is $y^2 = 4ax \rightarrow (1)$

Let OAB is an equilateral Triangle inscribed in the parabola



Let OB=l = OA = AB

$\therefore PA = PB = \frac{l}{2}$, $\angle BOA = 60^0 \Rightarrow \angle BOP = 30^0$

In a Triangle BOP $\sin 30^0 = \frac{PB}{l} \Rightarrow \frac{1}{2} = \frac{PB}{l}$

$\therefore PB = \frac{l}{2} \rightarrow (2)$

and $\cos 30^0 = \frac{OP}{OB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{OP}{l} \Rightarrow OP = \frac{\sqrt{3}l}{2}$

$\therefore B = (OP, PB) = (\frac{\sqrt{3}l}{2}, \frac{\sqrt{l}}{2})$ *willsatisfies* $y^2 = 4ax$

$\therefore \frac{l^2}{4} = 4a\frac{\sqrt{3}l}{2} \Rightarrow l = 8\sqrt{3}a$

$\therefore$ length of the side of the equilateral Triangle is $8\sqrt{3}a$.

<u>MCQ's Circles</u>

1. By solving the lines 2x+y-7=0 and x+3y-11=0 we get the centre of the circle C(2,3) = (-g,-f). Equations of the required circle $x^2+y^2-4x-6y+c=0$ since it passes through (5,7) we get C = -12

   $\therefore$ Required circle is $x^2+y^2-4x-6y-12=0$

2. Given centre A(2,1), P (10,7)

   AP = $\sqrt{8^2+6^2} = \sqrt{100} = 10$

   Given PQ = 5 $\Rightarrow$ since Q lies on circle

   Then the radius of circle AQ = AP-PQ =10-5=5

   $\therefore$r = 5

   Then radius $\sqrt{g^2+f^2-c} = r \Rightarrow g^2+f^2-c=r^2 \Rightarrow$ 4+1-C=25, $\therefore$C = -20.

3. Required equation of the circle $x^2+y^2+2ax+2py-b^2-q^2=0$

$\therefore$ radius $r=\sqrt{g^2+f^2-c} \Rightarrow \sqrt{a^2+p^2+b^2+q^2}$

4. The equation of the circle concentric with given circle be $x^2+y^2-6x+12y+C=0$.

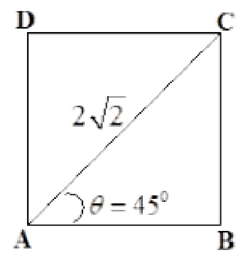
Radius of the given circle $r=\sqrt{9^2+36^2-15}=\sqrt{30}$.

If area is double then $A_1=2A \Rightarrow \Pi R^2=2.\Pi r^2 \Rightarrow R^2 = 2r^2=2x30 = 60$

$R^2=g^2+f^2-c \Rightarrow 60=9+36-C \Rightarrow C = -15$

$\therefore$ Required circle is $x^2+y^2-6x+12y-15=0$.

5. **Let ABCD be square inscribed in circle centre G = (1,-2), radius** $=\sqrt{2}$ **, diameter =** $2\sqrt{2}$ **.**



Let 'a' be length of side of square then $a^2+a^2=\left(2\sqrt{2}\right)^2=8$

$2a^2=8 \Rightarrow a^2=4 \Rightarrow a=2$ square sides are parallel to coordinate axis there coordinates of vertex of square are

$\left(1\pm\sqrt{2}\ \text{Cos}\,45^0,-2\pm\sqrt{25}\ \sin\,45^0\right)=(2,-1)\ and\ (0,-3)$,

other two vertexes of square are $(2,-1+2)$ and $(0,-3+2)$

$= (2,1),(0,-1)$

These are not in given options.

6. **The equation of the circle with centre C** $(\text{Cos}\,\alpha, \text{Sin}\,\alpha)$ and radius 1 is.

$(x-\text{Cos}\,\alpha)^2+(y-\text{Sin}\,\alpha)^2=1^2$

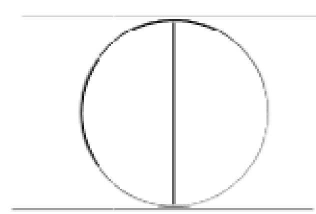
$x^2+y^2-2x\,\text{Cos}\,\alpha-2y\,\text{Sin}\,\alpha=0$

7. $r_1=\sqrt{4+16+44}=8$

$r_2=\sqrt{9+16+96}=\sqrt{121}=11$

$\therefore r_1:r_2=8:11$

8. **Given lines are parallel distance between parallel lines is diameter of the circle.**



$d=\left|\dfrac{C_1-C_2}{\sqrt{a^2+b^2}}\right|=\dfrac{10+9}{\sqrt{36+64}}=\dfrac{19}{10}$

$r={d}/{2}=\dfrac{19}{20}$

9. **Given circle** $x^2+y^2={16}/{3} \Rightarrow r=\sqrt{{16}/{3}}=\dfrac{4}{\sqrt{3}}$ perimeter of the circle = $2\pi r$

$2\pi\ {4}/{\sqrt{3}} = {8}/{\sqrt{3}}\ \pi$

10. **Given circle** $x^2+y^2-x+3y+{10}/{3}=0$, $g={-1}/{2}, f={3}/{2}, c={10}/{3}$,

$$r=\sqrt{g^2+f^2-c}$$

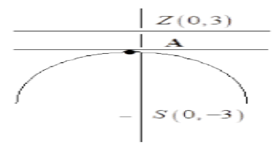
$$r=\sqrt{1/4+9/4-10/3}=\sqrt{\frac{10}{4}-\frac{10}{3}}=\sqrt{\frac{-10}{12}}<0$$

Parabola MCQ'S

1) $S(0,0), Z(2,0)$

Mid point of SZ = vertex $=(1,0)$

2) $S(0,-3), Z(0,3)$



Required parabola

$x^2=-4ay=-4(3y)$

$x^2=-12y$

3) Since the given parabola $y^2=4ax$ passes through $(3,2)$

$$\Rightarrow 2^2=4a.3$$

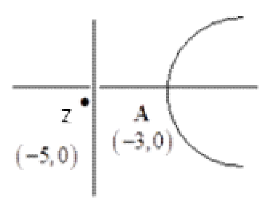
$$\Rightarrow 4a=4/3.$$

$\therefore$ Length of the latusrectum $=4a=4/3$

4) Given vertex $\mathbf{A}=(-3,0)$

Directrix is $x+5=0 \Rightarrow x=-5$

$Z=(-5,0)$ then $a=AZ=2$



Equation of the required parabola

$(y-0)^2=4\times 2(x+3)$

$(y-k)^2=4a(x-h)$

$y^2=8(x+3)$

Ellipse MCQ'S

1) $e=1/2$

$a/e = 4 \Rightarrow a = 4e \Rightarrow a = 4/2 \Rightarrow a = 2$

$b^2 = a^2\left(1-e^2\right) = 4\left(1-1/4\right) = \frac{4\times 3}{4} = 3$

Required equation of ellipse

$x^2/4 + y^2/3 = 1 \Rightarrow 3x^2 + 4y^2 = 12$

2) $ae = 4$, $e = 4/5$

$ae = 4 \Rightarrow a\,4/5 = 4 \Rightarrow a = 5$

$b^2 = a^2\left(1-e^2\right) = 25\left(1-16/25\right) = 25\frac{(9)}{25} = 9$

$\therefore$ Required equation of the ellipse is

$\frac{x^2}{25} + y^2/9 = 1 \Rightarrow x^2/5^2 + y^2/3^2 = 1$

3) The Ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point $(4,0)$. Then the equation of the ellipse is

Sol:- Given Ellipse $x^2 + 4y^2 = 4$

$\frac{x^2}{4} + \frac{y^2}{1} = 1 \Rightarrow a^2 = 4 \Rightarrow a = 2,\ b^2 = 1 \Rightarrow b = 1$

Let $E = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$ be required Ellipse

Given E passes through $(4,0)$ then $\frac{16}{a^2} + \frac{0}{b^2} = 1 \Rightarrow a^2 = 16$

And E passes through $(2,1)$ then $\frac{4}{16} + \frac{1}{b^2} = 1 \Rightarrow b^2 = 4/3$

Required Ellipse is $x^2 + 12y^2 = 16$

4) Since focus is at the origine. The distance from the focus to the directrix is given by

$\left|a/e - ae\right| = 4$

Put $e = 1/2$

$\left|2a - a/2\right| = 4 \Rightarrow 3a/2 = 4 \Rightarrow a = 8/3$

5) Major axis = 3 (Minor axes) of ellipse

$2a = 3(2b) \Rightarrow a = 3b$

Centricity $= e = \sqrt{\dfrac{a^2 - b^2}{a^2}} = \sqrt{\dfrac{9b^2 - b^2}{9b^2}} = \sqrt{\dfrac{8b^2}{9b^2}} = \sqrt{8}/3 = \dfrac{2\sqrt{2}}{3}$

6) $x^2 + 3y^2 = 6 \rightarrow$ Given Ellipse

$x^2 + 3y^2 = 6 \Rightarrow x^2/6 + y^2/2 = 1$

$e = \sqrt{\dfrac{a^2 - b^2}{a^2}} = \sqrt{\dfrac{6-2}{6}} = \dfrac{\sqrt{4}}{\sqrt{6}} = \sqrt{\dfrac{2}{3}}$

<u>MCQ's Hyperbola</u>

1) Let P (x,y) be any point on the hyperbola,

Here $\dfrac{SP}{PM} = e = \sqrt{2}, \qquad S = (1, -1), \qquad l \equiv x - y + 1 = 0$

$SP^2 = 2(PM)^2$

$\Rightarrow (x-1)^2 + (y+1)^2 = 2\left|\dfrac{x - y + 1}{\sqrt{2}}\right|^2$

$\Rightarrow x^2 + y^2 - 2x + 2y + 2 = x^2 + y^2 + 1 - 2xy - 2y + 2x$

$\Rightarrow 2xy - 4x + 4y + 1 = 0$

2) $\dfrac{SP}{PM} = \sqrt{5}, \qquad S = (0,0), \qquad l \equiv 3x + 4y + 1 = 0(1)$

$(x-0)^2 + (y-0)^2 = (\sqrt{5})^2 \quad \left|\dfrac{3x + 4y + 1}{\sqrt{25}}\right|^2$

$\Rightarrow 5x^2 + 5y^2 = 9x^2 + 16y^2 + 1 + 24xy + 8y + 6x$

$\Rightarrow 4x^2 + 11y^2 + 24xy + 6x + 8y + 1 = 0$

3) $(\pm a$, c) = $(\pm 2, 0)$, $(\pm c, 0) = (\pm 3, 0)$ (2)

a =2, c=3, $c^2 = a^2 + b^2$

$9 = 4 + b^2 \Rightarrow b^2 = 5, a^2 = 4$

Now equation $\dfrac{x^2}{4} - \dfrac{y^2}{5} = 1$

4) Given hyperbola $9y^2 - 4x^2 = 36$

$$\frac{y^2}{4}-\frac{x^2}{9}=1$$

$a^2=4 \Rightarrow a=2, \qquad b^2=9 \ \Rightarrow b=3$

$c^2-a^2+a^2=4+9=\sqrt{13}$

foci $\quad (o, \ \pm c)=(o, \ \pm\sqrt{13})$

5) Equation of hyperbola will be $\frac{(x-2)^2}{12}-\frac{(y+1)^2}{4}=1$

Here a^2 = 12, b^2 =4

Focal length of the hyperbola is $2\sqrt{a^2+b^2}=8$

6) $\quad \frac{x^2}{9-c}-\frac{y^2}{c-5}=1$

9-c$>0, \quad c-5>0$

c<9, c>5

$5<c<9$

7) $\quad$ Given ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$

$e=\sqrt{\frac{a^2-b^2}{a^2}} \quad =\sqrt{\frac{4}{9}} \quad = \quad \frac{2}{3}$

Given hyperbola $\frac{x^2}{9}-\frac{y^2}{5}=1$

$e^{'}=\frac{\sqrt{14}}{3}$

$ee^{'}=\frac{2\sqrt{14}}{9}$

8) $\quad c^2=a^2-b^2=25-16=9$, foci (3,0) and $c^2=a^2+b^2=4+b^2$, foci $(\sqrt{4+b^2},0)$

$\therefore \sqrt{4+b^2}=3 \ \Rightarrow b^2=5$

9) $\quad e^2=\cos^2 2+\sin^2\alpha \ \Rightarrow c=1$

Foci $=(\pm 1, 0)$

Abscissae of foci is constant.

Multiple Choice Questions (KEY)

CIRCLES:

1) 3	2) 4	3) 1	4) 1	5) 4	6) 2	7) 1	8) 4	9) 3	10) 4

PARABOLA:

1) 1	2) 1	3) 2	4) 1

ELLIPSE:

1) 4	2) 2	3) 1	4) 4	5) 1	6) 2

HYPERBOLA:

1) 3	2) 1	3) 2	4) $(o, \pm\sqrt{13})$	5) 3	6)1	7) $\frac{2\sqrt{14}}{9}$	8)2	9)4

11. INTRODUCTION TO THREE DIMENSIONAL GEOMETRY

Exercise 11 (a)

1. A Point is on the x-axis. What are its y- coordinate and z co ordinates ?

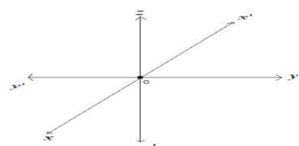
Sol: The Co- ordinates of any point on the x-axis is the form of $(x,0,0)$. So its y- co ordinate and z- co-ordinates are zero.

2. A point is in the XZ- plane . What can you say about its y- Co-ordinate ?

Sol: Any point on the XZ- plane will have the Co- ordinates $(x,0,z)$ then its y- co- ordinate is zero.

3.Name the octants in which the following points lie $(1,2,3)$, $(4,-2,3)$, $(4,-2,-5)$,(4,2,-5), $(-4,2,-5)$, $(-4,2,5)$, $(-3,-1,6)$, $(-2,-4,-7)$.

Sol: Given points are $(1,2,3)$, $(4,-2,3)$,(4,-2,-5), $(4,2,-5)$, $(-4,2,-5)$, $(-4,2,5)$, $(-3,-1,6)$, $(-2,-4,-7)$.



Octants Coordinate	I	II	III	IV	V	VI	VII	VIII
x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	-	-

Therefore the given points lies in I,IV,VIII,V,VI,II,III,VII

4. Fill in the blanks :-

(i) The x- axis and y-axis taken together determine a plane known as <u>*XY*-Plane</u>

(ii) The Co-ordinates of points in the *XY*- Plane are of the form $(x,y,0)$

(iii) Three Co-ordinate planes divides the space into <u>eight</u> octants

5. (i) The point (x,o,o) lies on <u>X-axis</u>

(ii) The point $(0,y,0)$ lies on <u>Y- axis</u>

(iv)The point $(0,0,z)$ **lies on <u>Z-axis</u>**

(v) The point $(x,y,0)$ **lies on <u>XY - plane</u>**

(vi) The point $(x,0,z)$ **lies on <u>XZ- plane</u>**

EXERCISE: 11 (b)

I) 1) Find the distance between the following pairs of

(i) $(2,3,5)$ and $(4,3,1)$ **(ii)** $(-3,7,2)$ and $(2,4,-1)$

(iii) $(-1,3,-4)$ and (1,-3,4) **(iv)** $(2,-1,3)$ and $(-2,1,3)$

Sol: (i) Let the given points are $A=(2,3,5)=(x,y,z)$ and $B=(4,3,1)=(x_2,y_2,z_2)$

Required distance $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$\sqrt{(2+3)^2+(4-7)^2+(-1-2)^2}$$

$$=\sqrt{4+0+16} \quad =\sqrt{20}=2\sqrt{5}$$

(ii) Let the given points are $\mathbf{A}=(-3,7,2)=(x,y,z)$ and

$B=(2,4,-1) = (x_2,y_2,z_2)$

Required distance $\mathbf{AB}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$$=\sqrt{(2+3)^2+(4-7)^2+(-1-2)^2} =\sqrt{43}$$

(iii) Let the given points are $\mathbf{A}=(-1,3,-4)=(x_{,},y_1,z_1)$ and $\mathbf{B}=(1,-3,4)=(x_2 y_2,z_2)$

Required distance $\mathbf{AB}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$$=\sqrt{(1+1)^2+(-3-3)^2+(4+4)^2}$$

$$=\sqrt{4+36+64}=\sqrt{10\ 4}=2\sqrt{26}$$

(iv) Let the given points are $\mathbf{A}=(2,-1,3)=(x_1+y_1,z_1)$ and $B=(-2,1,3) =(x_2,y_2,z_2)$

Required distance $\mathbf{AB}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$$=\sqrt{(-2-2)^2+(1+1)^2+(3-3)^2}$$

$$=\sqrt{16+4+0} \quad =\sqrt{20} =2\sqrt{5}$$

2) Find the distance of $P(3,-2,4)$ from the origin

Sol: Given point is $\mathbf{p}(3,-2,4)=(x_1,y_1,z_1)$ co-ordination of origin $O=(0,0,0)$

The distance $\mathbf{OP}=\sqrt{x_1^2+y_1^2+z_1^2} =\sqrt{9+4+16} = \sqrt{29}$

3) Find the value of x if the distance between $(5,-1,7)$ and $(x,5,1)$ is 9 units.

Sol: Let the given points are $\mathbf{A}=(5,-1,7)=(x_1,y_1,z_1)$ and $\mathbf{B}=(x,5,1)=(x_2,y_2,z_2)$

and also given that $\mathbf{AB}=9$ $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}=9$

$\Rightarrow \sqrt{(x-5)^2+(5+1)^2+(1-7)^2}=9 \quad \Rightarrow \sqrt{(x-5)^2+36+36}=9$

Squaring on both sides, we get $(x-5)^2=9$

$\therefore \quad x-5=\pm 3 \Rightarrow x=8$ (or) $x=2$

II) 1) Show that the points $(-2,3,5),(1,2,3)$ *and* $(7,0,-1)$ are collinear.

Sol: Let the given points are $\mathbf{A}=(-2,3,5),\mathbf{B}=(1,2,3)$ *and* $\mathbf{C}=(7,0,-1)$

$$\mathbf{AB}=\sqrt{(1+2)^2+(2-3)^2+(3-5)^2}=\sqrt{9+1+4}=\sqrt{14}$$

$$\mathbf{BC}=\sqrt{(7-1)^2+(0-2)^2+(-1-3)^2}=\sqrt{36+4+16}=2\sqrt{14}$$

$$\mathbf{AC}=\sqrt{(7+2)^2+(0-3)^2+(-1-5)^2}=\sqrt{81+9+36}=\sqrt{126}=3\sqrt{14}$$

Clearly AB+BC=AC

$\mathbf{AB}+\mathbf{BC}=\sqrt{14}+2\sqrt{14}=3\sqrt{14}$, $\mathbf{AC}=3\sqrt{14}$

Hence the given points are collinear.

2) Verify the following :

(i) $(0,7,-10),(1,6,-6)$ and $(4,9,-6)$ are the vertices of an isosceles triangle .

(ii) $(0,7,10),(-1,6,6)$ and $(-4,9,6)$ are the vertices of a right angled triangle.

(iii) $(-1,2,1),(1-,2,5),(4,-7,8)$ and $(2,-3,4)$ are vertices of a parallelogram.

Sol: **(i)** Let $\mathbf{A}=(0,7,-10)$¸ $\mathbf{B}=(1,6,-6)$ and $\mathbf{C}=(4,9,-6)$ are the vertices of a triangle

$$\mathbf{AB}=\sqrt{(1-0)^2+(6-7)^2+(-6+10)^2}=\sqrt{18}=3\sqrt{2}$$

$$\mathbf{BC}=\sqrt{(4-1)^2+(9-6)^2+(-6+6)^2}=\sqrt{18}=3\sqrt{2}$$

Clearly $\mathbf{AB}=\mathbf{BC}=\mathbf{3\sqrt{2}}$. Hence triangle is an Isosceles triangle.

(ii) Let $\mathbf{A}=(0,7,10),\mathbf{B}=(-1,6,6)$ and $C(-4,9,6)$ are the vertices of a triangle then side

$$\mathbf{AB}=\sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}=\sqrt{1+1+16}=\sqrt{18}=3\sqrt{2}$$

$$\mathbf{BC}=\sqrt{(-4+1)^2+(9-6)^2+(6-6)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$$

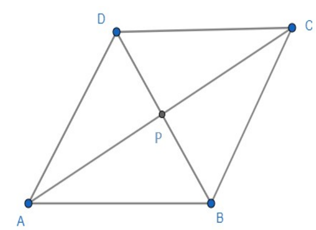
$$\mathbf{CA}=\sqrt{(-4-0)^2+(9-7)^2+(6-10)^2}=\sqrt{16+4+16}=\sqrt{36}=6$$

None $AB^2+BC^2=18+18=36\Rightarrow\ CA^2=\left(\sqrt{36}\right)^2=36$

$\therefore AB^2+BC^2=CA^2$

$\therefore\Delta$ ABC is right-angled Triangle at B.

(iii) Let $\mathbf{A}=(-1,2,3)$, $\mathbf{B}=(1,-2,5)$, $\mathbf{C}=(4,-7,8)$ and $D=(2,-3,4)$ are vertices of quadrilateral. ABCD.



$$\mathbf{AB}=\sqrt{(1+1)^2(-2-2)^2+(5-3)^2}=\sqrt{4+16+4}=\sqrt{24}=2\sqrt{6}$$

$$\mathbf{BC}=\sqrt{(4-1)^2+(-7+2)^2+(8-5)^2}=\sqrt{9+25+9}=\sqrt{43}$$

$$\mathbf{CD}=\sqrt{(4+2)^2+(-7+3)^2+(4-8)^2}=\sqrt{4+16+4}=\sqrt{24}=2\sqrt{6}$$

$$\mathbf{DA}=\sqrt{(2+1)^2+(-3-2)^2+(4+1)^2}=\sqrt{9+25+9}=\sqrt{43}$$

$$\mathbf{AC}=\sqrt{(4+1)^2+(-7-2)^2+(8-3)^2}=\sqrt{25+81+25}=\sqrt{131}$$

$$\mathbf{BD}=\sqrt{(2-1)^2+(-3+2)^2+(4-5)^2}=\sqrt{1+1+1}=\sqrt{3}$$

None = AB = DC, BC=DA, and AC $\neq$ BD

$\therefore$ ABCD quadrilateral is parallelogram .

3) Find the equitation of the set of points. Which are equidistant from the points (1,2,3) and (3,2,-1)

Sol: Let P (x,y,z) be any point on the locus. Let the given points are A= (1,2,3) and B (3,2,-1)

Given geometric condition is $PA=PB\ \Rightarrow PA^2=PB^2$

$$\Rightarrow(x-1)^2+(y-2)^2+(z-3)^2=(x-3)^2+(y-2)^2+(z+1)^2$$

$\Rightarrow x^2+1-2x-y^2+4-4y+z^2+9-6z=x^2+9-6x+y^2+4-4y+z^2+1-2z$

Therefore $x-2z=0$.

4) Find the equation of set of points P, the sum of whose distance from $\mathbf{A}=(4,0,0)$ and $B=(-4,0,0)$ is equal to 10.

Sol: Let P (x,y,z) be any point on the locus. Let the given points are $\mathbf{A}=(4,0,0)$ and $B=(-4,0,0)$

Given geometrics condition is $PA+PB=10 \Rightarrow PA=10-PB$

Squaring on both sides $PA^2=100+PB^2-20PB$

$\Rightarrow (x-4)^2+y^2+z^2=100+1(x+4)^2+y^2+z^2-20\,PB$

$\Rightarrow x^2+16-8x+y^2+z^2=100+x^2+16+8x+y^2+z^2-20\,PB$

$\Rightarrow -16x-100=-20\,PB \Rightarrow 4x+25=5PB$

Squaring on both sides

$\Rightarrow 16x^2+625+200\,x$

$=25\left((x+4)^2+y^2+z^2\right)$

$\Rightarrow 16x^2+625+200\,x=25x^2+400+200x+25y^2+25z^2$

$\Rightarrow 9x^2+25y^2+25z^2-225=0$

5) If a variable point which moves such that 3PA=2PB if A= (-2,2,3). And B =(13,-3,13). Prove that P satisfies the equation $x^2+y^2+z^2+28x-12y+10z-247=0$.

Sol: Let $P(x,y,z)$ be any point on the locus. Given points are $A=(-2,2,3)$, $B(13,-3,13)$ and given geometric condition is $3PA=2PB \Rightarrow 9PA^2=4PB^2$

$$9\left[(x+2)^2+(y-2)^2+(z-3)^2\right]=4\left[(x-13)^2+(y+3)^2+(z-13)^2\right]$$

$$9[x^2+4+4x+y^2+4-4y+z^2+9-6z]=4[x^2+169-26x+y^2+9+6y+z^2+169-26z]$$

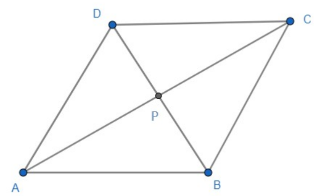
$$\Rightarrow 5x^2+5y^2+5z^2+140x-60y+50z-1235=0$$

$$\Rightarrow x^2+y^2+z^2+28x-12y+10z-247=0$$

EXERCISE : 11 (C)

II) 1) Three vertices of paroled gram ABCD are $A=(3,-1,2)$ $B(1,2,-4)$ $C(-1,1,2)$. Find the fourth vertex .

Sol: Let ABCD is a parallelogram.



Where $A=(3,-,1,2)$, $B=(1,2,-4)$,

$C=(-1,1,2)$ and $D(a,b,c)$

Mid point of AC = mid point of BD $\left(\frac{3-1}{1},\frac{-1+1}{2},\frac{2+2}{2}\right)=\left(\frac{1+9}{2},\frac{2+b}{2},\frac{-4+c}{2}\right)$

$\Rightarrow(1,0,2)=\left(\frac{a+1}{2},\frac{b+2}{2},\frac{c-4}{2}\right)\Rightarrow\frac{a+1}{2}=1,\ \frac{b+2}{2}=0,\ \frac{c+4}{2}=2$

$a=2-1, b=0-2,\ c=4+4$

$a=1,\ b=-2, c=8$

Coordinates of point $D=(1,-2,8)$

2) If the origin is the centroid of the triangle $P\theta R$ with Vertices $P=(2a,2,6)$, $Q(-4,3b-10)$, $R(8,14,2c)$ then find the values of a,b,c

Sol: Centroid of ΔPQR is $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$

$=\left(\frac{2a-4+8}{3},\frac{2+3b+14}{3},\frac{6-10+2c}{3}\right)$

Given origin is the centroid of ΔPQR

$\therefore\left(\frac{2a+4}{3},\frac{3b+16}{3},\frac{2c-4}{3}\right)=(0,0,0)\Rightarrow\ 2a+4=0,\ 3b+16=0,\ 2c-4=0$

Therefore $a=-2, b=\frac{-16}{3}, c=2$

3) If A and B be the points $(3,4,5)$ and $(-1,3-7)$ respectively, find the equation of the set of points P such that $PA^2-PB^3=K^2$, where K is constant.

Sol: Let the point P be (x,y,z) $A=(3,4,5)$ and $B=(-1,3,-7)$.

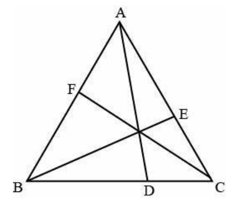
Given condition is $PA^2-PB^3=K^2$

$$\Rightarrow (x-3)^2+(y-4)^2+(z-5)^2-(x+1)^2-(y-3)^2-(z+7)^2=k^2$$

$$\Rightarrow -8x-2y-24z-9=k^2$$

$$\Rightarrow 8x+2y+24z+9+k^2=0$$

III) 1) Find the lengths of the medians of the triangle with vertices $A=(0,0,6)$ $B=(0,4,0)$ and $C=(6,0,0)$.



Sol: Let the points D, E and F are the mid points of BC, AC and AB respectively.

So $\overline{AD},\overline{BE},\overline{CF}$ will be the medians of the triangle.

The point D= Mid point of $BC=\left(\frac{0+6}{2},\frac{4+0}{2},\frac{0+0}{2}\right)$ = $(3,2,0)$

The point E= Mid point of $CA=\left(\frac{6+0}{2},\frac{0+0}{2},\frac{0+0}{2}\right)$ = $(3,0,3)$

The point F = Mid point of $AB=\frac{0+1}{2},\frac{0+4}{2},\frac{6+0}{2}$ = $(0,2,3)$

$$AD=\sqrt{(3-0)^2+(0-4)^2+(3-0)^2}\quad=\sqrt{9+16+9}\quad=\sqrt{49}=7$$

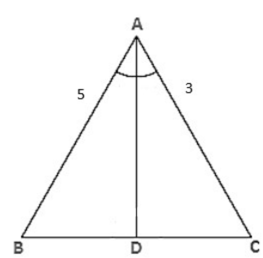
$$BE=\sqrt{(3-0)^2+(0-4)^2(3-0)^2}\quad=\sqrt{9+16+9}\quad=\sqrt{34}$$

$$CF=\sqrt{(0-6)^2+(2-0)^2+(3-0)^2}\quad=\sqrt{36+4+9}\quad=\sqrt{49}=7$$

2. $A(5,4,6)$, $B(1,-1,3)$, $C(4,3,2)$ are three points. Find the co-ordinates of the point in which the bisector of $\underline{|BAC}$ meets the side $\overline{BC}$.

Sol: We know that the bisector of $\underline{|BAC}$ divides $\overline{BC}$ in the ratio $AB:AC$.

Given $A=(5,4,6)$, $B=(1,-,1,3)$, $C=(4,3,2)$



$$AB=\sqrt{(5-1)^2+(4+1)^2+(6-3)^2}\quad=\sqrt{50}\quad=5\sqrt{2}$$

$$AC=\sqrt{(5-4)^2+(4-3)^2+(6-2)^2}\quad=\sqrt{18}\quad=3\sqrt{2}$$

$$AB:AC=5\sqrt{2}:3\sqrt{2}=5:3$$

If the bisector of $\underline{|BAC}$ meets the side $\overline{BC}$ at D, Then D divides $\overline{BC}$ in the ratio 5:3

$$\therefore D = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)$$

$$= \left(\frac{(5)(4)+(3)(1)}{5+3}, \frac{(5)(3)+(3)(-1)}{5+3}, \frac{(5)(2)+(3)(3)}{5+3} \right)$$

$$= \left(\frac{20+3}{8}, \frac{15-3}{8}, \frac{10+9}{8} \right) = \left(\frac{23}{8}, \frac{3}{2}, \frac{19}{8} \right)$$

3) Show that the points $O(0,0,0)$, $A(2,-3,3)$, $B(-2,3,-3)$ are collinear. Find the ratio in which each point divider the segment joining other two.

Sol: Given points are $O=(0,0,0)$, $A=(2,-3,3)$ and $B=(-2,3-3)$.

Now $OA = \sqrt{4+9+9} \;\; = \sqrt{22}$

$$AB = \sqrt{(2+2)^2 + (3+3)^2 + (-3-3)^2} \quad = \sqrt{78} \;\; = 2\sqrt{22}$$

$$BO = \sqrt{(0+2)^2 + (0-3)^2 + (0+3)^2} = \sqrt{22}$$

$$OA + BO = \sqrt{22} + \sqrt{22} = 2\sqrt{22}$$

$$AB = 2\sqrt{22}$$

$\therefore O, A, B$ are collinear points

The Ratio in which $P(x,y,z)$ divides $Q(x,y,z)$ and $R(x_2,y_2,z_2)$ is $\frac{x_1 - x}{x - x_2}$ (or) $\frac{y_1 - y}{y - y_2}$ (or) $\frac{z_1 - z}{z - z_2}$

Now $\frac{OA}{AB} = \frac{0-2}{2+2} = \frac{-2}{4} = \frac{-1}{2}$

$$\frac{AB}{BO} = \frac{2+2}{-2-0} = \frac{4}{-2} = \frac{-2}{1}$$

and $\frac{OB}{OA} = \frac{2-0}{0+2} = \frac{2}{2} = \frac{1}{1}$

MULTIPLE CHOICE QUESTIONS AND ANSWERS

1) **On $Y-axis$, $x\ and\ z$ - co-ordinates of every point are zero so the co-ordinates of the perpendicular are $(0,7,0)$.**
2) **The length of the perpendicular drawn from the point $P(a,b,c)$ on the co-ordinate axes are** $\sqrt{b^2+c^2},\sqrt{b^2+c^2},\sqrt{a^2+b^2}$ so. Required length $\sqrt{5^2+12^2}=13$.
3) **If a parallelepiped is formed by planes drawn through the points (x,y,z) and (x_2,y_2,z_2) paroled to the co-ordinate planes , then the length of the diagonal of the parallelepiped is** $\sqrt{(x_2-x_1)^2+(y_2-y_1)+2+(z_2-z_1)^2}$. Than length of the diagonal = $\sqrt{(4-1)^2+(5-2)^2+(z_2-z_1)^2}=\sqrt{3}$
4) **The y- Co- ordinate of every point on** ZX -plane is zero the locus of the point (x,y,z) for which $y=0$ is XZ –plane.
5) $PQ=\sqrt{41}\ \Rightarrow\sqrt{(0-3)^2+(a-0)^2+(3-7)^2}=\sqrt{41}\ \ 9+a^2+16=16\Rightarrow a=\pm 4$.
6) YZ – plane divides in the ratio $=\frac{-x_1}{x_2}=\frac{-2}{3}$.
7) $A=(1,1,1)$, $B(4,1,1)$ $C=(4,5,1)$, $D=(1,5,1)$
   $AB=\sqrt{9+0+0}=3$, $BC=\sqrt{0+16+0}=4$, $CD=\sqrt{9+0+0}=3$, $DA=\sqrt{0+16+0}=4$,
   $AC=\sqrt{9+16+0}=5$, $BD=\sqrt{9+16+1}=5$
   $AB=CD, BC=AD, AC=BD$
   $\therefore$ ABCD is a rectangle
8) X-axis
9) XZ- plane divides in the ratio = $\frac{-y_1}{y_2}=\frac{-3}{7}\ \Rightarrow$ ratio $=-3:7$
10) The point on the yz - plane x - co- ordination mint be zero $(0,4,5)$
11) Statement (I) : (A) if a = then $a^2-1=0$, and $a^2-3a+2=0$ the point is $(3,0,0)$ lies on x - axis.
   Statement (II) : (R) in a correct explanation for statement – I. Hence option 1 is correct .

ANSWERS

1) 2	**2) 1**	**3) 4**	**4) 3**	**5) 3**	**6) 3**	**7) 1**	**8) 1**
9) 3	**10)2**	**11)1**					

12. LIMITS AND DERIVATIVES

Exercise – 12(a)

I.

1. $\lim_{x\to 3}(x+3)$

Sol.: $\lim_{x\to 3}(x+3)$ **= 3 + 3 = 6**

2. $\lim_{x\to \pi}\left(x-\frac{22}{7}\right)$

Sol.: $\lim_{x\to \pi}\left(x-\frac{22}{7}\right)=\pi-\frac{22}{7}$

3. $\lim_{r\to 1}\pi r^2$

Sol.: $\lim_{r\to 1}\pi r^2=\pi(1)^2=\pi$

4. $\lim_{x\to 4}\frac{4x+3}{x-2}$

Sol.: $\lim_{x\to 4}\frac{4x+3}{x-2}=\frac{4(4)+3}{4-2}=\frac{16+3}{2}=\frac{19}{2}$

5. $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1}$

Sol.: $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1}=\frac{(-1)^{10}+(-1)^5+1}{-1-1}=\frac{1-1+1}{-2}=\frac{-1}{2}$

6. $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$

Sol.: $\lim_{x\to 0}\frac{(x+1)^5-1}{x}=\lim_{x\to 0}\frac{(x+1)^5-(1)^5}{(x+1)-1}$

$=\lim_{(x+1)\to 0}\frac{(x+1)^5-(1)^5}{(x+1)-1}$ **= 5(1)^4 = 5**

7. $\lim_{x\to 2}\frac{3x^2-x-10}{x^2-4}$ $\left(\frac{0}{0}\text{ form}\right)$

Sol.: $\lim_{x\to 2}\frac{3x^2-x-10}{x^2-4} = \lim_{x\to 2}\frac{3x^2-6x+5x-10}{(x+2)(x-2)} = \lim_{x\to 2}\frac{3x(x-2)+5(x-2)}{(x+2)(x-2)}$

$= \lim_{x\to 2}\frac{(3x+5)(x-2)}{(x+2)(x-2)} = \lim_{x\to 2}\frac{3x+5}{x+2} = \frac{6+5}{2+2} = \frac{11}{4}$

8. $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$ $\left(\frac{0}{0}\text{ form}\right)$

Sol.: $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3} = \lim_{x\to 3}\frac{(x^2)^2-(9)^2}{2x^2 6x+x-3}$

$= \lim_{x\to 3}\frac{(x^2-9)(x^2+9)}{2x(x-3)+1(x-3)} = \lim_{x\to 3}\frac{(x+3)(x^2+9)}{(2x+1)} = \frac{6(9+9)}{6+1} = \frac{108}{7}$

9. $\lim_{x\to 0}\frac{ax+b}{cx+1}$

Sol.: $\lim_{x\to 0}\frac{ax+b}{cx+1} = \frac{a(0)+b}{c(0)+1} = \frac{b}{1}$ = **b**

10. $\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$

Sol.: $\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1} = \lim_{z\to 1}\frac{(z)^{\frac{1}{6}}-1}{z-1} \times \frac{z-1}{(z)^{\frac{1}{6}}-1} = \frac{\frac{1}{3}(1)^{\frac{1}{3}-1}}{\frac{1}{6}(1)^{\frac{1}{6}-1}} = \frac{1}{3}\times\frac{6}{1}$ = **2**

11. $\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}, a+b+c\neq 0$

Sol.: $\lim_{x\to 1}\left(\frac{ax^2+bx+c}{cx^2+bx+a}\right) = \frac{a(1)^2+b(1)+c}{c(1)^2+b(1)+a} = \frac{a+b+c}{c+b+a}$ **=1**

12. $\lim\limits_{x\to -2}\dfrac{\frac{1}{x}+\frac{1}{2}}{x+2}$

Sol.: $\lim\limits_{x\to -2}\dfrac{\frac{1}{x}+\frac{1}{2}}{x+2} = \lim\limits_{x\to -2}\left(\frac{2+x}{2x}\right)\left(\frac{1}{x+2}\right) = \lim\limits_{x\to -2}\left(\frac{1}{2x}\right) = \frac{1}{-4}$

13. $\lim\limits_{x\to 0}\dfrac{\sin ax}{bx}$

Sol.: $\lim\limits_{x\to 0}\dfrac{\sin ax}{bx} = \lim\limits_{x\to 0}\dfrac{a.\sin ax}{b\,ax}$ **(multiply and divide with a)**

$$= \frac{a}{b}\lim_{ax\to 0}\frac{\sin ax}{ax} = \frac{a}{b}(1) = \frac{a}{b}$$

14. $\lim\limits_{x\to 0}\dfrac{\sin ax}{\sin bx}$**, a,b $\neq$ 0**

Sol.: $\lim\limits_{x\to 0}\dfrac{\sin ax}{\sin bx} = \lim\limits_{x\to 0}\left(\dfrac{\sin ax}{ax}.\dfrac{bx}{\sin bx}.\dfrac{ax}{bx}\right) = \dfrac{a}{b}\left(\lim\limits_{ax\to 0}\dfrac{\sin ax}{ax}\right)\left(\lim\limits_{bx\to 0}\dfrac{bx}{\sin bx}\right) = \dfrac{a}{b}(1)(1) = \dfrac{a}{b}$

15. $\lim\limits_{x\to \pi}\dfrac{\sin(\pi-x)}{\pi(\pi-x)}$

Sol.: $\lim\limits_{x\to \pi}\dfrac{\sin(\pi-x)}{\pi(\pi-x)} = \lim\limits_{x\to \pi}\dfrac{1}{\pi}\dfrac{\sin(\pi-x)}{(\pi-x)}$ **(As $x\to\pi \Rightarrow (\pi-x)\to 0$)**

$$= \frac{1}{\pi}\lim_{(\pi-x)\to 0}\frac{\sin(\pi-x)}{(\pi-x)} = \frac{1}{\pi}(1) = \frac{1}{\pi}$$

16. $\lim\limits_{x\to 0}\dfrac{\cos x}{\pi-x}$

Sol.: $\lim\limits_{x\to 0}\dfrac{\cos x}{\pi-x} = \dfrac{\cos 0}{\pi-0} = \dfrac{1}{\pi}$

17. $\lim\limits_{x\to 0}\dfrac{ax+x\cos x}{b\sin x}$

Sol.: $\lim\limits_{x\to 0}\dfrac{ax+x\cos x}{b\sin x} = \lim\limits_{x\to 0}\dfrac{(a+\cos x)}{b}\dfrac{x}{\sin x} = \lim\limits_{x\to 0}\dfrac{x}{\sin x}.\lim\limits_{x\to 0}\dfrac{(a+\cos x)}{b} = (1)\dfrac{a+cos}{b} = \dfrac{a+1}{b}$

18. $\lim\limits_{x\to 0} x\sec x$

Sol.: $\lim\limits_{x\to 0} x\sec x = \lim\limits_{x\to 0}\dfrac{x}{\cos x} = \dfrac{0}{1} = \mathbf{0}$

19. $\lim\limits_{x\to 0}\left(\dfrac{\sin ax+bx}{ax+\sin bx}\right)$ **a,b, a + b $\neq$ 0**

Sol.: $\lim\limits_{x\to 0}\left(\dfrac{\sin ax+bx}{ax+\sin bx}\right) = \dfrac{\lim\limits_{x\to 0}\dfrac{\sin ax}{x}+\lim\limits_{x\to 0}\dfrac{bx}{x}}{\lim\limits_{x\to 0}\dfrac{ax}{x}+\lim\limits_{x\to 0}\dfrac{\sin bx}{x}} = \dfrac{\lim\limits_{x\to 0}\dfrac{a\sin ax}{ax}+b}{a+\lim\limits_{x\to 0}\dfrac{b\sin bx}{bx}} = \dfrac{a+b}{a+b} = \mathbf{1}$

20. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\tau an2x}{x-\dfrac{\pi}{2}}$

Sol.: Let $x-\dfrac{\pi}{2}=h$ **as** $x\to\frac{\pi}{2}\Rightarrow h\to 0$

$\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\tan 2x}{x-\dfrac{\pi}{2}} = \lim\limits_{h\to 0}\dfrac{tan2\left(\frac{\pi}{2}+h\right)}{h} = \lim\limits_{h\to 0}2\dfrac{\tan 2h}{2h} = 2\lim\limits_{2h\to 0}\dfrac{tan2h}{2h} = \mathbf{2\ (1)}$

21. $\lim\limits_{x\to 2^-}\dfrac{|x-2|}{x-2}=-1$

Sol.: **As** $x\to 2^-$ $\Rightarrow x<2\Rightarrow$ **x – 2 < 0 and** $|x-2|=-(x-2)$

$\therefore \lim\limits_{x\to 2^-}\dfrac{|x-2|}{x-2}=\lim\limits_{x\to 2^-}\dfrac{-(x-2)}{(x-2)}=-1$

22. **Compute** $\lim\limits_{x\to 2^+}([x]+x)$ **and** $\lim\limits_{x\to 2^-}([x]+x)$ **where [.] denotes integral part.**

Sol.: **(a)** As $x\to 2^+\Rightarrow 2\le x<3$ $\Rightarrow$ [x] = 2

$\lim\limits_{x\to 2^+}([x]+x) = \lim\limits_{x\to 2^+}[x] + \lim\limits_{x\to 2^+}x = 2+2 = 4$ and $\lim\limits_{x\to 2^-}([x]+x)$

(b) As $x\to 2^-\Rightarrow 1\le x<2$ $\Rightarrow$ [x] = 1

$\lim\limits_{x\to 2^-}([x]+x) = \lim\limits_{x\to 2^-}[x]+\lim\limits_{x\to 2^-}x = 1+2=3$

Note : $\lim\limits_{x\to 2}([x]+x)$ does not exist

23. **Compute** $\lim\limits_{x\to 0}\dfrac{a^x-1}{b^x-1}\left(a>0, b>0, b\neq 1\right)$

Sol.: $\lim\limits_{x\to 0}\dfrac{a^x-1}{b^x-1} = \lim\limits_{x\to 0}\dfrac{\frac{a^x-1}{x}}{\frac{b^x-1}{x}} = \dfrac{\lim\limits_{x\to 0}\frac{a^x-1}{x}}{\lim\limits_{x\to 0}\frac{b^x-1}{x}} = \dfrac{\log a}{\log b} = \log_b^a$

24. **Compute** $\lim\limits_{x\to 0}\dfrac{e^x-\sin x-1}{x}$

Sol.: $\lim\limits_{x\to 0}\dfrac{e^x-\sin x-1}{x} = \lim\limits_{x\to 0}\left(\dfrac{e^x-1}{x}-\dfrac{\sin x}{x}\right) = \lim\limits_{x\to 0}\dfrac{e^x-1}{x} - \lim\limits_{x\to 0}\dfrac{\sin x}{x}$ **= 1 – 1 = 0**

25. **Compute** $\lim\limits_{x\to 0}\dfrac{e^{3+x}-e^3}{x}$

Sol.: $\lim\limits_{x\to 0}\dfrac{e^{3+x}-e^3}{x} = \lim\limits_{x\to 0}\dfrac{e^3\left(e^x-1\right)}{x} = e^3\lim\limits_{x\to 0}\dfrac{\left(e^x-1\right)}{x} = e^3\left(1\right) = e^3$

26. **Compute** $\lim\limits_{x\to 0}\dfrac{e^{4x}-1}{x}$.

Sol. $\lim\limits_{x\to 0}\dfrac{e^{4x}-1}{x} = 4\lim\limits_{4x\to 0}\dfrac{e^{4x}-1}{4x} = 4(1) = 4.$

27. **Compute** $\lim\limits_{x\to 0}\dfrac{e^{2+x}-e^2}{x}$

Sol.: $\lim\limits_{x\to 0}\dfrac{e^{2+x}-e^2}{x} = \lim\limits_{x\to 0}\dfrac{e^2\left(e^x-1\right)}{x} = e^2\left(\lim\limits_{x\to 0}\dfrac{\left(e^x-1\right)}{x}\right) = e^2\,(1) = e^2$

28. **Compute** $\lim\limits_{x\to 5}\dfrac{e^x-e^5}{x-5}$.

Sol.: $\lim\limits_{x\to 5}\dfrac{e^x-e^5}{x-5} = \lim\limits_{x-5\to 0}\dfrac{e^5\left(e^{x-5}-1\right)}{x-5} = e^5\,(1) = e^5$

29. **Compute** $\lim\limits_{x\to 0}\dfrac{e^{\sin x}-1}{x}$

Sol.: $\underset{x\to 0}{Lim}\dfrac{e^{\sin x}-1}{x}$ = $\underset{x\to 0}{Lim}\left(\dfrac{e^{\sin x}-1}{\sin x}.\dfrac{\sin x}{x}\right)$ = $\underset{\sin x\to 0}{Lim}\left(\dfrac{e^{\sin x}-1}{\sin x}\right)\underset{x\to 0}{Lim}\left(\dfrac{\sin x}{x}\right)$ = (1)(1) = 1

30. **Compute** $\underset{x\to 3}{Lim}\dfrac{e^{x}-e^{3}}{x-3}$

Sol.: $\underset{x\to 3}{Lim}\dfrac{e^{x}-e^{3}}{x-3}$ = $\underset{x-3\to 0}{Lim}\dfrac{e^{3}\left(e^{x-3}-1\right)}{x-3}$ = e^3 (1) = e^3

31. **Compute** $\underset{x\to 0}{Lim}\dfrac{x\left(e^{x}-1\right)}{1-\cos x}$.

Sol.: $\underset{x\to 0}{Lim}\dfrac{x\left(e^{x}-1\right)}{1-\cos x}$ = $\underset{x\to 0}{Lim}\dfrac{\left(\dfrac{x\left(e^{x}-1\right)}{x^{2}}\right)}{\left(\dfrac{1-\cos x}{x^{2}}\right)}$ = $\dfrac{\left(\underset{x\to 0}{Lim}\dfrac{\left(e^{x}-1\right)}{x}\right)}{\left(\underset{\frac{x}{2}\to 0}{Lim}\dfrac{2\sin^{2}\frac{x}{2}}{4.\frac{x^{2}}{4}}\right)}$ = $\dfrac{1}{\left(\frac{1}{2}\right)(1)^{2}}=2$

32. **Compute** $\underset{x\to 0}{Lim}\dfrac{\log_{e}\left(1+2x\right)}{x}$.

Sol.: $\underset{x\to 0}{Lim}\dfrac{\log_{e}\left(1+2x\right)}{x}=\underset{x\to 0}{Lim}\dfrac{2.\log_{e}\left(1+2x\right)}{2x}$ =2. $\underset{x\to 0}{Lim}\dfrac{\log_{e}\left(1+2x\right)}{2x}$ =(2)(1)=2

33. **Compute** $\underset{x\to 0}{Lim}\dfrac{\log_{e}\left(1+x^{3}\right)}{\sin^{3}x}$

Sol.: $\underset{x\to 0}{Lim}\dfrac{\log_{e}\left(1+x^{3}\right)}{\sin^{3}x}=\underset{x\to 0}{Lim}\dfrac{\log_{e}\left(1+x^{3}\right)}{x^{3}}.\dfrac{x^{3}}{\sin^{3}x}$

= $\left(\underset{x^{3}\to 0}{Lim}\dfrac{\log\left(1+x^{3}\right)}{x^{3}}\right).\left(\underset{x\to 0}{Lim}\dfrac{x}{\sin x}\right)^{3}$ = (1).(1) = 1

II.

1. **Compute** $\lim_{x\to0}\left(\frac{\cos 2x-1}{\cos x-1}\right)$

Sol.: $\lim_{x\to0}\frac{\cos 2x-1}{\cos x-1} = \lim_{x\to0}\frac{1-\cos 2x}{1-\cos x} = \lim_{x\to0}\frac{2\sin^2 x}{2\sin^2\frac{x}{2}} = \lim_{x\to0}\left(\frac{\sin^2 x}{x^2}\cdot\frac{x^2}{\sin^2\frac{x}{2}}\right)$

$= \lim_{x\to0}\frac{\sin^2 x}{x^2}\lim_{x\to0}\left(\frac{4\frac{x^2}{4}}{\sin^2\frac{x}{2}}\right) = 4\lim_{x\to0}\left(\frac{\sin x}{x}\right)^2\cdot\lim_{\frac{x}{2}\to0}\left(\frac{\frac{x}{2}}{\sin\frac{x}{2}}\right)^2$ **= 4(1)^2 (1)^2 = 4**

2. **Compute** $\lim_{x\to0}(\operatorname{cosec} x-\cot x)$**.**

Sol.: $\lim_{x\to0}(\operatorname{cosec}x-\cot x)=\lim_{x\to0}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right) = \lim_{x\to0}\left(\frac{1-\cos x}{\sin x}\right)$

$= \lim_{x\to0}\frac{(2\sin^2\frac{x}{2})}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \lim_{x\to0}\left(\tan\frac{x}{2}\right) = \tan 0 =$ **0**

3. **Find** $\lim_{x\to0}$ **f(x) and** $\lim_{x\to1}$ **f(x) where** $f(x)=\begin{cases}2x+3 & , x\le0\\ 3(x+1) & , x>1\end{cases}$

Sol.: Given $f(x)=\begin{cases}2x+3 & , x\le0\\ 3(x+1) & , x>1\end{cases}$

At x = 0,

R.H.L = $\lim_{x\to0^+}f(x)=\lim_{h\to0}f(o+h)=\lim_{h\to0}3(0+h+1)$ **= 3 (0 + 0 + 1) = 3**

L.H. L = $\lim_{x\to0^-}f(x)=\lim_{h\to0}f(0-h)=\lim_{h\to0}2(0-h)+3$ **= 2(0- 0) + 3 = 0 + 3 = 3**

R.H.L = L.H.L = f(0) = 3

Here at x = 0 Limit exist.

4. **Find** $\lim_{x\to1}$ **f(x), where f(x) =** $\begin{cases}x^2-1, & x\le1\\ -x^2-1, & x>1\end{cases}$

Given f(x) = $\begin{cases}x^2-1, & x\le1\\ -x^2-1, & x>1\end{cases}$

At $x = 1$**,** **R.H. L** = $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} -(1+h)^2 - 1$

$$= -(1+0)^2 - 1 = -1 - 1 = -2$$

L.H.L = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} [(1-h)^2 - 1]$

$$= (1-0)^2 - 1 \quad = 1 - 1 = 0$$

$\therefore$ **RHL** $\neq$ **LHL**

Hence at $x = 1$ limit does not exists

5. **Evaluate** $\lim_{x \to 0} f(x)$**, where** $f(x) = \begin{cases} \frac{|x|}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$

Sol.: Given $f(x) = \begin{cases} \frac{|x|}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$

We know that $|x| = \begin{cases} x, & if \quad x \geq 0 \\ -x, & if \quad x < 0 \end{cases}$

At x = 0, RHL = $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} f(0+h) = \lim_{x \to 0^+} \frac{|0+h|}{0+h}$

$$= \lim_{x \to 0^+} \frac{h}{h} = 1$$

LHL $= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h)$

$$= \lim_{h \to 0} \frac{|0-h|}{(0-h)}$$

$$= \lim_{h \to 0} \frac{-(0-h)}{h} = -1$$

$\therefore$ **RHL** $\neq$ **LHL**

Hence limit does not exist at x = 0

6. **Find** $\lim_{x \to 0} d(x)$ **where** $f(x) = \begin{cases} \frac{x}{|x|} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$

Sol.: Given $f(x) = \begin{cases} \frac{x}{|x|} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$

At x = 0, RHL= $\lim_{x \to 0^+} f(x) = \lim_{h \to 0^+} f(0+h) = \lim_{h \to 0} \frac{0+h}{|0+h|} = \lim_{h \to 0^+} \frac{h}{h}$ **= 1**

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0^-} f(0-h) = \lim_{h \to 0} \frac{0-h}{|0-h|} = \lim_{h \to 0} \frac{-h}{h}$ **= - 1**

$\therefore$ **RHL $\neq$ LHL**

Hence at x = 0, limit does not Exist

7. Find $\lim_{x\to 5} f(x)$**, where f(x) =** $|x|-5$

Sol.: Given f(x) = $|x|-5$

We know that $|x| = \begin{cases} x, & if \quad x>0 \\ -x, & if \quad x<0 \end{cases}$

As $x \to 5 \Rightarrow |x| = x$

$$\lim_{x \to 5} f(x) = \lim_{x \to 5} (|x| + 5) = \lim_{x \to 5} (x-5) = 5-5 = 0$$

8. Let $a_1, a_2, \ldots\ldots a_n$ be fixed real numbers and define a function f(x) = $(x-a_1)(x-a_2) \ldots. (x-a_n)$ what is $\lim_{x \to a_1}$**? For same** $a \neq a_1, a_2 \ldots.. a_n$ **compute.** $\lim_{x \to a} f(x)$

Sol.: Given f(x) = $(x - a_1)(x - a_2) \ldots..(x-a_n)$

$$\lim_{x \to a_1} f(x) = \lim_{x \to a_1} (x - a1)(x - a2) \ldots\ldots (x - an)$$

$$= (a_1 - a_1)(a_1 - a_2) \ldots..(a_1-a_n) = 0(a_1 - a_2) \ldots..(a_1-a_n) = 0$$

Again $\lim_{x \to a} f(x) = \lim_{x \to a} (x - a_1)(x - a_2) \ldots..(x-a_n) = (a-a_1)(a-a_2) \ldots..(a-a_x)$

9. Compute $\lim_{x\to 3} \frac{x^4-81}{2x^2-5x-3}$

Sol.: $\lim_{x \to 3} \frac{x^4-81}{2x^2-5x-3} \left(\frac{0}{0} form\right) = \lim_{x\to 3} \frac{(x^2-9)(x^2+9)}{2x^2-6x+x-3} = \lim_{x\to 3} \frac{(x-3)(x+3)(x^2+9)}{(x-3)(2x+1)}$

$$= \lim_{x\to 3} \frac{(x+3)(x^2+9)}{(2x+1)} = \frac{(3+3)(9+9)}{(6+1)} = \frac{108}{7}$$

10. **If f(x) =** $-\sqrt{25-x^2}$ **then find** $\lim\limits_{x\to 1}\dfrac{f(x)-f(1)}{x-1}$

Sol.: Given $f(x)=-\sqrt{25-x^2}$ **, then f(1) =** $-\sqrt{25-1}=-\sqrt{24}$

$$\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1}\frac{-\sqrt{25-x^2}+\sqrt{24}}{x-1}=\lim_{x\to 1}\frac{(\sqrt{24}-\sqrt{25-x^2})(\sqrt{24}+\sqrt{25-x^2})}{(x-1)(\sqrt{24}+\sqrt{25-x^2})}$$

$$=\lim_{x\to 1}\frac{(24-25+x^2)}{(x-1)\left(\sqrt{24}+\sqrt{25-x^2}\right)}$$

$$=\lim_{x\to 1}\frac{(x^2-1)}{(x-1)\left(\sqrt{24}+\sqrt{25-x^2}\right)}$$

$$=\lim_{x\to 1}\frac{(x+1)}{\left(\sqrt{24}+\sqrt{25-x^2}\right)}=\frac{1+1}{\sqrt{24}+\sqrt{24}}=\frac{1}{\sqrt{24}}$$

11. **Compute** $\lim\limits_{x\to a}\dfrac{x\sin a-a\sin x}{x-a}$

Sol.: $\lim\limits_{x\to a}\dfrac{x\sin a-a\sin x}{x-a}=\lim\limits_{x\to a}\dfrac{x\sin a-a\sin a+a\sin a-a\sin x}{(x-a)}$

$$=\lim_{x\to a}\frac{(x-a)\sin a-a(\sin x-\sin a)}{(x-a)}$$

$$=\lim_{x\to a}\frac{(x-a)\sin a}{(x-a)}-a.\lim_{x\to a}\frac{\sin x-\sin a}{(x-a)}$$

$$=\sin a-a\lim_{x\to a}\frac{2\cos\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)}{2\left(\frac{x-a}{2}\right)}$$

$$=\sin a-a\lim_{x\to a}\cos\left(\frac{x+a}{2}\right).\lim_{\frac{(x-a)}{2}\to 0}\frac{\sin\left(\frac{x-a}{2}\right)}{\left(\frac{x-a}{2}\right)}$$

= sin a – a cos a (1) = sin a – a cos a

12. **Compute** $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$

Sol.: $\lim_{x\to 0}\frac{\sin\left(\pi\cos^2 x\right)}{x^2} = \lim_{x\to 0}\frac{\sin\left(\pi\left(1-\sin^2 x\right)\right)}{x^2}$

$= \lim_{x \to 0} \frac{\sin(\pi - \pi\sin^2 x)}{x^2} = \lim_{x \to 0} \frac{\sin(\pi \sin^2 x)}{\pi \sin^2 x} . \frac{\pi \sin^2 x}{x^2}$

$= \pi \lim_{x\to 0}\frac{\sin\left(\pi\sin^2 x\right)}{\left(\pi\sin^2 x\right)}.\lim_{x\to 0}\left(\frac{\sin^2 x}{x^2}\right) = \pi(1)^2(1)^2 = \pi$

13. **Compute** $\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$ **[$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]**

Sol.: **Given** $\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x} = \lim_{x\to 0}\frac{\left[(1+x)^{\frac{1}{3}}-(1-x)^{\frac{1}{3}}\right]\left[(1+x)^{\frac{2}{3}}+(1+x)^{\frac{1}{3}}(1+x)^{\frac{1}{3}}+(1-x)^{\frac{2}{3}}\right]}{x\left[(1+x)^{\frac{2}{3}}+(1+x)^{\frac{1}{3}}(1-x)^{\frac{1}{3}}+(1-x)^{\frac{2}{3}}\right]}$

$= \lim_{x\to 0}\frac{(1+x-1+x)}{x\left[(1+x)^{\frac{2}{3}}+(1+x)^{\frac{1}{3}}(1-x)^{\frac{2}{3}}\right]} = \frac{2}{1+(1)(1)+(1)} = \frac{2}{3}$

III.

1.Suppose $f(x)=\begin{cases} a+bx & , \quad x<1 \\ 4 & , \quad x=1 \\ b-ax & , \quad x>1 \end{cases}$ **and if** $\lim_{x\to 1} f(x) = f(1)$ **what are possible values of** a **andb.**

Sol.: We have $f(x)=\begin{cases} a+bx & , \quad x<1 \\ 4 & , \quad x=1 \\ b-ax & , \quad x>1 \end{cases}$

At x = 1, $\lim_{x\to 1} f(x) = f(1)$**, given f(1) = 4**

i.e RHL = LHL = f(1)

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = 4$

$\lim_{h \to 0} f(1+h) = f(1-h) = 4$

$\Rightarrow \lim_{h \to 0}[\boldsymbol{b} - \boldsymbol{a}(1+h)] = \lim_{h \to 0}[\boldsymbol{a} + \boldsymbol{b}(1-h)] = 4$

$\Rightarrow b-a(1+0) = a+b(1-0) = 4 \Rightarrow b-a = a+b = 4$

b –a = 4, a + b = 4 on solving we get b = 4, a = 0

2. **If** $f(x)=\begin{cases}|x|+1 & , \quad x<0 \\ 0 & , \quad x=0 \\ |x|-1 & , \quad x>0\end{cases}$

For what values of a does $\lim\limits_{x\to a} f(x)$ **exist ?**

Sol.: Given $f(x)\begin{cases}|x|+1 & , \quad x<0 \\ 0 & , \quad x=0 \\ |x|-1 & , \quad x>0\end{cases}$

We know that $|x|=\begin{cases}x, & if\ x>0 \\ -x, & if\ x<0\end{cases}$

At x = 0

R H L = $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{h\to 0} f(0+h) = \lim\limits_{h\to 0}(0+h-1)$

$= \lim\limits_{h\to 0}(h-1) = 0 - 1 = -1$

L H L = $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} f(0-h)$

$= \lim\limits_{h\to 0}(|0-h|+1) = \lim\limits_{h\to 0} -(0-h)+1 = \lim\limits_{h\to 0} h+1 = 1$

RHL $\neq$ LHL

At x = 0 limit does not exist

Hence $\lim\limits_{x\to a} f(x)$ exists for all $a \neq 0$

3. **If the function f(x) satisfies** $\lim\limits_{x\to 1} \frac{f(x)-f(2)}{x^2-1} = \pi$ **evaluate** $\lim\limits_{x\to 1} f(x)$

Sol.: Given $\lim\limits_{x\to 1} \dfrac{f(x)-f(2)}{x^2-1} = \pi$

We know that $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$

Now $\lim\limits_{x\to 1} \frac{f(x)-f(2)}{x^2-1} = \pi \quad \Rightarrow \dfrac{\lim\limits_{x\to 1}\left(f(x)-f(2)\right)}{\lim\limits_{x\to 1}\left(x^2-1\right)} = \pi$

$\Rightarrow \lim\limits_{x\to 1} f(x) - \lim\limits_{x\to 1} f(2) = \pi \lim\limits_{x\to 1}(x^2-1)$

$\Rightarrow \lim\limits_{x\to 1} f(x) - \lim\limits_{x\to 1} f(2) = \pi(1^2-1) \Rightarrow \lim\limits_{x\to 1} f(x) - f(2) = 0 \therefore \lim\limits_{x\to 1} f(x) = f(2)$

4. **If** $f(x)\begin{cases} mx^2+n & , \quad x<0 \\ nx+m & , \quad 0\le x\le 1 \\ nx^3+m & , \quad x>1 \end{cases}$ **for what integers m and n does both** $\underset{x\to 0}{Lim}\, f(x)$ **and** $\underset{x\to 1}{Lim}\, f(x)$ **exist ?**

Sol.: At x = 0, limit exists then

RHL = LHL

$\Rightarrow \underset{x\to 0^+}{L\text{im}}\, f(x) = \underset{x\to 0^-}{Lim}\, f(x)$

$\Rightarrow \underset{h\to 0}{L\text{im}}\, f(0+h) = \underset{h\to 0}{Lim}\, f(0-h) \Rightarrow \underset{h\to 0}{L\text{im}}\big(n(0+h)+m\big) = \underset{h\to 0}{Lim}\big(m(0-h)^2+n\big)$

$\Rightarrow n(0+0)+m = m(0-0)^2+n$

m = n ------(1)

Again at x = 1 limit exists

RHL = LHL

$\Rightarrow \underset{x\to 1^+}{Lim}\, f(x) = \underset{x\to 1^-}{Lim}\, f(x)$

$\Rightarrow \underset{h\to 0}{Lim}\, f(1+h) = \underset{h\to 0}{Lim}\, f(1-h) \Rightarrow \underset{h\to 0}{Lim}\, f\big(n(1+h)^3+m\big) = \underset{h\to 0}{Lim}\big(n(1-h)+m\big)$

$\Rightarrow n(1+0)^3+m = n(1-0)+m \quad \Rightarrow n+m = m+n$ ----(2)

Hence from (1) and (2) for lim f(x) to be existed we need m = n and $\underset{x\to 1}{Lim}\, f(x)$ exist for any integral value of m and n.

EXERCISE - 12(b)

I. 1. **Find the derivative of** x^2-2 **at** x **= 10**

Sol.: **Let f(** x **) =** x^2-2 **and** $f^1(x) = \underset{h\to 0}{Lim}\dfrac{f(x+h)-f(x)}{h}$

$$f^1(x) = \underset{h\to 0}{Lim}\frac{\left[(x+h)^2-2\right]-\left[x^2-2\right]}{h}$$

$$= \underset{h\to 0}{Lim}\frac{x^2+h^2+2hx-2-x^2+2}{h}$$

$$= \underset{h\to 0}{Lim}\frac{h^2+2hx}{h} = \underset{h\to 0}{Lim}(h+2x) = 2x$$

$\therefore$ At x = 10, $f^1(10) = 2(10) = 20$

2. Find the derivation of x at $x = 1$

Sol.: Let f(x) = x and $f^1(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$f^1(x) = \lim_{h\to 0} \frac{x+h-x}{h} = \lim_{h\to 0} \frac{h}{h} = 1$$

$\therefore$ At $x = 1$, $f^1(1) = 1$

3. Find the derivative of 99 x at $x = 100$

Sol.: Let f x) = 99 x and $f^1(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{99x+99h-99x}{h} = \lim_{h\to 0} \frac{99h}{h} = 99$$

$\therefore$ At x = 100, $f^1(100) = 99$

4. For some constants a and b find the derivative of

(i) $(x - a)(x - b)$ **(ii)** $(ax^2+b)^2$ **(iii)** $\frac{x-a}{x-b}$

Sol.: **(i)** Let f(x) = $(x - a)(x - b) = x^2 - bx - ax + ab = x^2 - (a+b)x + ab$

Differentiate w.r.t x

$$f^1(x) = 2x - (a+b)$$

(ii) Let f(x) = $(ax^2+b)^2 = a^2x^4 + b^2 + 2abx^2$

Differentiate w.r.t x

$$f^1(x) = a^2(4x^3) + 0 + 2ab(2x) = 4a^2x^3 + 4abx = 4ax(ax^2+b)$$

(iii) **Let** $f(x) = \frac{x-a}{x-b}$

$$\left[f^1(x) = \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{dg(x)}{dx}}{(g(x))^2} \right]$$

$$\therefore f^1(x) = \frac{(x-b)\frac{d}{dx}^{(x-a)} - (x-a)\frac{d}{dx}(x-b)}{(x+b)^2} = \frac{x-b-x+a}{(x-b)^2} = \frac{a-b}{(x-b)^2}$$

5. Find the derivative of $\frac{x^n - a^n}{x-a}$ for some constant a

Sol.: Let f(x) = $x^n - a^n$, $g(x) = x - a$

$$\left[f^{|}(x) = \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{dg(x)}{dx}}{(g(x))^2} \right]$$

$$\frac{d}{dx}\left(\frac{x^n - a^n}{x - a}\right) = \frac{(x-a)\frac{d}{dx}(x^n - a^n) - (x^n - a^n)\frac{d}{dx}(x-a)}{(x-a)^2}$$

$$= \frac{(x-a)nx^{x-1}(x^x - a^x)(1)}{(x-a)^2}$$

$$= \frac{xx^n - axx^{x-1}x^n + a^n}{(x-a)^2} = \frac{(x-1)x^n - anx^{x-1} + a^n}{(x-a)^2}$$

6. Find the derivative of

(i) $2x - \frac{3}{4}$ **(ii)** $(5x^3 + 3x - 1)(x - 1)$ **(iii)** $x^{-3}(5 + 3x)$ **(iv)** $x^5(3 - 6x^{-9})$

(v) $y = x^{-4}(3 - 4x^{-5})$ **vi)** $\frac{2}{x+1} - \frac{x^2}{3x-1}$

Sol.: (i) Let y = $2x$ - $\frac{3}{4}$

Differentiate w.r.t 'x', we get

$$\frac{dy}{dx} = 2\frac{dx}{dx} - \frac{d}{dx}\left(\frac{3}{4}\right) = 2(1) - 0 = 2$$

(ii) Let $y = (5x^3 + 3x - 1)(x - 1)$ differentiate w.r.t 'x' both side we get

$$\frac{dy}{dx} = (5x^3 + 3x - 1)\frac{d}{dx}(x - 1) + (x - 1)\frac{d}{dx}(5x^3 + 3x - 1)$$

$$= (5x^3 + 3x - 1)(1 - 0) + (x - 1)(15x^2 + 3)$$

$$= 5x^3 + 3x - 1 + 15x^3 + 3x - 15x^2 - 3 = 20x^3 - 15x^2 + 6x - 4$$

(iii) Let y = $x^{-3}(5 + 3x)$

Differentiate w.r.t 'x' on both sides we get

$$\frac{dy}{dx} = \frac{d}{dx}(5x^{-3} + 3x^{-2}) = 5(-3)x^{-3-1} + 3(-2)x^{-2-1} = -15x^{-4} - 6x^{-3}$$

iv) Let $y = x^5(3 - 6x^{-9})$ = $y = 3x^5 - 6x^{-4}$

Differentiate w.r.t 'x' on both sides we get $\frac{dy}{dx}$

$$\frac{dy}{dx}= 15x^4+24x^{-5}$$

v) Let $y=x^{-4}\left(3-4x^{-5}\right) = 3x^{-4}-4x^{-9}$

Differentiate w.r.t 'x' on both sides

$$\frac{dy}{dx}=3(-4)x^{-5}-4(-9)x^{-10}$$

vi) Let $y=\frac{2}{(x+1)}-\frac{x^2}{3x-1}$

Differentiate w.r.t 'x' on both sides

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{2}{x+1}\right)-\frac{d}{dx}\left(\frac{x^2}{3x-1}\right)=\frac{d}{dx}\left(\frac{2}{x+1}\right)-\frac{d}{dx}\left(\frac{x^2}{3x-1}\right)$$

$$=\frac{(x+1)\frac{d}{dx}2-2\frac{d(x+1)}{dx}}{(x+1)^2}=\frac{(3x-1)\frac{d}{dx}x^2-x^2\frac{d(3x-1)}{dx}}{(3x-1)^2}$$

$$=\frac{(x+1)(0)-2(1+0)}{(x+1)^2}-\frac{(3x-1)(2x)-x^2(3-0)}{(3x-1)^2}$$

$$=\frac{-2}{(x+1)^2}-\frac{\left(6x^2-2x-3x^2\right)}{(3x-1)^2}=\frac{-2}{(x+1)^2}-\frac{\left(3x^2-2x\right)}{(3x-1)^2}=\frac{-2}{(x+1)^2}-\frac{x(3x-2)}{(3x-1)^2}$$

7. Find the derivatives of the following functions

i) sin x cos x ii) sec x iii) 5 sec x + 4 cos x iv) cosec x

v) 3 cot x + 4 cosec x vi) 5 sin x – 6 cos x +7

Sol.: i) Let $y=\sin x.\cos x$

Differentiate w.r.t 'x' on both sides

$$\frac{dy}{dx}=\sin x\frac{d}{dx}\cos x+\cos x+\frac{d}{dx}\sin x =-\sin x^2+\cos^2 x= \cos^2 x-\sin^2 x= \cos 2x$$

ii) Let y = sec x

Differentiate w.r.t 'x' both sides

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{\cos x}\right)=\frac{\cos x\frac{d(1)}{dx}-1\frac{d\cos x}{dx}}{\cos^{\frac{2}{x}}}=\frac{(\cos x)(0)-1(-\sin x)}{\cos^2 x}=\frac{\sin x}{\cos^2 x}=\sec x \tan x$$

iii) Let $y=5\sec x+4\cos x$

Differentiating w.r.t 'x' we get

$$\frac{dy}{dx}=5\frac{d}{dx}\sec x+4\frac{d}{dx}\cos x = 5\sec x\tan x+4\left(-\sin x\right)= 5\sec x\tan x+4\sin x$$

iv) Let $y=Cosec\ x$

Differentiating w.r.t x we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{\sin x}\right) = \frac{\sin x\frac{d(1)}{dx}-1\frac{d}{dx}\sin x}{\left(\sin^2 x\right)}=\frac{-\cos x}{\sin x}.\frac{1}{\sin x}=-Cosec\ x.Cot\ x$$

v) Let $y=3Cotx+5Cosec\,x$

Differentiate w.r.t x on both sides we get

$$\frac{dy}{dx}=3\frac{d}{dx}Cot\,x+5\frac{d}{dx}Co\sec x = -3Co\sec^2 x-5Cosec\ x.Cot\,x$$

vi) Let $y=5\sin x-6\cos x+7$

Differentiate w.r.t 'x' on both side we get

$$\frac{dy}{dx}=5\frac{d}{dx}(\sin x)-6\frac{d}{dx}(\cos x)+\frac{d}{dx}(7)= 5\cos x+6\sin x$$

vii) Let $y=2\tan x-7\sec x$

Differentiate w.r.t 'x' on both sides

$$\frac{dy}{dx}=2\frac{d}{dx}\tan x-7\frac{d}{dx}\sec x = 2\sec^2 x-7\sec x.\tan x.$$

8. Find the derivative of $5\sin x+e^x\log x$

Sol.: $\frac{d}{dx}\left(5\sin x+e^x\log x\right) = 5\frac{d}{dx}(\sin x)+\frac{d}{dx}\left(e^x\log x\right)$

$$=5\cos x+e^x\frac{d}{dx}(\log x)+\log x\frac{d}{dx}\left(e^x\right) = 5\cos x+\frac{e^x}{x}+e^x\log x$$

9. Find the derivative of $5^x+\log x+x^3e^x$

Sol.: $\frac{d}{dx}\left(5^x+\log x+x^3e^x\right)=\frac{d}{dx}5^x+\frac{d}{dx}logx+\frac{d}{dx}x^3e^x$

$$= 5^x\log 5+\frac{1}{x}+e^xx^3+3x^2e^{3x}$$

10. If $f(x)=1+x+x^2+......+x^{100}$ **then find** $f^1(x)$

Sol.: Given $f(x)=1+x+x^2+.....+x^{100}$

$f^1(x)=0+1+2x+....+100.x^{99}$

$f^1(1) = 1 + 2 + 3 + \ldots + 100 = \dfrac{(100)(101)}{2} = 50\,(101) = 5050$

Note : sum of n natural numbers $\dfrac{n(n+1)}{2}$

11. **If** $f(x) = 2x^2 + 3x - 5$ **then prove that** $f^1(0) + 3f^1(-1) = 0$

Sol.: Given $f(x) = 2x^2 + 3x - 5$ and $f^1(x) = 4x + 3$

Now $f^1(0) + 3f^1(-1) = [4(0) + 3] + 3[-4 + 3] = 3 - 3 = 0$

II. **1) Find the derivative of the following functions**

i) $x^3 - 27$ ii) (x – 1) x – 2) iii) $\dfrac{1}{x^3}$ iv) $\dfrac{x+1}{x-1}$

Sol.: i) Let f(x) = $x^3 - 27$

First principle $f^1(x) = \underset{h\to 0}{Lt} \dfrac{f(x+h) - f(x)}{h}$

$$f^1(x) = \underset{h\to 0}{Lt} \frac{\left((x+h)^3 - 27\right) - \left(x^3 - 27\right)}{h}$$

$$= \underset{h\to 0}{Lim} \frac{x^3 + h^3 + 3x^2h + 3xh^2 - 27 - x^3 + 27}{h}$$

$$= \underset{h\to 0}{Lim} \frac{h^3 + 3x^2h + 3xh^2}{h} = \underset{h\to 0}{Lim}\left(h^2 + 3x^2 + 3xh\right) = 0 + 3x^2 + 0 = 3x^2$$

ii) Let $f(x) = (x-1)(x-2) = x^2 - 2x - x + 2 = x^2 - 3x + 2$

$$f^1(x) = \underset{h\to 0}{Lim} \frac{f(x+h) - f(x)}{h}$$

$$f^1(x) = \underset{h\to 0}{Lim} \frac{\left[(x+h)^2 - 3(x+h) + 2\right] - \left[x^2 - 3x + 2\right]}{h}$$

$$= \underset{h\to 0}{Lim} \frac{x^2 + h^2 + 2xh - 3x - 3h + 2 - x^2 + 3x - 2}{h}$$

$$= \underset{h\to 0}{Lim} \frac{h^2 + 2hx - 3h}{h} = \underset{h\to 0}{Lim}\left(h + 2x - 3\right) = 2x - 3$$

iii) Let $f(x) = \dfrac{1}{x^3}$

$$f^1(x) = \underset{h\to 0}{Lim} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h} = \lim_{h\to 0} \frac{x^3-(x+h)^3}{h(x+h)^3 x^3}$$

$$= \lim_{h\to 0} \frac{x^3-x^3-h^3-3x^2h-3xh^2}{h(x+h)^3 x^3} = \lim_{h\to 0} \frac{-h^3-3x^2h-3h^2}{h(x+h)^3 x^3}$$

$$= \lim_{h\to 0} \frac{-h\left(h^2+3x^2+3xh\right)}{h(x+h)^3 x^3} = \lim_{h\to 0} \frac{-\left(h^2+3x^2+3xh\right)}{(x+h)^3 x^3}$$

$$= = \frac{-\left(0+3x^2+0\right)}{(x+0)^3 x^3} = \frac{-3x^2}{x^6} = = \frac{-3}{x^4}$$

iv) Let f(x) = $\frac{x+1}{x-1}$

$$f^1(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0} \frac{\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}}{h} = \lim_{h\to 0} \frac{-2h}{h(x+h-1)(x-1)} = \frac{-2}{(x-1)^2}$$

2. For the function $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x + 1$ **prove that** $f^1(1) = 100 f^1(0)$

Sol.: Given $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x + 1$

$$f^1(x) = \frac{100.x^{99}}{100} + \frac{99.x^{98}}{99} + ... + \frac{2x}{2} + 1 + 0 = x^{99} + x^{98} + ... + x + 1$$

[f(x) = x^n, $f^1(x) = nx^{n-1}$]

Put x = 1 $\quad f^1(1) = 1^{99} + 1^{98} + ... + 1 + 1$

$= 1+1+......+1+1(100\ times) = 100$ ----(i)

Again putting x = 0 we get

$f^1(0) = 0+0+....+0+1 = 1$ ---------(ii)

From (i) and (ii) $f^1(1) = 100 f^1(0)$

3.Find the derivative of $x^n + a.x^{n-1} + a.x^{n-1} + a^2x^{n-2} + ... + a^{n-1}x + a^n$ **for some fixed real number a.**

Sol.: Let $f(x) = x^n + a.x^{n-1} + a.x^{n-1} + a^2x^{n-2} + ... + a^{n-1}x + a^n$

$$f^1(x) = nx^{n-1} + a(n-1)x^{n-2} + (n-2)a^2x^{n-3} + ... + a^{n-1}(1) + 0$$

$$f^1(x) = nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + ... + a^{n-1}$$

$$\because f(x) = ax^n \Rightarrow f^1(x) = an.x^{n-1}$$

4. Find the derivative of cos x from the first principle

Sol.: Let f(x) = cos x , $f(x+h) = \cos(x+h)$

$$f^1(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h} = \lim_{h\to 0}\frac{-2\sin\left(\frac{x+h+x}{2}\right).\sin\left(\frac{x+h-x}{2}\right)}{h}$$

$$= \lim_{h\to 0}\frac{-2\sin\left(x+\frac{h}{2}\right).\sin\left(\frac{h}{2}\right)}{2\left(\frac{h}{2}\right)} = -\lim_{h\to 0}\sin\left(x+\frac{h}{2}\right).\lim_{\frac{h}{2}\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$$

$$= -\sin(x-0).(1) = \text{-sin x} = \frac{d}{dx}(\cos x)$$

5. Find the derivatives of the following functions from the first principal.

i) $\sqrt{x+1}$ **ii) sin 2x** **iii) cosax** **iv) sec 3x** **v) x sinx** **vi)** $\cos^2 x$

Sol.: i) Let $f(x) = \sqrt{x+1}$, $f(x+h) = \sqrt{x+h+1}$

$$\frac{d}{dx}\sqrt{x+1} = \lim_{h\to 0}\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}$$

$$f^1(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\left(\sqrt{x+h+1}-\sqrt{x+1}\right)\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$

$$= \lim_{h\to 0}\frac{(x+h+1-x-1)}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)} = = \lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$

$$= \lim_{h\to 0}\frac{1}{\sqrt{x+h+1}+\sqrt{x+1}} = = \frac{1}{\sqrt{x+0+1}+\sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}$$

ii) Let $f(x) = \sin 2x$, $f(x+h) = \sin(2x+2h)$

$$f^1(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(\sin 2x) = \underset{h\to 0}{Lim}\frac{\sin(2x+2h)-\sin 2x}{h}$$

$$= \underset{h\to 0}{Lim}\frac{2.\cos\left(\frac{2x+2h+2x}{h}\right)\sin\left(\frac{2x+2h-2x}{2}\right)}{h}$$

$$= \underset{h\to 0}{Lim}\frac{2.\cos(2x+h)\sinh}{h} = \underset{h\to 0}{Lim}\,2\cos(2x+h).\ \underset{h\to 0}{Lim}\frac{\sinh}{h}$$

$$= 2\cos(2x+0) \text{ --- } (1) \quad = 2\text{Cos } 2x$$

iii) Let $f(x) = \cos ax$, $f(x+h) = \cos(ax+ah)$

$$f^1(x) = \underset{h\to 0}{Lim}\frac{f(x+h)-f(x)}{h} = \underset{h\to 0}{Lim}\frac{\cos(ax+ah)-\cos ax}{h}$$

$$= \underset{h\to 0}{Lim}\frac{-2\sin\left(\frac{ax+ah+ax}{2}\right).\sin\left(\frac{ax+ah-ax}{2}\right)}{h}$$

$$= \underset{h\to 0}{Lim}\frac{-2\sin\left(ax+\frac{ah}{2}\right)\sin\left(\frac{ah}{2}\right)}{h}$$

$$= \underset{h\to 0}{Lim}-2.\sin\left(ax+\frac{ah}{2}\right).\underset{\frac{ah}{2}\to 0}{Lim}\frac{\left(\frac{9}{2}\right)\sin\frac{ah}{2}}{\frac{ah}{2}}$$

$$= -2\sin(ax+0)\left(\frac{9}{2}\right).(1) = -\ a\sin ax$$

iv) Let $f(x) = \sec 3x$, $f(x+h) = se(3x+3h)$,

$$f^1(x) = \underset{h\to 0}{Lim}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d(\sec 3x)}{dx} = \underset{h\to 0}{Lim}\frac{\sec(3x+3h)-\sec 3x}{h}$$

$$= \underset{h\to 0}{Lim}\frac{1}{h}\left[\frac{1}{\cos(3x+3h)}-\frac{1}{\cos 3x}\right] = \underset{h\to 0}{Lim}\frac{\cos 3x-\cos)(3x+3h)}{h\cos(3x+3h)\cos 3x}$$

$$= \underset{h\to 0}{Lim}\frac{-2\sin\left(\frac{3x+3h+3x}{2}\right)\sin(3x-3x-3h)}{h\cos(3x+3h).\cos 3x}$$

$$= \lim_{h\to 0}\frac{+2\sin\left(3x+3\frac{h}{2}\right)}{\cos(3x+3h)\cos 3x}\left(\frac{3}{2}\right)\lim_{3h\to 0}\frac{\sin\frac{3h}{2}}{\frac{3h}{2}} = \frac{(2)\sin(3x+0)}{\cos(3x+0)\cos 3x}\left(\frac{3}{2}\right)(1)$$

$$= 3.\frac{\sin 3x}{\cos 3x}\frac{1}{\cos 3x} = 3.\ \text{Tan}\ 3x\ \text{Sec}\ 3x = 3\ \text{Sec}\ 3x\ \text{Tan}\ 3x$$

v) Let $f(x) = x\sin x, f(x+h) = (x+h)\sin(x+h)$

$$f^1(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(x\sin x) = \lim_{h\to 0}\frac{(x+h)\sin(x+h)-x\sin x}{h}$$

$$= \lim_{h\to 0}\frac{x\sin(x+h)+h\sin(x+h)-x\sin x}{h}$$

$$= x\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}+\lim_{h\to 0}\sin(x+h)$$

$$= \lim_{h\to 0}\frac{2\cos\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{2}+\sin(x+0)$$

$$= \lim_{h\to 0}2\cos\left(x+\frac{h}{2}\right)\frac{\sin\left(\frac{h}{2}\right)}{2.\frac{h}{2}}+\sin x$$

$$= x\cos(x+0)+\sin x = x\cos x+\sin x$$

vi) Let $f(x) = \cos^2 x, f(x+4) = \cos^2(x+h)$

$$f^1(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d(\cos^2 x)}{dx} = \lim_{h\to 0}\frac{\cos^2 x+h-\cos^2 x}{h} \qquad \begin{pmatrix} a^2-b^2 \\ =(a+b)(a-b)\end{pmatrix}$$

$$= \lim_{h\to 0}\frac{(\cos(x+h)+\cos x)(\cos(x+h)-\cos x)}{h}$$

$$= \lim_{h\to 0}(\cos(x+h)+\cos x).\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$$

$$= (\cos(x+0)+\cos x).\lim_{h\to 0}\frac{-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h}$$

$$= -2\cos x.\sin\left(x+0\right).\left(1\right)$$

$$= \text{-2Cos } x \ \lim_{h\to 0}\left[2\sin\left(x+\frac{h}{2}\right).\frac{\sin\frac{h}{2}}{2.\frac{h}{2}}\right] = -2S\text{in}\, x\, Cosx = -Sin2x$$

Exercise – 12(C)

I. **Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non – zero constants and m and n are integers)**

1) $(x + a)$

Sol.: Let y = $x + a$

Differentiating w. r. t. ' x '

$$\frac{dy}{dx} = \frac{d}{dx}\left(x+a\right) = \frac{dx}{dx} + \frac{da}{dx} = 1 + 0 = 1$$

2. $\left(px+q\right)\left(\frac{r}{x}+s\right)$

Sol.: Let $y = \left(px+q\right)\left(\frac{r}{x}+s\right)$

Differentiate w. r. t ' x ' on both sides

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(px+q\right)\left(\frac{r}{x}+s\right)\right] \quad \text{(By product rule)}$$

$$= \left(px+q\right)\frac{d}{dx}\left(\frac{r}{x}+s\right) + \left(\frac{r}{x}+s\right)\frac{d}{dx}px+q$$

$$= \left(px+q\right)\left(\frac{-r}{x^2}\right) + \left(\frac{r}{x}+s\right)P$$

$$= \frac{-\Pr}{x} - \frac{qr}{x^2} + \frac{pr}{x} + sp \;=\; Ps - \frac{qr}{x^2}$$

3. $\left(ax+b\right)\left(cx+d\right)^2$

Sol.: Let $y = \left(ax+b\right)\left(cx+d\right)^2$

Differentiate w.r.t 'x' on both sides

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(ax+b\right)\left(cx+d\right)^2\right] \quad \text{(by product rule)}$$

$$= \left(ax+b\right)\frac{d}{dx}\left(c^2x^2+d^2+2cdx\right) + \left(cx+d\right)^2\left(\frac{dax}{dx}+\frac{d}{dx}\right)\left(b\right)$$

$= (ax+b)(2c^2x+0+2cd)+(cx+d)^2(a) = (ax+b)2c(cx+d)+a(cx+d)^2$

$= (cx+d)[2acx+2bc+acx+ad] = (cx+d)[3acx+2bc+ad]$

4. $\dfrac{ax+b}{cx+d}$

Sol.: Let $y=\dfrac{ax+b}{cx+d}$

Differentiating w. r. t 'x"

$\dfrac{dy}{dx}=\dfrac{d}{dx}\left(\dfrac{ax+b}{cx+d}\right)$ (by quotient rule)

$= \dfrac{(cx+d)\dfrac{d}{dx}(ax+b)-(ax+b)\dfrac{d}{dx}(cx+d)}{(cx+d)^2} = \dfrac{(cx+d)(a(1)+0)-(ax+b)(c(1)+0)}{(cx+d)^2}$

$= \dfrac{(cx+d)(a)-(ax+b)(c)}{(cx+d)^2} = \dfrac{acx+da-axc-bc}{(cx+d)^2} = \dfrac{ad-bc}{(cx+d)^2}$

5) $\dfrac{1+\dfrac{1}{x}}{1-\dfrac{1}{x}}$

Sol.: Let $y=\dfrac{1+\dfrac{1}{x}}{1-\dfrac{1}{x}}=\dfrac{x+1}{x-1}$

Differentiating w. r. t 'x'

$\dfrac{dy}{dx}=\dfrac{d}{dx}\left(\dfrac{x+1}{x-1}\right) = \dfrac{(x-1)\dfrac{d}{dx}(x+1)-(x+1)\dfrac{d}{dx}(x-1)}{(x-1)^2}$ (By Quotient rule)

$= \dfrac{(x-1)(1+0)-(x+1)(1-0)}{(x-1)^2} = \dfrac{x-1-x-1}{(x-1)^2} = \dfrac{-2}{(x-1)^2}$

6. $\dfrac{1}{ax^2+bx+c}$

Sol.: Let $y=\dfrac{1}{ax^2+bx+c}$

Differentiating y w. r. t 'x'

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{ax^2+bx+c}\right)\text{(by Quotient rule)}$$

$$=\frac{\left(ax^2+bx+c\right)\frac{d(1)}{dx}-1.\frac{d}{dx}(ax^2+bx+c)}{\left(ax^2+bx+c\right)^2}$$

$$=\frac{\left(ax^2+bx+c\right)(0)-1(2xa+b+0)}{\left(ax^2+bx+c\right)^2} \quad =\frac{-\left(2ax+b\right)}{\left(ax^2+bx+c\right)^2}$$

7. $\dfrac{ax+b}{px^2+qx+r}$

Sol.: Let $y=\dfrac{ax+b}{px^2+qx+r}$

Differentiate w. r. t. ‘ x ’ on both sides

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{ax+b}{px^2+qx+r}\right)=\frac{\left(px^2+qx+r\right)\frac{d}{dx}(ax+b)-(ax+b)\frac{d}{dx}\left(px^2+qx+r\right)}{\left(px^2+qx+r\right)^2}$$

$$=\frac{\left(px^2+qx+r\right)(a)-(ax+b)(2px+q)}{\left(px^2+qx+r\right)^2}$$

$$=\frac{apx^2+aqx+ar-2apx^2-aqx-2bpx-bq}{\left(px^2+qx+r\right)^2} \quad =\frac{-apx^2-2bpx+ra-bq}{\left(px^2+qx+r\right)^2}$$

8. $\dfrac{px^2+qx+r}{ax+b}$

Sol.: Let $y=\dfrac{px^2+qx+r}{ax+b}$

Differentiate w. r. t ‘ x ’ on both sides

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{px^2+qx+r}{ax+b}\right)=\frac{(ax+b)\frac{d}{dx}\left(px^2+qx+r\right)-\left(px^2+qx+r\right)\frac{d}{dx}(ax+b)}{\left(ax+b\right)^2}$$

$$=\frac{\left(ax+b\right)\left(2px+q\right)-\left(px^2+qx+r\right)\left(a\right)}{\left(ax+b\right)^2}$$

$$=\frac{apx^2+2bpx+bq-ar}{\left(ax+b\right)^2}=\frac{apx^2+2bpx+bq-ar}{\left(ax+b\right)^2}$$

9. $\frac{a}{x^4}-\frac{b}{x^2}+\cos x$

Sol.: Let $y=\frac{a}{x^4}-\frac{b}{x^2}+C\text{os}\,x \Rightarrow \quad y=ax^{-4}-bx^{-2}+\text{Cos}\,x$

Now $\frac{dy}{dx}=a\frac{d}{dx}x^{-4}-b\frac{d}{dx}x^{-2}+\frac{d}{dx}\text{Cos}\,x$

$$=a.(-4)x^{-4-1}-(-2)x^{-2-1}-S\text{in}\,x \quad =-4ax^{-5}+2x^{-3}-S\text{in}\,x=\frac{-4a}{x^5}+\frac{2}{x^3}-S\text{in}\,x$$

10. $4\sqrt{x}-2$

Sol.: Let $y=4\sqrt{x}-2$

Differentiating w. r. t 'x' we get

$$\frac{dy}{dx}=4\frac{d}{dx}\sqrt{x}-\frac{d}{dx}2 \quad =4\left(\frac{1}{2}x^{\frac{1}{2}-1}\right)-0 \quad =2x^{-\frac{1}{2}}=\frac{2}{\sqrt{x}}$$

Note: For the following problems we have the function of a function as y=f {g(x)}. Here f is a function of g and g in a function of x the derivation of such a functions are given as $\frac{dy}{dx}=f'\left(g\left(x\right)\right)g'\left(x\right)$

11) $\left(ax+b\right)^n$

Sol.: Let $y=\left(ax+b\right)^n$

Differentiating w. r. t 'x' we get

$$\frac{dy}{dx}=n\left(ax+b\right)^{n-1}\frac{d}{dx}\left(ax+b\right)=n(ax+b)^{n-1}(a) \quad =na(ax+b)^{n-1}$$

12) $\left(ax+b\right)^n(cx+d)^m$

Sol.: Let $y=\left(ax+b\right)^n(cx+d)^m$

Differentiating w. r. t 'x' we get

$$\frac{dy}{dx}=\left(ax+b\right)^n\frac{d}{dx}\left(cx+d\right)^m+(cx+d)^m\frac{d}{dx}(ax+b)^n$$

$$=(ax+b)^n\,m(cx+d)^{m-1}(c)+(cx+d)^m\,n(ax+b)^{n-1}(a)$$

$$=mc(ax+b)^n(cx+d)^{m-1}+na(cx+d)^m(ax+b)^{n-1}$$

$$=(ax+b)^{n-1}(cx+d)^{m-1}\left[\left(ax+b\right)mc+(cx+d)na\right]$$

13) $S\text{in}(x+a)$

Sol.: Let $y = S\,\text{in}(x+a)$

Differentiating w. r. t 'x' on both sides

$$\frac{dy}{dx} = \frac{d}{dx} S\,\text{in}(x+a) \quad = C\,\text{os}(x+a).\frac{d}{dx}(x+a) = C\,\text{os}(x+a)(1+0) = C\,\text{os}(x+a)$$

14) Find the derivatives of the following functions:

$(i)\, C\,\text{ot}^n x$ $\quad (ii)\, Co\sec^4 x$ $\quad (iii)\, S\,\text{in}^m x\; C\,\text{os}^n x$ $\quad (iv)\, S\,\text{in}\, mx\, C\,\text{os}\, nx$

$(v)\, \log(Tan\, 5x)$ $\quad (vi)\, \log\left(\frac{x^2+x+2}{x^2-x+2}\right)$

$(vii)\, C\,\text{os}(\log x + e^{x)}$

Sol.: (i) Let $y = C\,\text{ot}^n x$

Differentiating w. r. t x on both sides

$$\frac{dy}{dx} = \frac{d}{dx} C\,\text{ot}^n x \; = n.C\,\text{ot}^{n-1} x.\frac{d}{dx} C\,\text{ot}\, x \; = nC\,\text{ot}^{n-1} x(-C\,\text{os}\,ec^2 x)$$

$$= -nC\text{ot}^{n-1}x\;.\text{Cos}\,ec^2 x$$

(ii) Let $y = Co\sec^4 x$

Differentiating w. r. t 'x' we get

$$\frac{dy}{dx} = \frac{d}{dx} Co\sec^4 x \quad = 4.Co\sec^{4-1} x.\frac{d}{dx} Co\sec x$$

$$= -4Co\sec^3 x\; Co\sec x\;.\; C\,\text{ot}\, x = -4Co\sec^4 x\;.\; C\,\text{ot}\, x$$

(iii) Let $y = S\text{in}^m\; x\text{Cos}^n\, x$

Differentiating w. r. t 'x' we get

$$\frac{dy}{dx} = S\text{in}^m x\, \frac{d}{dx} C\text{os}^n x + C\text{os}^n x\, \frac{d}{dx} S\text{in}^m x$$

$$= S\text{in}^m\; x\, n\, C\text{os}^{n-1} x.\frac{d}{dx} \text{Cos}\, x + C\text{os}^n\; x\, m.S\text{in}^{m-1} x\, \frac{d}{dx} \text{Sin}\, x$$

$$= -nS\text{in}^m\; x\, C\text{os}^{n-1}\; x.S\text{in}\, x + C\text{os}^n\, x\, m.\, S\text{in}^{m-1} x\, C\text{os}\, x$$

$$= S\text{in}^{m-1} x\; C\text{os}^{n-1} x\left[m\, C\text{os}^2 x - n\, S\text{in}^2 x\right]$$

(iv) Let $y = S\text{in}\, mx\;\; \text{Cos}\, nx$

Differentiating w. r. t 'x' we get

$$\frac{dy}{dx} = S\text{in}\, mx\, \frac{d}{dx} C\text{os}\, nx + \; C\text{os}\, nx\, \frac{d}{dx} S\text{in}\, mx$$

$$= -n\, S\,\text{in}\, mx\, C\text{os}\, x + m\, C\text{os}\, mx\;\, C\text{os}\, nx$$

(v) Let $y = \log(Tan\, 5x)$

Differentiating w. r. t 'x' we get

$$\frac{dy}{dx}=\frac{1}{Tan\,5x}\frac{d}{dx}Tan\,5x \quad =\frac{Sec^2\,5x}{Tan5x}\frac{d}{dx}(5x)$$

$$=\frac{5.Sec^2\;5x}{Tan\,5x} \quad =5.\frac{1}{Cos^2\,5x}.\frac{Cos\,5x}{Sin\,5x}$$

$$=\frac{10}{2\sin 5x\cos 5x} \quad =\frac{10}{S\,in\,10x} \quad =10Cosec10x$$

(vi) Let $y=\log\left(\frac{x^2+x+2}{x^2-x+2}\right)$

Differentiating w. r. t 'x' we get

$$\frac{dy}{dx}=\frac{d}{dx}\log\left(\frac{x^2+x+2}{x^2-x+2}\right)$$

$$=\frac{d}{dx}\left[\log\left(x^2+x+2\right)-\log\left(x^2-x+2\right)\right]$$

$$=\frac{(2x+1)}{\left(x^2+x+2\right)}-\frac{(2x-1)}{\left(x^2-x+2\right)}$$

$$=\frac{(2x+1)\left(x^2-x+2\right)-(2x-1)\left(x^2+x+2\right)}{\left(x^2+x+2\right)\left(x^2-x+2\right)}$$

$$=\frac{2x^3-2x^2+4x+x^2-x+2-2x^3-2x^2-4x+x^2+x+2)}{x^4+3x^2+4}$$

$$=\frac{-2x^2+4}{x^4+3x^2+4}$$

(vii) Let $y=C\,os(\log x+e^x)$

Differentiate w. r. t x on both sides

$$\frac{dy}{dx}=\frac{d}{dx}C\,os(\log x+e^x) \quad =-S\,in(\log x+e^x)\left(\frac{d}{dx}\log x+\frac{d}{dx}e^x\right)$$

$$=-S\,in(\log x+e^x)\left(\frac{1}{x}+e^x\right)$$

II.

1. Find the derivations of the following functions from the first principle.

$(i)-x$ $\qquad(ii)(-x)^{-1}$ $\qquad(iii)S\,in(x+1)$ $\qquad(iv)C\,os\left(x-\frac{\pi}{8}\right)$

Sol.: (i) Let $f(x)=-x, f(x+h)=-x-h$

$$\boxed{f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}$$

$$\frac{d}{dx}(-x)=\lim_{h\to 0}\frac{-x-h+x}{h}$$

$$=\lim_{h\to 0}\frac{-h}{h}\quad=\lim_{h\to 0}(-1)\quad=-1$$

(ii) Let $f(x)=(-x)^{-1}=-\frac{1}{x}$, $f(x+h)=\frac{-1}{x+h}$

$$\frac{d}{dx}(-x)^{-1}=\lim_{h\to 0}\frac{\frac{-1}{x+h}+\frac{1}{x}}{h}=\lim_{h\to 0}\frac{-x+x+h}{h(x+h)x}\quad=\lim_{h\to 0}\frac{h}{h(x+h)x}$$

$$=\lim_{h\to 0}\frac{1}{(x+h)x}\quad=\frac{1}{(x+0)x}=\frac{1}{x^2}$$

(iii) Let $f(x)=S\text{in}(x+1)$, $f(x+h)=S\text{in}(x+h+1)$

$$\frac{d}{dx}S\text{in}(x+1)=\lim_{h\to 0}\frac{S\text{in}(x+h+1)-S\text{in}(x+1)}{h}$$

$$=\lim_{h\to 0}\frac{2C\text{os}\left(\frac{x+h+1+x+1}{2}\right)S\text{in}\left(\frac{x+h+1-x-1}{2}\right)}{h}$$

$$=\lim_{h\to 0}C\text{os}\left(x+1+\frac{h}{2}\right).\lim_{\frac{h}{2}\to 0}\frac{S\text{in}\frac{h}{2}}{\frac{h}{2}}\quad=C\text{os}(x+1+0).(1)\quad=C\text{os}(x+1)$$

(iv) Let $f(x)=C\text{os}(x-\frac{\pi}{8})$, $f(x+h)=C\text{os}(x+h-\frac{\pi}{8})$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{C\text{os}(x+h-\frac{\pi}{8})-C\text{os}(x-\frac{\pi}{8})}{h}$$

$$=\lim_{h\to 0}\frac{-2S\text{in}\left(\frac{x+h-\frac{\pi}{8}+x-\frac{\pi}{8}}{2}\right)S\text{in}\left(\frac{x+h-\frac{\pi}{8}-x+\frac{\pi}{8}}{2}\right)}{h}$$

$$=\lim_{h\to 0}\frac{-2S\text{in}\left(x-\frac{\pi}{8}+\frac{h}{2}\right)S\text{in}\frac{h}{2}}{2.\frac{h}{2}}\quad=-S\text{in}\left(x-\frac{\pi}{8}+0\right).(1)=-S\text{in}(x-\frac{\pi}{8})$$

2) $Co\sec x\,C\text{ot}\,x$

Sol.: Let $y = Co\sec x\,C\text{ot}\,x$

Differentiate with respect to x on both sides

$$\frac{dy}{dx} = C\text{os}\,ecx.\frac{d}{dx}C\text{ot}\,x + C\text{ot}\,x.\frac{d}{dx}C\text{os}\,ecx$$

$$= -C\text{os}\,ecx\,C\text{os}\,ec^2 x + C\text{ot}\,x(-C\text{os}\,ecx\,C\text{ot}\,x)$$

$$= -C\text{os}\,ec^3 x - C\text{ot}^2\,x\,C\text{os}\,ecx$$

3) $\dfrac{C\text{os}\,x}{1+S\text{in}\,x}$

Sol.: Let $y = \dfrac{C\text{os}\,x}{1+S\text{in}\,x}$

Differentiating w. r. t 'x'

$$\frac{dy}{dx} = \frac{(1+S\text{in}\,x)\frac{d}{dx}C\text{os}\,x - C\text{os}\,x\frac{d}{dx}(1+S\text{in}\,x)}{(1+S\text{in}\,x)^2}$$

$$= \frac{-(1+S\text{in}\,x)S\text{in}\,x - C\text{os}\,x(0+C\text{os}\,x)}{(1+S\text{in}\,x)^2}$$

$$= \frac{-S\text{in}\,x - S\text{in}^2 x - C\text{os}^2 x}{(1+S\text{in}\,x)^2} = \frac{-S\text{in}\,x-(1)}{(1+S\text{in}\,x)^2} = -\frac{(1+S\text{in}\,x)}{(1+S\text{in}\,x)^2} = \frac{-1}{1+S\text{in}\,x}$$

4) $\dfrac{Sinx + C\text{os}\,x}{S\text{in}\,x - C\text{os}\,x}$

Sol.: Let $y = \dfrac{Sinx + C\text{os}\,x}{S\text{in}\,x - C\text{os}\,x}$

Differentiating w. r. t 'x'

$$\frac{dy}{dx} = \frac{(S\text{in}\,x - C\text{os}\,x)\frac{d}{dx}(S\text{in}\,x + C\text{os}\,x) - (S\text{in}\,x + C\text{os}\,x)\frac{d}{dx}(S\text{in}\,x - C\text{os}\,x)}{(S\text{in}\,x - C\text{os}\,x)^2}$$

$$= \frac{(S\text{in}\,x - C\text{os}\,x)(C\text{os}\,x - S\text{in}\,x) - (S\text{in}\,x + C\text{os}\,x)(C\text{os}\,x + S\text{in}\,x)}{(S\text{in}\,x - C\text{os}\,x)^2}$$

$$= \frac{-(S\text{in}\,x - C\text{os}\,x)^2 - (S\text{in}\,x + C\text{os}\,x)^2}{(S\text{in}\,x - C\text{os}\,x)^2} = \frac{-\left[2\left(S\text{in}^2 x + C\text{os}^2 x\right)\right]}{(S\text{in}\,x - C\text{os}\,x)^2} \qquad = \frac{-2}{(S\text{in}\,x - C\text{os}\,x)^2}$$

5) $\dfrac{S\text{ec}\,x - 1}{S\text{ec}\,x + 1}$

Sol.: Let $y=\frac{S\,\mathrm{ec}\,x-1}{S\,\mathrm{ec}\,x+1}$

Differentiating w. r. t 'x' we get

$$\frac{dy}{dx}=\frac{\left(S\,\mathrm{ec}\,x+1\right)\frac{d}{dx}\left(S\,\mathrm{ec}\,x-1\right)-\left(S\,\mathrm{ec}\,x-1\right)\frac{d}{dx}\left(S\,\mathrm{ec}\,x+1\right)}{\left(S\,\mathrm{ec}\,x+1\right)^{2}}$$

$$=\frac{\left(S\,\mathrm{ec}\,x+1\right)\left(S\,\mathrm{ec}\,xxTanx-0\right)-\left(S\,\mathrm{ec}\,x-1\right)\left(S\,\mathrm{ec}\,xTanx+0\right)}{\left(S\,\mathrm{ec}\,x+1\right)^{2}}$$

$$=\frac{S\,\mathrm{ec}^{2}\,xTanx+S\,\mathrm{ec}\,xTanx-S\,\mathrm{ec}^{2}\,xTanx+S\,\mathrm{ec}\,xTanx}{\left(S\,\mathrm{ec}\,x+1\right)^{2}}\qquad=\frac{2S\,\mathrm{ec}\,xTanx}{\left(S\,\mathrm{ec}\,x+1\right)^{2}}$$

6) $S\,\mathrm{in}^{n}\,x$

Sol.: Note: Here we will use chain rule of differentiation:

Let $y=S\,\mathrm{in}^{n}\,x$

Differentiating w. r. t x

$$\frac{dy}{dx}=n.S\,\mathrm{in}^{n-1}\,x\frac{d}{dx}S\,\mathrm{in}\,x\qquad=nS\,\mathrm{in}^{n-1}\,xC\,\mathrm{os}\,x$$

7) $\frac{a+bS\,\mathrm{in}\,x}{c+dC\,\mathrm{os}\,x}$

Sol.: Let $y=\frac{a+bS\,\mathrm{in}\,x}{c+dC\,\mathrm{os}\,x}$

Differentiating w. r. t 'x'

$$\frac{dy}{dx}=\frac{\left(c+d\cos x\right)\frac{d}{dx}\left(a+b\sin x\right)-\left(a+b\sin x\right)\frac{d}{dx}\left(c+d\cos x\right)}{\left(c+d\cos x\right)^{2}}$$

$$=\frac{\left(c+d\cos x\right)(b\cos x)-(a+b\sin x)(-d\sin x)}{\left(c+d\cos x\right)^{2}}$$

$$=\frac{bc\cos x+bd\cos^{2}x+ad\sin x+bd\sin^{2}x}{\left(c+d\cos x\right)^{2}}$$

$$=\frac{bd(S\,\mathrm{in}^{2}\,x+C\,\mathrm{os}^{2}\,x)+bcC\,\mathrm{os}\,x+adS\,\mathrm{in}\,x}{\left(c+dC\,\mathrm{os}\,x\right)^{2}}=\frac{bd+bcC\,\mathrm{os}\,x+adS\,\mathrm{in}\,x}{\left(c+dC\,\mathrm{os}\,x\right)^{2}}$$

8) $\frac{S\,\mathrm{in}(x+a)}{C\,\mathrm{os}\,x}$

Sol.: Let $y=\frac{S\text{in}(x+a)}{C\text{os}\,x}$

Differentiating w. r. t 'x'

$$\frac{dy}{dx}=\frac{C\text{os}\,x\frac{d}{dx}S\text{in}(x+a)-S\text{in}(x+a)\frac{d}{dx}C\text{os}\,x}{C\text{os}^2 x}=\frac{C\text{os}\,x.C\text{os}(x+a)(1)+S\text{in}(x+a)S\text{in}\,x}{C\text{os}^2 x}$$

$$=\frac{C\text{os}(x+a-x)}{C\text{os}^2 x}=\frac{C\text{os}\,a}{C\text{os}^2 x}$$

9) $x^4(5S\text{in}\,x-3C\text{os}\,x)$

Sol.: Let $y=x^4(5S\text{in}\,x-3C\text{os}\,x)$

Differentiating w. r. t 'x'

$$\frac{dy}{dx}=x^4\frac{d}{dx}(5S\text{in}\,x-3C\text{os}\,x)+(5S\text{in}\,x-3C\text{os}\,x)\frac{d}{dx}x^4$$

$$=x^3\left[5xC\text{os}\,x+3xS\text{in}\,x+20S\text{in}\,x-12C\text{os}\,x\right]$$

10) $\left(x^2+1\right)C\text{os}\,x$

Sol.: Let $y=\left(x^2+1\right)C\text{os}\,x$

Differentiating w. r. t 'x'

$$\frac{dy}{dx}=\left(x^2+1\right)\frac{d}{dx}C\text{os}\,x+C\text{os}\,x\frac{d}{dx}\left(x^2+1\right)$$

$$=-\left(x^2+1\right)S\text{in}\,x+(C\text{os}\,x)(2x+0)=-x^2S\text{in}\,x-S\text{in}\,x+2xC\text{os}\,x$$

11) $\left(ax^2+S\text{in}\,x\right)\left(p+q\,C\text{os}\,x\right)$

Ans) Let $y=\left(ax^2+S\text{in}\,x\right)\left(p+qC\text{os}\,x\right)$

Differentiating w. r. t 'x'

$$\frac{dy}{dx}=\left(ax^2+S\text{in}\,x\right)\frac{d}{dx}\left(p+qC\text{os}\,x\right)+\left(p+qC\text{os}\,x\right)\frac{d}{dx}\left(ax^2+S\text{in}\,x\right)$$

$$=-qS\text{in}\,x\left(ax^2+S\text{in}\,x\right)+\left(p+qC\text{os}\,x\right)(2ax+C\text{os}\,x)$$

12) $\left(x+C\text{os}\,x\right)(x-T\text{an}\,x)$

Sol.: Let $y=\left(x+C\text{os}\,x\right)(x-T\text{an}\,x)$

Differentiating w. r. t 'x'

$$\frac{dy}{dx}=\left(x+C\text{os}\,x\right)\frac{d}{dx}(x-T\text{an}\,x)+(x-T\text{an}\,x)\frac{d}{dx}\left(x+C\text{os}\,x\right)$$

$$=\left(x+C\text{os}\,x\right)(1-S\text{ec}^2 x)+(x-T\text{an}\,x)(1-S\text{in}\,x)$$

13) $\frac{4x+5S\text{in}\,x}{3x+7C\text{os}\,x}$

Sol.: Let $y=\frac{4x+5S\text{in}\,x}{3x+7C\text{os}\,x}$

Differentiating w. r. t 'x'

$$\frac{dy}{dx}=\frac{\left(3x+7C\text{os}\,x\right)\frac{d}{dx}\left(4x+5S\text{in}\,x\right)-\left(4x+5S\text{in}\,x\right)\frac{d}{dx}\left(3x+7C\text{os}\,x\right)}{\left(3x+7C\text{os}\,x\right)^2}$$

$$=\frac{(3x+7Cos\,x)(4+5Cos\,x)-(4x+5Sin\,x)(3-7Sin\,x)}{(3x+7Cos\,x)^2}$$

$$=\frac{12x+15x\,Cos\,x+28\,Cos\,x+35Cos^2\,x-12x+28xSin\,x-15Sin\,x+35Sin^2\,x}{(3x+7Cos\,x)^2}$$

$$=\frac{35+15x\,Cos\,x+28Cos\,x+28x\,Sin\,x-15Sin\,x}{(3x+7Cos\,x)^2}$$

14) $\dfrac{x^2 Cos\left(\frac{\pi}{4}\right)}{Sin\,x}$

Sol.: Let $y=\dfrac{x^2 Cos\left(\frac{\pi}{4}\right)}{Sin\,x}$

Differentiating w. r. t 'x'

$$\frac{dy}{dx}=\frac{Cos\frac{\pi}{4}\left[Sin\,x\frac{dx^2}{dx}-x^2\frac{d}{dx}Sin\,x\right]}{Sin^2\,x}$$

$$=\frac{Cos\frac{\pi}{4}\left(2x\,Sin\,x-x^2\,Cos\,x\right)}{Sin^2\,x}=\frac{x\cos\frac{\pi}{4}\left(2Sin\,x-xCos\,x\right)}{Sin^2\,x}$$

15) $\dfrac{x}{1+Tanx}$

Sol.: Let $y=\dfrac{x}{1+Tanx}$

Differentiating w. r. t 'x'

$$\frac{dy}{dx}=\frac{(1+Tanx)\frac{dx}{dx}-x\frac{d}{dx}(1+Tanx)}{(1+Tanx)^2}=\frac{1+Tanx-x\,Sec^2\,x}{(1+Tanx)^2}$$

16) $(x+Sec\,x)(x-Tan\,x)$

Sol.: Let $y=(x+Sec\,x)(x-Tan\,x)$

Differentiating w. r. t 'x'

$$\frac{dy}{dx}=(x+Sec\,x)\frac{d}{dx}(x-Tan\,x)+(x-Tan\,x)\frac{d}{dx}(x+Sec\,x)$$

$$=(x+Sec\,x)(1-Sec^2\,x)+(x-Tan\,x)(1+Sec\,x\,Tanx)$$

17) $\dfrac{x}{Sin^n\,x}$

Sol.: Let $y=\dfrac{x}{Sin^n\,x}$

Differentiating w. r. t 'x' on both sides

$$\frac{dy}{dx}=\frac{Sin^n x\frac{dx}{dx}-x\frac{d}{dx}(Sin x)^n}{(Sin^n x)^2}=\frac{Sin^n x-x.nSin^{n-1}x\frac{d}{dx}Sin x}{Sin^{2n}x}$$

$$=\frac{Sin^n x-nx\,Sin^{n-1}x\,Cos x}{Sin^{2n}x}$$

MULTIPLE CHOICE QUESTIONS

1. $\lim\limits_{x\to1}\frac{x^2-1}{|x-1|}=\lim\limits_{x\to1}\frac{(x+1)(x-1)}{|x-1|}$ (Where x-1=y)

$=\lim\limits_{x\to1}(x+1)\lim\limits_{x\to1}\frac{(x-1)}{|x-1|}$ $=2.\lim\limits_{y\to0}\frac{y}{|y|}$ = Does not exist

2. $\lim\limits_{x\to0^+}\frac{1}{x}Cos^{-1}\frac{(1-x^2)}{1+x^2}=\lim\limits_{x\to0^+}2\frac{Tan^{-1}x}{x}=2$ and $\lim\limits_{x\to0^-}Cos^{-1}\frac{(1-x^2)}{1+x^2}=\lim\limits_{x\to0^-}-2\frac{Tan^{-1}x}{x}=-2$

∴ The limit does not exist.

3. $\lim\limits_{x\to0}(1+Tan^2x)^{\frac{1}{2x^2}}=\lim\limits_{x\to0}(1+Tan^2x)^{\frac{1}{Tan^2x}\cdot\frac{Tan^2x}{2x^2}}=\left(\lim\limits_{x\to0}(1+Tan^2x)^{\frac{1}{Tan^2x}}\right)^{\frac{Tan^2x}{2x^2}}=e^{\frac{1}{2}}$

4. $\lim\limits_{x\to2}\frac{(Cos\alpha)^x(Sin\alpha)^x-Cos^2\alpha-Sin^2\alpha}{(x-2)}$

$$=Cos^2\alpha.\lim_{(x-2)\to0}\frac{(Cos\alpha)^{x-2}-1}{x-2}+Sin^2\alpha.\lim_{(x-2)\to0}\frac{(Sin\alpha)^{x-2}-1}{x-2}$$

$$=Cos^2\alpha.\log Cos\alpha+Sin^2\alpha.\log Sin\alpha$$

5. $\Delta(x)=e^x+\sin x-1 \Rightarrow \lim\limits_{x\to0}\frac{\Delta x}{x}=\lim\limits_{x\to0}\left(\frac{e^x-1}{x}+\frac{\sin x}{x}\right)=1+1=2$

6. $s\,e^{\lim\limits_{x\to0}\frac{1}{x}\left(\frac{a^x+b^x}{2}-1\right)}=e^{\frac{1}{2}\lim\limits_{x\to0}\left(\frac{a^x-1}{x}+\frac{b^x-1}{x}\right)}=e^{\frac{1}{2}(\log a+\log b)}=\sqrt{ab}$

7. $\lim\limits_{x\to0}\frac{\sin x(\sec x-1)}{x.x^2}=\lim\limits_{x\to0}\frac{\sin x}{x}\lim\limits_{x\to0}\left(\frac{\sec x-1}{x^2}\right)=1.\left(\frac{1}{2}\right)=\frac{1}{2}$

8. $\lim\limits_{x\to0}\frac{\left(\frac{a^x-1}{x}\right)-\left(\frac{b^x-1}{x}\right)}{\left(\frac{d^x-1}{x}\right)}=\frac{\log a-\log b}{\log d}=\log_d\left(\frac{a}{b}\right)$

9. $e^{\lim_{x\to 0}\frac{1}{\sin x}\left(\frac{1+Tanx}{1+\sin x}-1\right)} = e^{\lim_{x\to 0}\frac{\tan x-\sin x}{\sin x(1+\sin x)}} = e^{\lim_{x\to 0}\frac{\sec x-1}{1+\sin x}} = e^0 = 1$

10. $\lim_{x\to 0}\left(\frac{(1+2x)-(1-2x)}{\sin^{-1}2x}\right).\left(\frac{1}{\sqrt{1+2x}+\sqrt{1-2x}}\right) = 2\lim_{2x\to 0}\frac{2x}{\sin^{-1}2x}.\lim_{x\to 0}\frac{1}{\sqrt{1+2x}+\sqrt{1-2x}}$

$= (2)(1)\frac{1}{(1+1)} = 1$

11. Clear ans.- 2

12. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\frac{|x|}{x} = 1 \qquad (|x| = x)$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\frac{|x|}{x} = -1 \qquad (|x| = -x)$

Does not exist.

Range f={-1,1} Sol.:4

13. $f(x) = x+[x] = \begin{cases} x, & 0\le x<1 \\ x+1, & 1\le x<2 \end{cases}$

$f'(x) = \frac{d}{dx}(x+1) = 1,\ 1<x<2 \Rightarrow f'\left(\frac{3}{2}\right) = 1$

14. $f(x) = x\,Cos\,x \Rightarrow f'(x) = Cos\,x - x\,Sin\,x$

$f'\left(\frac{\pi}{2}\right) = Cos\frac{\pi}{2} - \frac{\pi}{2}Sin\frac{\pi}{2} = \frac{-\pi}{2}$

15. $f(x) = 1+x+x^2+x^3+x^4+..............+x^n$

$f'(x) = 0+1+2x+3x^2+4x^3+..........+nx^{n-1}$

$f'(1) = 1+2+3+4+...........+n \quad = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$

Multiple Choice Questions Key

21) 4	**22) 4**	**23) 1**	**24) 3**	**25) 3**	**26) 2**	**27) 4**	**28) 1**	**29) 1**	**30) 3**
31) 2	**32) 4**	**33) 2**	**34) 1**	**35) 4**					

13.STATISTICS

Exercise13.a

I)

1. Find the mean deviation about the mean for the following

(a) 4,7,8,9,10,12,13,17

Sol: - Mean deviation about mean M.D $= \dfrac{\sum_{i=1}^{n}\left|x_i - \bar{x}\right|}{n}$

$\bar{x} = \text{Mean} = \dfrac{1}{n}\sum_{i}^{n} x_i \ . = \dfrac{1}{8}(4+7+8+9+10+12+13+17) = \dfrac{80}{8} = 10$

$\therefore \text{M.D} = \dfrac{1}{8}\left[|4-10|+|7-10|+|8-10|+|9-10|+|10-10|+|12-10|+|13-10|+|17-10|\right]$

$= \dfrac{1}{8}[6+3+2+1+0+2+3+7] = \dfrac{24}{8} = 3$

(b)38,70,48,40,42,55,63,46,54,44

Sol: M.D $= \dfrac{1}{n}\sum_{i=1}^{n}\left|x_i - \bar{x}\right|$ where $\bar{x} = \dfrac{1}{n}\sum_{i=1}^{n} x_i$

$\therefore \bar{x} = \dfrac{1}{10}[38+70+48+40+42+55+63+46+54+49] = \dfrac{1}{10}[500] = 50$

$\therefore \text{M.D} = \left(\dfrac{1}{10}\left[|38-50|+|70-50|+|48-50|+|40-50|+|42-50|+|55-50|+|63-50|+|46-50|+|54-50|+|44-50|\right]\right)$

$= \dfrac{1}{10}[12+20+2+10+8+5+13+4+4+6] = \dfrac{84}{10} = 8.4$

2.Find the Mean deviation about the median for the following

(a)13,17,16,14,11,13,10,16,11,18,12,17

Sol:- Mean deviation about the median M.D (M)$= \dfrac{1}{n}\sum_{i=1}^{n}\left|x_i - \mathbf{M}\right|$ Where M= Median.

Number of observation $n = 12$

The ascending order of the data 10,11,11,12,13,**13**,**14**,16,16,17,17,18

$\because n$ is even, Median $= \dfrac{1}{2}\left[\left(\dfrac{n}{2}\right)+\left(\dfrac{n}{2}+1\right)\right]$ observation $= \dfrac{13+14}{2} = 13.5$

$\therefore \text{M.D} \ = \dfrac{1}{12}[|13-13.5|+|17-13.5|+|16-13.5|+|14-13.5|+|11-13.5|+|13-13.5|+|10-13.5|+|16-13.5|$
$|11-13.5|+|18-13.5|+|12-13.5|+|17-13.5|]$

$= \dfrac{1}{12}(28) = 2.33\bar{3} = 2.33$

(b) 36,72,46,42,60,45,53,46,51,49.

Sol: :- Here number of observations $n = 10$

The ascending order of the data 36,42,46,46, **46,49**,51,53,60,72

Median M = $average\ of\ \frac{n}{2}, \frac{n}{2}+1$ observations $= \frac{46+49}{2} = 47.5$

Mean deviation about median M.D (M) $= \frac{1}{n}\sum_{c=1}^{n}\left|x_i - M\right|$ When M = Median

$$\therefore \text{M.D} = \frac{1}{10}[|36-47.5|+|72-47.5|+|46-47.5|+|14.2-47.5|+|60-47.51|+|45-47.5|+$$
$$|53-47.5|+|46-47.5|+|51-47.5|+|49-47.5|]$$

$$= \frac{1}{10}[70] = 7$$

II)

1. Find the mean deviation about the mean for the following data

(a)

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Sol::- M.D $= \frac{1}{N}\sum_{i=1}^{n} f_i\left|x_i - \bar{x}\right|$ where $\bar{x}$ in the mean $= \frac{1}{N}\sum_{i=1}^{n} f_i\ x_i$

x_i	f_i	$f_i.x_i$	$\left\|x_i - \bar{x}\right\|$	$f_i\left\|x_i - \bar{x}\right\|$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	**350**		**158**

$N = \sum_{i=1}^{n} f_i = 25.\ \sum_{i=1}^{n} f_i\ x_i = 350$

Mean $\left(\bar{x}\right) = \frac{1}{N}\sum_{i=1}^{n} f_i\ x_i = \frac{1}{25}(350) = 14$

M.D $= \frac{1}{25}(158) = 6.32$

(b)

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Sol:M.d $= \frac{1}{N}\sum_{i=1}^{n} f_i \left|x_i - \bar{x}\right|$ where $\bar{x}$ is the mean $= \frac{1}{N}\sum_{i=1}^{n} f_i\, x_i$

x_i	f_i	$f_i\, x_i$	$\left\|x_i - \bar{x}\right\|$	$fi\left\|x_i - \bar{x}\right\|$
10 30 50 70 90	4 24 28 16 8	40 720 1400 1120 720	40 20 0 20 40	160 480 0 320 320
	80	**4000**		**1280**

$N = \sum_{i=1}^{n} f_i = 80 \;\; N = \sum_{i=1}^{n} f_i\, x_i = 4000$

$\therefore \;\; \bar{x} = \frac{1}{N}\sum_{i=1}^{n} f_i\, x_i = \frac{4000}{80} = 50$

M.D $= \frac{1}{N}\sum_{i=1}^{n} f_i \left|x_i - \bar{x}\right| = \frac{1280}{80} = 16$.

2. Find the mean deactivation about the median for the following data

(a)

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Sol:

x_i	f_i	$c.f$	$\left\|x_i - M\right\|$	$fi\left\|x_i - M\right\|$
5 7 9 10 12 15	8 6 2 2 2 6	8 8+6=14 14+2=16 16+2=18 18+2=20 20+6=26	2 0 2 3 5 8	16 0 4 6 10 48
	26			**84**

$N = \sum_{i=1}^{6} f_i = 26, \quad \frac{N}{2} = 13 \qquad \therefore$ Median = 7

$$\therefore \text{ M.D } = \frac{1}{N}\sum_{i=1}^{n} f_i\left|x_i - M\right| = \frac{1}{26}(84) = 3.23.$$

b)

x_i	15	21	27	30	35
f_i	3	5	6	7	8

Sol:

x_i	f_i	$c.f$	$\left\|x_i - M\right\|$	$f_i\left\|x_i - M\right\|$
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40
	29			**148**

$$N = \sum_{i=1} f_i = 29 \Rightarrow \frac{N}{2} = \frac{29}{2} = 14.5$$

Median = 30

$$\therefore \text{ M.D} = \frac{1}{N}\sum_{i=1}^{n} f_i\left|x_i - M\right| = \frac{1}{29}(148) = 5.103 = 5.1$$

III.

1.Find the mean deviation about the mean for the following data

(a)

Income for day in ₹	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800
Number of persons	4	8	9	10	7	5	4	3

Sol:

Income per day in Rs ₹	**Number of persons** f_i	**Mid – points** x_i	$f_i\ x_i$	$\left\|x_i - \bar{x}\right\|$	$f_i\left\|x_i - \bar{x}\right\|$
0-100	4	50	200	308	1232
100-200	8	150	1200	208	1664
200-300	9	250	2250	108	972
300-400	10	350	3500	8	80
400-500	7	450	3150	92	644
500-600	8	550	2750	192	960
600-700	4	650	2600	292	1168
700-800	3	750	2250	392	1176
	50		**17900**		**7896**

$$N = \sum_{i=1}^{n} f_i = 50, \ \sum_{i=1}^{n} f_i\ x_i = 17900 \text{ and } \quad \therefore \text{ Mean } \bar{x} = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{17900}{50} = 358$$

$$\therefore \text{ M. D} = \frac{1}{N}\sum_{i=1}^{n} f_i \left|x_i - \bar{x}\right| = \frac{1}{50}(7896) = 157.92$$

Step deviation Method

Income for day in ₹	Number of persons f_i	Mid-Points x_i	Devotions $d_i = \frac{x_i - a}{n}$	$f_i d_i$	$\left|x_i - \bar{x}\right|$	$fi\left|x_i - \bar{x}\right|$
0-100	4	50	-4	-16	308	1232
100-200	8	150	-3	-24	208	1664
200-300	9	250	-2	-18	108	972
300-400	10	350	-1	-10	8	80
400-500	7	450	0	0	92	644
500-600	5	550	1	5	192	960
600-700	4	650	2	8	292	1168
700-800	3	750	3	9	392	1176
	50			-46		7896

$N = \sum_{i=1}^{n} f_i = 50$ Take assumed mean a = 450, h =100

$$\therefore \; \bar{x} = a + \frac{\sum_{i=1}^{n} f_i \, d_i}{N}.h = 450 + \frac{-46}{50}\times 100 = 450 - 92 = 358$$

$$\therefore \text{M.D} = \frac{1}{N}\sum f_i \left|x_i - \bar{x}\right| = \frac{1}{50}(7896) = 157.92$$

(b)

Height in cms	95-105	105-115	115-125	125-135	135-145	145-155
Number of Boys	9	13	26	30	12	10

Sol: : -

Height in Cms	Number of Boys f_i	Mid- Points x_i	$f_i.x_i$	$\left|x_i - \bar{x}\right|$	$f_i\left|x_i - \bar{x}\right|$
95-105	9	100	900	25.3	227.7
105-115	13	110	1430	15.3	198.9
115-125	26	1230	3120	5.3	137.8
125-135	30	130	3900	4.7	141.0
135-145	12	140	1680	14.7	176.4
145-155	10	150	1500	24.7	247.0
	100		**12530**		**1128.8**

$N = \sum_{i=1}^{n} f_i = 100$

Mean $\bar{x} = \frac{1}{N}\sum_{i=1}^{n} f_i\, x_i = \frac{12530}{100} = 125.3$

M.D $= \frac{1}{N}\sum_{i=1}^{n} f_i \left|x_i - \bar{x}\right| = \frac{1}{100}(1128.8) = 11.28$

Step – Deviation Method

Height in cms	**Number of Boys f_i**	**Mid-Points x_i**	**Deviations** $d_i = \frac{x_i - a}{n}$	$f_i\, d_i$	$\left\|x_i - \bar{x}\right\|$	$f_i\left\|x_i - \bar{x}\right\|$
95-105	9	10	-3	-27	25.3	227.7
105-115	13	110	-26	-26	15.3	198.9
115-125	26	120	-1	-26	5.3	137.8
125-135	30	130	0	0	4.7	141.0
135-145	12	140	1	12	14.7	176.4
145-155	10	150	2	20	24.7	247.0
	100			-47		**1128.8**

$N = \sum_{i=1}^{n} f_i = 100$ Take a = 130, h=10

Mean $\bar{x} = a + \frac{\sum_{i=1}^{n} f_i\, d_i}{N}.h = 130 + \frac{-47}{100}\times 10 = 130 - 4.7 = 125.3$

M.D$= \frac{1}{N}\sum_{i=1}^{n} f_i\left|x_i - \bar{x}\right| = \frac{1}{100}(1128.8) = 11.28$

2. Find the deviation about median for the following data.

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**
Number of Girls	**6**	**8**	**14**	**16**	**4**	**2**

Sol:

Marks	f_i **Frequency number of Girls**	**Cumulative Frequency** cf	**Mid – Points** x_i	$\left\|x_i - M\right\|$	$f_i\left\|x_i - M\right\|$
0-10	6	6	5	22.5	137.10
10-20	8	14 c	15	12.85	102.80
20-30	14 f	28 ↑	25	2.85	39.90
30-40	16	44	35	7.15	114.40
40-50	4	48	45	17.15	68.60
50-60	2	50	55	27.15	54.30
	50				517.10

$N = \sum_{i=1}^{n} f_i = 50 \quad \frac{N}{2} = \frac{50}{2} = 25$

Median class is (25^{th}item) = 20-30 $\Rightarrow l = 20$, h = 10

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c}{f}\right) h = 20 + \frac{25-14}{14} \times 10 = 20 + \frac{110}{14} = 20.7.85 = 27.85$$

$$\text{M.D} = \frac{1}{N}\sum_{i=1}^{n} f_i \left|x_i - M\right| = \frac{1}{50}(517.10) = 10.342 = 10.34$$

3. Calcualate the mean deviation about median age for the age distribution of 100 persons given below.

Age (in Yrs)	**16-20**	**21-25**	**26-30**	**31-35**	**36-40**	**41-45**	**46-50**	**51-55**
Number	5	6	12	14	26	12	16	9

Sol: Convert the given data into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class internal.

Age in Yrs	**Number** f_i	**Cumulative frequency** cf	**Mid-Point** x_i	$\left\|x_i - M\right\|$	$f_i\left\|x_i - M\right\|$
15.5-20.5	5	5	18	20	100
20.5-25.5	6	11	23	15	90
25.5-30.5	12	23	28	10	120
30.5-35.5	14	37 c	33	5	70
35.5-40.5	26 f	63 ↑	38	0	0
40.5-45.5	12	75	43	5	60
45.5-50.5	16	91	48	10	160
50.5-55.5	9	100	53	15	135
	100				**735**

$$N = \sum_{i=1}^{n} f_i = 100 \quad \frac{N}{2} = \frac{100}{2} = 50$$

Median class $= 35.5 - 40.5 \Rightarrow l = 35.5, h = 5$

$$\text{Median M} = l + \left(\frac{\frac{N}{2} - C}{f}\right).h$$

$$= 35.5 + \frac{50-37}{26} \times 5 = 35.5 + \frac{65}{26} = 35.5 + 2.5 = 38$$

$$\text{Mean deviation} = \frac{1}{N}\sum_{i=1}^{n} f_i \left|x_i - M\right|$$

$$= \frac{1}{100}(735) = 7.35$$

EXERCISE : 13.b

I) **1. Find the mean and variance for each of following the data**
(a)6,7,10,12,13,4,8,12.

Sol: $\bar{x} = \frac{\sum x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

Variance $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_1 - \bar{x}\right)^2$

$$= \frac{1}{8}\left[(6-9)^2 + (7-9)^2 + (10-9)^2 + (12-9)^2 + (13-9)^2 + (4-9)^2 + (8-9)^2 + (12-9)^2\right]$$

$$= \frac{1}{8}\left[9+4+1+9+16+25+1+9\right] = \frac{74}{8} = 9.25$$

(b) First 'n' natural numbers .

$1,2,3,.....n$

$$\bar{x} = \frac{1+2+3+.....+n}{n} = \frac{\sum n}{n} = \frac{n(n+1)}{2.n} = \frac{n+1}{2}$$

Variance $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n}\left[\sum_{i=1}^{n}\left(x_i^2 - 2x_i\bar{x} + \left(\bar{x}\right)^2\right)\right]$

$$= \frac{1}{n}\sum_{i=1}^{n}x_i^2 - 2\bar{x}\sum_{i=1}^{n}\frac{x_i}{n} + \frac{1}{n}\left(\bar{x}\right)^2\sum 1$$

$$= \frac{1}{n}\sum_{i=1}^{n}x_i^2 - 2\bar{x}\bar{x} + \frac{1}{n}\left(\bar{x}\right)^2.n$$

$$= \frac{1}{n}\sum_{i=1}^{n}x_i^2 - 2\left(\bar{x}\right)^2 + \left(\bar{x}\right)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}x_i^2 - \left(\bar{x}\right)^2$$

$$= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 \quad \left(\because \bar{x} = \frac{\sum x_i}{n}\right)$$

Alternative formula $\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$$\therefore \sigma^2 = \frac{1}{n}\left(1^2 + 2^2 + 3^2 + ...n^2\right) - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{1}{n}\frac{n(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = \frac{(n+1)}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$$

$$= \frac{(n+1)}{2}\left(\frac{4n+2-3n-3}{6}\right) = \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12}$$

(c)First 10 multiples of 3

Sol:3,6,9,12,15,18,21,24,27,30

$$\bar{x} = \frac{\sum x_i}{n} = \frac{3+6+9+12+15+18+21+24+27+30}{10} = \frac{165}{10} = 16.5$$

Variance $\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$$= \frac{1}{10}\left(3^2+6^2+9^2+12^2+15^2+18^2+21^2+27^2+30^2\right) - \left(\frac{165}{10}\right)^2$$

$$= \frac{1}{10}\left(3^2\left(1^2+2^2+3^2+.....10^2\right)\right) - \frac{27225}{100}$$

$$= \frac{9}{10}\frac{\left(10(10+1)(20+1)\right)}{6} - 272.25$$

$$= 346.5 - 272.25 = 74.25$$

II)

1. Find the mean and variance for each of the following data

(a)

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

Sol:

x_i	f_i	$f_i\ x_i$	$x_i - \bar{x}$	$\left(x_i - \bar{x}\right)^2$	$f_i\left(x_i - \bar{x}\right)^2$
6	2	12	-13	169	338
10	4	40	-9	81	324
14	7	98	-5	25	175
18	12	21 6	-1	1	12
24	8	19 2	5	25	200
28	4	11 2	9	81	324
30	3	90	11	121	363
	40	760			1736

$$N = \sum_{i=1}^{n} f_i = 40$$

Mean $\bar{x} = \frac{1}{N}\sum_{i=1}^{n} f_i\ x_i = \frac{760}{40} = 19$

Variance $\sigma^2 = \frac{1}{N}\sum_{i=1}^{n} f_i\left(x_i - \bar{x}\right)^2 = \frac{1736}{40} = 43.4$

<u>Alternative method</u>

x_i	f_i	x_i^2	$f_i\ x_i$	$f_i\ x_i^2$
6	2	36	12	72
10	4	100	40	400
14	7	196	98	1372
18	12	324	216	3888
24	8	576	192	4608
28	4	784	112	3136
30	3	900	90	2700
	40		**760**	**16176**

$$N=\sum_{i=2}^{n} f_i = 40$$

$$\text{Mean } \bar{x}=\frac{1}{N}\sum_{i=1}^{n} f_i\ x_i = \frac{760}{40}=19$$

$$\text{Variance } \sigma^2 = \frac{\sum f_i x_i^2}{N}-\left(\frac{\sum f_i x_i}{N}\right)^2$$

$$=\frac{16176}{40}-\left(\frac{760}{40}\right)^2 = 404.4-(19)^2 = 404.4-361 = 43.4$$

(b)

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

Sol:

x_i	f_i	$f_i x_i$	$x_i-\bar{x}$	$\left(x_i-\bar{x}\right)^2$	$f_i\left(x_i-\bar{x}\right)^2$
92	3	276	-8	64	192
93	2	186	-7	49	98
97	3	291	-3	9	27
98	2	196	-2	4	8
102	6	612	2	4	24
104	3	312	4	16	48
109	3	327	9	81	243
	22	2200			640

$$N=\sum_{i=1}^{n} f_i = 22$$

$$\text{Mean } \bar{x}=\frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{2200}{22}=100$$

$$\text{Variance } \sigma^2=\frac{1}{N}\sum_{i=1}^{n} f_i\left(x_i-\bar{x}\right)^2 = \frac{1}{22}(640) = 29.09$$

2nd method

x_i	f_i	x_i^2	$f_i x_i$	$f_i x_i^2$
92 93 97 98 102 104 109	3 2 3 2 6 3 3	8464 8649 9409 9604 10404 10816 11881	276 186 291 196 612 312 327	25392 17298 28227 19208 62424 32448 35643
	22		2200	220640

$$N=\sum_{i=1}^{n} f_i = 22$$

Mean $\bar{x}=\frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{2200}{22}=100$

Variance $\sigma^2 = \frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2$

$$=\frac{220640}{22}-\left(\frac{2200}{22}\right)^2 = 10029.09-(100)^2$$

$$=10029.09-10000 = 29.09$$

2. Find the mean and standard deviation using short-cut method

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Sol:

x_i	f_i	**Deviation from mean** $y_i=\frac{x_i-A}{h}$	y_i^2	$f_i\ y_i$	$f_i\ y_i^2$
60 61 62 63 64 65 66 67 68	2 1 12 29 25 12 10 4 5	-4 -3 -2 -1 0 1 2 3 4	16 9 4 1 0 1 4 9 16	-8 -3 -24 -29 0 12 20 12 20	32 9 48 29 0 12 40 36 80
	100			**0**	**286**

$$N=\sum_{i=1}^{n} f_i = 100, h=1$$

Assumed mean = $A = 64$

Mean $\bar{x} = \mathbf{A} + \dfrac{\sum f_i\, y_i}{N}.h = 64 + \dfrac{0}{100}.1 = 64$

Standard deviation $\sigma = SD = \dfrac{h}{N}\sqrt{N.\sum f_i\, y_i^2 - \left(\sum f_i\, y_i\right)^2}$

$$= \frac{1}{100}\sqrt{100(286)-(0)^2} = \frac{10(16.91)}{100} = 1.691 = 1.69$$

Alternative formula for S.D $\sigma = \sqrt{\dfrac{\sum f_i y_i^2}{N} - \left(\dfrac{\sum f_i y_i}{N}\right)^2} \quad = \sqrt{\dfrac{286}{100} - \left(\dfrac{0}{100}\right)^2} = \sqrt{2.86} = 1.69$

III.

1. Find the mean and variance for the following frequency distributions

(a)

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

Sol:

Classes	**Frequencies** f_i	**Mid- Points** x_i	$f_i x_i$	$\left(x_i - \bar{x}\right)^2$	$f_i\left(x_i - \bar{x}\right)^2$
0-30	2	15	30	8464	16928
30-60	3	45	135	3844	11532
60-90	5	75	375	1024	5120
90-120	10	105	1050	4	40
120-150	3	135	405	784	2352
150-180	5	165	825	3364	16820
180-210	2	195	390	7744	15488
	30		3210		68280

$N = \sum f_i = 30$

Mean $\bar{x} = \dfrac{1}{N}\sum f_i x_i = \dfrac{3210}{30} = 107$

Variance $\sigma^2 = \dfrac{1}{N}\sum f_i\left(x_i - \bar{x}\right)^2 = \dfrac{68280}{30} = 2276$

(*We can solve this using step – deviation method)

(b)

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Sol:

Classes	**Frequencies** f_i	**Mid- Point** x_i	$f_i x_i$	$\left(x_i - \bar{x}\right)^2$	$f_i\left(x_i - \bar{x}\right)^2$
0-10	5	5	25	484	2420
10-20	8	15	120	144	1152
20-30	15	25	375	4	60
30-40	16	35	560	64	1024
40-50	6	45	270	324	1944
	50		1350		6600

$$N = \sum f_i = 50$$

$$\text{Mean } \bar{x} = \frac{1}{N}\sum f_i x_i = \frac{1350}{50} = 27$$

$$\text{Variance } \sigma^2 = \frac{1}{N}\sum f_i \left(x_i - \bar{x}\right)^2 = \frac{6600}{50} = 132$$

2.Find the mean , variance and standard deviation using short –cut method

Height in cms	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. Of children	3	4	7	7	15	9	6	6	3

Sol:

Class	**Frequency** f_i	**Mid-Points** x_i	**Deviation** $y_i = \frac{x_i - A}{n}$	y_i^2	$f_i y_i$	$f_i\ y_i^2$
70-75	3	72.5	-4	16	-12	48
75-80	4	77.5	-3	9	-12	36
80-85	7	82.5	-2	4	-14	28
85-90	7	87.5	-1	1	-7	7
90-95	15	92.5	0	0	0	0
95-100	9	97.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	60				**6**	**254**

Let the assumed mean A=92.5, h= 5, $N = \sum f_i = 60$

$$\text{Mean } \bar{x} = \mathbf{A} + \frac{\sum f_i y_i}{N} h = 92.5 + \frac{6(5)}{60} = 93.$$

$$\text{Variance } \sigma^2 = \frac{h^2}{N^2}\left[N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right] = \frac{5^2}{60^2}\left[60(254) - (6)^2\right]$$

$$= \frac{25}{3600}(15240 - 36) = 105.58$$

Stranded deviation $\sigma = \sqrt{105.58} = 10.27$

3. The diameter of circles (in mm) draw in a design are given below.

Diameters	33-36	37-40	41-44	45-48	49-52
No.of Circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circle.

Sol: Frequency distribution table is to be prepared by making the data continuous .

Class	Frequency f_i	Mid-point x_i	Deviation $y_i = \frac{x_i - \mathbf{A}}{h}$	y_i^2	$f_i\ y_i$	$f_i\ y_i^2$
33 - 36	15	34.5	-2	4	-30	60
37 - 40	17	38.5	-1	1	-17	17
41 - 44	21	42.5	0	0	0	0
45 - 48	22	46.5	1	1	22	22
49 - 52	25	50.5	2	4	50	100
	100				25	199

Let the assumed mean A=42.5, $h = 4$, $N = \sum_{i=1}^{n} f_i = 100$

Mean $\bar{x} = \mathbf{A} + \frac{\sum f_i y_i}{\mathbf{N}}.h = 42.5 + \frac{25}{100} \times 4 = 43.5$

$= 42.5 + \frac{25}{100} \times 4 = 43.5$ Mean diameter of the circles = 43.5 (mm)

Standard deviation $\sigma = \frac{h}{N}\sqrt{N\sum_{i=1}^{n} f_i y_i^2 - \left(\sum_{i=1}^{n} f_i y_i\right)^2}$

$= \frac{4}{100}\sqrt{100.(199) - (25)^2} = \frac{4.5}{100}\sqrt{4(199) - 25}$

$= \frac{1}{5}\sqrt{796 - 25} = \frac{\sqrt{771}}{5} = \frac{27.76}{5} = 5.55$

EXERCISE 13.c

III)

1. The mean and variance of eight observations one 9 and 9.25 respectively. If six of the observations are 6,7,10,12,12 and 13, find the remaining two observations .

Sol: Let the other two observations be x and y then the series give be 6,7,10,12,12,13, $x\ y$.

Given that Mean $\bar{x} = 9 \Rightarrow \frac{6+7+10+12+12+13+x+y}{8} = 9$

$\Rightarrow x + y = 72 - 60 = 12$ ……………………………..(1)

Also given that $\sigma^2 = 9.25 \Rightarrow \frac{1}{n}\sum_{i=1}^{8}\left(x_i - \bar{x}\right)^2 = 9.25$

$\Rightarrow \frac{1}{8}[(6-9)^2 + (7-9)^2 + (10-9)^2 + (12-9)^2 + (12-9)^2$

$+(13-9)^2 + (x-9)^2 + (y-9)^2] = 9.25$

$\Rightarrow 9 + 4 + 1 + 9 + 9 + 16 + x^2 - 18x + 81 + y^2 - 18y + 81 = 74.00$

$\Rightarrow x^2 + y^2 - 18(x + y) = 74 - 210$

$\Rightarrow x^2 + y^2 - 18(12) = -136$

$\Rightarrow x^2 + y^2 = 216 - 136 = 80$ ………………………..(2)

$\Rightarrow x^2 + (12 - x)^2 = 80$ $(\because y = 12 - x)$ From ……… (1))

$$\Rightarrow x^2 + x^2 - 24x + 144 = 80$$
$$\Rightarrow 2x^2 - 24x + 64 = 0$$
$$\Rightarrow x^2 - 12x + 32 = 0$$
$$\Rightarrow (x-4)(x-8) = 0$$
$$\Rightarrow x = 4, x = 8 \Rightarrow y = 8, 4$$
$$\therefore\ x = 4, y = 8$$

2. The mean and variance of 7 observations are 8 and16 respectively .If five of the observations are $2, 4, 10, 12, 14$ **. Find the remaining two observations.**

Sol: Let the other two observations be x and y.

There the series given 2,4,10,12,14, x and y.

Given that $\bar{x} = 8 \Rightarrow \dfrac{2+4+10+12+14+x+y}{7} = 8$

$\Rightarrow x + y = 56 - 42 = 14$ ………………………...1

Also given that

$$\sigma^2 = 16 \Rightarrow \frac{1}{7}\left[(2-8)^2 + (4-8)^2 + (10-8)^2 - (12-8)^2 + (14-8)^2 + (x-8)^2 + (y-8)^2\right] = 16$$

$$\Rightarrow 36 + 16 + 4 + 16 + 36 + x^2 + y^2 - 16(x-y) + 64 + 64 = 112$$

$\Rightarrow x^2 + y^2 = 224 - 124 - 100$ …………………..2

$\Rightarrow x^2 + (14-x)^2 = 100$ (using (1))

$$\Rightarrow 2x^2 - 28x + 96 = 0$$
$$\Rightarrow x^2 - 14x + 48 = 0$$
$$\Rightarrow (x-8)(x-6) = 0$$
$$\Rightarrow x = 6, x = 8 \Rightarrow y = 8, y = 6$$
$$\therefore x = 6,\ y = 8$$

3. The mean and standard deviation of six observations and 8 and 4 respectively .If each observation in multiplied by 3, find the new mean and new slandered deviation of the resulting observations.

Sol::- Let the six observations be $x_1, x_2, x_3 .... x_6$ and $\bar{x}$ be their mean.

Given that $\bar{x} = 8 \Rightarrow \dfrac{1}{6}(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) = 8$

$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 48$ …………(1)

Let $y_i\ (i = 1, 2 ... 6)$ be the new observation where $y_i = 3x_i\ (1 \le i \le 6)$

New mean $\bar{y_i} = \dfrac{1}{6}\sum_{i=1}^{6} y_i = \dfrac{1}{6}[3x_1 + 3x_2 + ...3x_6] = \dfrac{3}{6}\left(\sum x_i\right) = \dfrac{3}{6}(48) = 24 = \left(\bar{x_i}\right)$

$y_i = 3x_i \Rightarrow x_i = \dfrac{1}{3} y_i$

Variance $\sigma^2 = \dfrac{1}{n}\sum_{i=1}^{6}\left(x_i - \bar{x}\right)^2 \Rightarrow 4^2 = \dfrac{1}{6}\sum_{i=1}^{6}\left(x_i - \bar{x}\right)^2 \Rightarrow \sum_{i=1}^{6}\left(x_i - \bar{x}\right)^2 = 96 \Rightarrow \sum_{i=1}^{6}\left(\dfrac{1}{3}y_i - \dfrac{1}{3}\bar{y_i}\right)^2 = 96$

$$\Rightarrow \frac{1}{9}\sum_{i=1}^{6}\left(y_i - \bar{y_i}\right)^2 = 96 \Rightarrow \sum_{i=1}^{6}\left(y_i - \bar{y_i}\right)^2 = 9 \times 96$$

$\therefore$ Variance of new observation $=\sigma_y^2=\frac{1}{6}(\sum_{i=1}^{6}\left(y_i-\overline{y}_i\right)^2=\frac{1}{6}\left(9\times16\right)=144$

$\therefore$ New S.D $=\sigma_y=\sqrt{144}=12$

Note: If each observation in multiplied by a constant K, the variance of the resulting observations becomes K^2 times the original variance.

4) Given that $\overline{x}$ in the mean and σ^2 in the variance of n observations $x_1,x_2,x_3.....x_n$. Prove that the mean and variance of the observations $ax_1,ax_2...ax_n$ are $a\overline{x}$ and $a^2\sigma^2$ respectively. $\left(a\neq0\right)$.

Sol:: Given that $\overline{x}=\frac{x_1+x_2+.....+x_n}{n}$ and $\sigma^2=\frac{1}{x}\sum\left(x_i-\overline{x}\right)^2$

The mean of $ax_1,ax_2.....ax_n=\frac{1}{n}\sum\left(ax_1+ax_2+....ax_n\right)$

$$=a\left[\frac{x_1+x_2+...+x_n}{n}\right]$$

$$=a.\overline{x}$$

The variance of $ax_1,ax_{2,}ax_3.......ax_n=\frac{1}{n}\sum\left(ax_i-a\overline{x}\right)^2$

$$=\frac{1}{n}\left[\left(ax_1-a\overline{x}\right)^2+(ax_2-a\overline{x})^2+.....(ax_n-a\overline{x})^2\right]$$

$$=\frac{1}{n}\left[a^2\left(x_1-\overline{x}\right)^2+a^2\left(x_2-\overline{x}\right)^2+.....+a^2\left(x_n-\overline{x}\right)^2\right]$$

$$=a^2\left[\frac{\left(x_1-\overline{x}\right)^2+\left(x_2-\overline{x}\right)^2+.....\left(x_n-\overline{x}\right)^2}{n}\right]=a^2\frac{\sum\left(x_1-\overline{x}\right)^2}{n}=a^2\sigma^2$$

5) The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On rechecking, it was found that an observation 8 was in correct. Calculate the correct mean and slandered deviation in each of the following cases. (i) if wrong item is omitted ii) If it is replaced by 12

Sol: Case (i) if wrong item is omitted

Given that $\overline{x}=10$, $\sigma=2$, $n=20$

$$\Rightarrow\frac{\sum x_i}{n}=10\Rightarrow\sum x_i=10\times20=200$$

If the wrong observation 8 is omitted then $n=19$

$$\sum x_i=200-8=192$$

$\therefore$ The correct Mean $=\frac{\sum x_i}{n}=\frac{192}{19}=10.105$

Now $\sigma^2=4\Rightarrow\frac{1}{n}\sum x_i^2-\left(\overline{x}\right)^2=4\Rightarrow\frac{1}{20}\left(\sum x_i^2\right)-\left(10^2\right)=4\Rightarrow\sum x_i^1=104\times20=2080$

If wrong observation 8 is omitted than

$$\sum x_i^2=208-8^2=2080-64=2016$$

$\therefore$ The correct standard deviation $\sigma = \sqrt{\frac{1}{n}\sum x_i^2 - \left(\bar{x}\right)^2} = \sqrt{\frac{1}{19}(2016) - (10.105)^2}$

$= \sqrt{106.105 - 102.111} = \sqrt{3.984} = 1.995 == 1.99$

Case ii: If it is replaced by 12

Given that $\bar{x} = 10$, $\sigma = 2$, $n = 20$

$\Rightarrow \frac{\sum x_i}{n} = 10 \Rightarrow \sum x_i = 10 \times 20 = 200$

If the observation 8 is replaced by 12 then

$\sum x_i = 200 - 8 + 12 = 204$

Now correct mean = $\frac{\sum x_i}{n} = \frac{204}{20} = 10.2$

Now $\sigma = 2 \Rightarrow \sigma^2 = 4 \Rightarrow \frac{1}{n}\sum x_i^2 - \left(\bar{x}\right)^2 = 4 \Rightarrow \frac{1}{20}.\sum x_i^2 - (10)^2 = 4 = 1\sum x_i^2 = 104 \times 20 = 2080$

If the observation 8 is replaced by 12

$$\sum x_i^2 = 2080 - 8^2 + 12^2 = 2080 - 64 + 144 = 2160$$

Now correct standard deviation $\sigma = \sqrt{\frac{1}{n}\sum x_i^2 - \left(\bar{x}\right)^2} = \sqrt{\frac{2160}{20} - (10.2)^2} = \sqrt{108 - 104.04}$

$$= \sqrt{3.96} = 1.98$$

6. The mean and standard deviation of a group of 100 observations were found to be 20 and 3 respectively. Later it was found that three observations were incorrect, which were recorded on 21,21 and 18.. Find the mean and standard deviation . if the incorrect observation are omitted.

Sol: Given $\bar{x} = 20$, $\sigma = 3$, $n = 100$

Three incorrect observations are 21,21 and 18.

$\bar{x} = \frac{\sum x_i}{n} \Rightarrow \frac{\sum x_i}{100} = 20 \Rightarrow \sum x_i = 20 \times 100 = 2000$

If the three incorrect observations are omitted then

$\sum \bar{x}_i = 2000 - 21 - 21 - 18 = 1940$

New mean $= \frac{\sum x_i}{n} = \frac{1940}{97} = 120$

Also $\sigma = 3 \Rightarrow \sigma^2 = 9 \quad \Rightarrow \frac{1}{n}\sum x_i^2 - \left(\bar{x}\right)^2 = 9 \quad \Rightarrow \frac{1}{100}\sum x_i^2 - (20)^2 = 9 \Rightarrow \sum x_i^2 = 409 \times 100 = 40900$

If the three incorrect observation are omitted then

$$\sum x_i^2 = 40900 - 21^2 - 21^2 - 18^2$$
$$= 40900 - 441 - 441 - 324 = 39694$$

New standard deviation $\sigma = \sqrt{\frac{1}{n}\sum x_i^2 - \left(\bar{x}\right)^2} = \sqrt{\frac{1}{97}(39694) - (20)^2}$

$$= \sqrt{409.216 - 400} = \sqrt{9.216}$$
$$= 3.0359 = 3.036$$

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14. PROBABILITY

Exercise 14(a)

I. 1) A die is rolled. Let E be the event "die shows 4" and F be event "die shows even number". Are E and F mutually exclusive?

Sol:- Let S be the sample space of rolling a die

$S=\{1,2,3,4,5,6\}$

E = die shown 4 ={4} F = die shown even number = {2,4,6}

$E\cap F=\{4\}\neq\phi$

$\therefore$ E and F are not mutually exclusive .

2) In the experiment of throwing a die, consider the following events.

$\mathbf{A}=\{1,3,5\}$ $\mathbf{B}=\{2,4,6\}$ $C=\{1,2,3\}$: Are these events equally likely?

Sol:- In the experiment of throwing a die , the chances of happening / occurring the events A, B and C are equal. Hence they are equally likely events.

3) In the experiment of throwing a die, consider the following events

$\mathbf{A}=\{1,3,5\}$ $\mathbf{B}=\{2,4\}$ $\mathbf{C}=\{6\}$ Are these events mutually exclusive?

Sol:- Since the happening of one of the given events A, B and C prevents the happening of other two. The events A, B and C are mutually exclusive (or) $\because$ $\mathbf{A}\cap\mathbf{B}=\phi,\mathbf{B}\cap\mathbf{C}=\phi$ and $\mathbf{A}\cap\mathbf{C}=\phi$, A, B and c are mutually exclusive events

4) In the experiment of throwing a die consider the events $\mathbf{A}=\{2,4,6\}$ $\mathbf{B}=\{3,6\}$ $\mathbf{C}=\{1,5,6\}$. Are these events exhaustive?

Sol:- Let S be the sample space for the random experiment of throwing a die . Then

$S=\{1,2,3,4,5,6\}$ Given. $A=\{2,4,6\}$, $B=\{3,6\}$ and $C=\{1,5,6\}$ .Clearly $\mathbf{A}\subset\mathbf{S},\mathbf{B}\subset\mathbf{S}$ and $\mathbf{C}\subset\mathbf{S}$ and $\mathbf{A}\cup\mathbf{B}\cup\mathbf{C}=S$.

$\therefore$ The events A, B and C are exhaustive events.

II. 1) An experiment involves rolling a pair of dice and recording the number that comes up. Describe the following events.

A: The sum is greater than 8.

B: 2 occurs on either die

C: The sum is atleast 7 and a multiple of 3.

Which pair of these events are mutually exclusive ?

Sol: -Let S be the sample space for the random experiment of rolling a pair of dice and recording the results.

$S=\{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,2).....(6,6)\}$

i) $A=$ The sum is greater than $8=\{(3,6)(4,5)(4,6)(5,4)(5,5)(5,6)(6,3)(6,4)(6,5)(6,6)\}$

ii) B = 2 occurs on the either die$=\{(1,2)(2,1)(3,2)(2,3)(2,4)(4,2)(2,5)(5,2)(6,2)(2,6)\}$

iii) C = The sum is at least 7 and a multiple of 3 i.e sum is 9 and 12

$=\{(3,6)(4,5)(5,4)(6,3)(6,6)\}$

Now $\mathbf{A}\cap\mathbf{B}=\phi\Rightarrow$ A and B are mutually exclusive .

$\mathbf{B}\cap\mathbf{C}=\phi\Rightarrow$ B and C are mutually exclusive .

2) Three coins are tossed once. Let A denote the events "three heads show". B denote the event " Two heads and one tail show" C denote the event ."Three tails show" and D denote the event " a head shows on the first coin". Which events are i) Mutually exclusive? ii) Simple? iii) Compound?

Sol:- Let S be the sample space for the random experiment of tossing 3 coins.

Then S = $\{(H\ H\ H)(H\ H\ T)(H\ T\ H)(T\ H\ H)(H\ T\ T)(T\ H\ T)(T\ T\ H)(T\ T\ T)\}$

A = ' Three heads shows' $=\{(H\ H\ H)\}$

B = Two heads and one tail $=\{(H\ H\ T)(H\ T\ H)(T\ H\ H)\}$

C = "Three tail show"= $\{(T\ T\ T)\}$

D = A head shows on the first coin $=\{(H\ H\ H)(H\ H\ T)(H\ T\ H)(H\ T\ T)\}$

i) $\mathbf{A}\cap\mathbf{B}=\phi,\mathbf{A}\cap\mathbf{C}=\phi,\mathbf{B}\cap\mathbf{C}=\phi,\mathbf{C}\cap\mathbf{D}=\phi$ A and B, A and C , B and C,C and D are mutually exclusive. Also $\mathbf{A}\cap\mathbf{B}\cap\mathbf{C}=\phi\Rightarrow$ A, B and C are mutually exclusive .

ii) An event is said to be simple if it has one sample point only

$\therefore$ The events A and C are simple events.

iii) An event is said to be compound if it has at least two sample points.

$\therefore$ The events B and D are compound.

3) Two dice are thrown. The events A, B and C are as follows.

A: Getting an even number on the first die.

B. Getting an odd number on the first die.

C: Getting the sum of the numbers on the dice ≤ 5.

State true or false (give reason for your answer).

i)A and B are mutually exclusive ii) A and B are mutually exclusive and exhaustive

iii) $\mathbf{A} = \mathbf{B}^{'}$ iv)A and C are mutually exclusive

v) A and $B^{'}$ are mutually exclusive vi) $\mathbf{A}^{'}, \mathbf{B}^{'}, C$ are mutually exclusive and exhaustive

Sol:- Let S be the sample space for the radom experiment of throwing two dice.

$$S = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,2)...(6,6)\}$$

A = getting an even number on the first die

$= \{(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$
$(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$

B = getting and odd number on the first die.

$= \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)$
$(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)\}$

C $= \{(1,1)(1,2)(1,3)(1,4)(2,1)(2,2)(2,3)(3,1)(3,2)(4,1)\}$

$\mathbf{B}^{'} =$ not B = Which are not in B = A

$\mathbf{A}^{'}$ = not A = which are not in A =B

i) $\mathbf{A} \cap \mathbf{B} = \phi$ (True)

ii) $\mathbf{A} \cup \mathbf{B} = S, \mathbf{A} \cap \mathbf{B} = \phi$ (True)

iii)True ($\therefore$ B = not getting B = getting A)

iv) False $\mathbf{A} \cap \mathbf{C} = \{(2,1)(2,2)(2,3)(4,1)\} \neq \phi$ $\therefore$ A and C are not mutually exclusive .

v) False $\mathbf{A} \cap \mathbf{B}^{'} = \mathbf{A} \cap \mathbf{A} = \mathbf{A} \neq \phi$ $\therefore$ A and $B^{'}$ and not mutually exclusive.

vi) False $\mathbf{A}^{'} \cap \mathbf{B}^{'} = \mathbf{B} \cap \mathbf{A} = \phi$

$\mathbf{A}^{'} \cap \mathbf{B}^{'} \cap \mathbf{C} = \phi \cap C = \phi$

Also $\mathbf{A}^{'} \cup \mathbf{B}^{'} \cup \mathbf{C} = \mathbf{B} \cup \mathbf{A} \cup C = S$

$\mathbf{A}^{'} \cap \mathbf{C} = \mathbf{B} \cap \mathbf{C} = \{(1,1)(1,2)(1,3)(1,4)(3,1)(3,2)\} \neq \phi$

$\mathbf{B}^{'} \cap \mathbf{C} = \mathbf{A} \cap \mathbf{C} = \{(2,1)(2,2)(2,3)(4,1)\} \neq \phi$

$\therefore \mathbf{A}^{'}, \mathbf{B}^{'}$ and C are not mutually exclusive.

III. 1) A die is thrown, Describe the following events.

i) A : a number less than 7 ii) B: a number greater than 7.

iii) C :a multiple of 3 iv) D: a number less than 4.

v) E: an even number greater than 4 vi) F: a number not less than 3.

Also find $\mathbf{A} \cup \mathbf{B}, \mathbf{A} \cap \mathbf{B}, \mathbf{B} \cup \mathbf{C}, \mathbf{E} \cap \mathbf{F}, \mathbf{D} \cap \mathbf{E}, \mathbf{A}-\mathbf{C}, \mathbf{D}-\mathbf{E}, \mathbf{E} \cap \mathbf{F}^{'}$ *and* $\mathbf{F}^{'}$

Sol:- Let S and sample space denotes the random experiment of throwing a die.

S: $=\{1,2,3,4,5,6\}$

i)A = a number less than 7 = { 1,2,3,4,5,6}

ii) B= a number greater than 7 ={ } $=\phi$

iii) C = a multiple of 3 = {3,6}

iv) D = a number less than 4 ={1,2,3}

v) E = an even number greater than 4 ={6}

vi) F= a number not less than 3 = {3,4,5,6}

$\mathbf{A} \cup \mathbf{B}$ = The elements of A and B ={1,2,3,4,5,6} {$\cup$ } ={1,2,3,4,5,6}= A

$\mathbf{A} \cap \mathbf{B}$ = The common elements of A and B ={1,2,3,4,5,6,)$\cap \phi$ = ϕ

$\mathbf{B} \cup \mathbf{C}$ = The elements of B or C = { } $\cup$ { 3,6}= {3,6}

$\mathbf{E} \cap \mathbf{F}$ = The common element of E and F = {6} $\cap${3,4,5,6}={6}= E

$\mathbf{D} \cap \mathbf{E}$ = The common element of D and E = {1,2,3}$\cap${6}=ϕ

A – C = The elements which are in A and not is C ={1,2,3,4,5,6}-{3,6}={1,2,4,5}

D – E = The elements which are in D and not in E = {1,2,3}- {6}= {1,2,3}

$\mathbf{E} \cap \mathbf{F}^{'}$ = The common elements of E and F' = {6}$\cap${1,2}={ } =ϕ

$\mathbf{F}^{'}$ = S – F = {1,2,3,4,5,6} –{3,4,5,6} ={ 1,2}

2) Three coins are tossed. Describe

i) Two events which are mutually exclusive

ii) Three events which are mutually exclusive and exhaustive

iii) Two events which are not mutually exclusive.

iv) Two events which are mutually inclusive but not exhaustive

v) Three events which are mutually excluessive but not exhaustive .

Sol:- Let S be the sample space denotes the random experiment of tossing Three coins.

S= $\mathbf{F}\{(H\ H\ H)(H\ H\ T)(H\ T\ H)(T\ H\ H)(T\ T\ H)(T\ H\ T)(H\ T\ T)(T\ T\ T)\}$

i) Let A be the event of "Three heads show" $\Rightarrow \mathbf{A}=\{(H\ H\ H)\}$

Let B be the event of 'Three tails show" $\Rightarrow \mathbf{B} = \{(T\ T\ T)\}$

$\mathbf{A} \cap \mathbf{B} = \phi \Rightarrow$ A and B are mutually exclusive events.

ii) Let C be the events of "at least one head"

$\Rightarrow C = \{(H\ T\ T)(T\ H\ T)(T\ T\ H)(H\ H\ T)(H\ T\ H)(T\ H\ H)(H\ H\ H)\}$

Let D be the event of "getting exactly two heads" D= $\{(H\ H\ T)(H\ T\ H)(T\ H\ T)\}$

$\mathbf{B} \cap \mathbf{C} \cap \mathbf{D} = \{\} = \phi$ and $\mathbf{B} \cup \mathbf{C} \cup \mathbf{D} = S$

B, C and D are mutually exclusive and exhaustive .

iii) Let E be the event of "at least two heads show"

= E = {(H H H)(H H T)(H T H) (THH)}

$\mathbf{A} \cap \mathbf{E} = \{(H\ H\ H)\} \neq \phi$

A and E are not mutually exclusive .

iv) $\mathbf{A} \cap \mathbf{B} = \phi$, $\mathbf{A} \cup \mathbf{B} \neq S$

$\Rightarrow$ A and B are mutually exclusive but not exhaustive .

v) $\mathbf{A} \cap \mathbf{B} \cap \mathbf{D} = \phi$, $A \cup B \cup C = \{(HHH),(HHT),(HTH),(THH),(TTT)\} \neq S$

$\therefore$ A, B and D are there mutually exclusive but not exhaustive

(* There may be other events also an answer to the above question)

3) Two dice are thrown The events A, B and C are as follows.

A: Getting an even number on the first die.,

B: Getting an odd number on the first die and

C: Getting the sum of the numbers on dice ≤ 5.

Describe the events (i) $\mathbf{A}'$ ii) not B iii) A or B iv) A and B v) A but not C

vi) B or C vii) B and C viii) $\mathbf{A} \cap \mathbf{B}' \cap \mathbf{C}'$

Sol:- Let S be the sample space for the random experiment of throwing two dice.

Then S = $\{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,2)......(6,6)\}$

A = Getting an even number on the first die

$= \{(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)(4,1)(4,2)(4,3)(4,4)(4,5)$
$(4,6)(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$

B: Getting an odd number on the first die=

$= \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(3,1)(3,2)(3,3)$
$(3,4)(3,5)(3,6)(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)\}$

C: Getting the sum of the numbers on the dice ≤ 5

$= \{(1,1)(1,2)(1,3)(1,4)(2,1)(2,2)(2,3)(3,1)(3,2)(4,1)\}$

i) $\mathbf{A}'$ = not A(i.e not in A) = S-A

$= \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(3,1)(3,2)(3,3)(3,4)$
$(3,5)(3,6)(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)\} = B$

ii) not B = S-B

$= \{(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)(4,1)(4,2)(4,3)(4,4)$
$(4,5)(4,6)(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\} = A$

iii) A or B $= \mathbf{A} \cup \mathbf{B}$ = All the elements of A and all the elements of B = S

iv) A and B = $\mathbf{A} \cap \mathbf{B}$ = The common elements of A and B $= \phi$

v) A but not C = $\mathbf{A - C}$ = The elements which are in A but not in C.

$= \{(2,4)(2,5)(2,6)(4,2)(4,3)(4,4)(4,5)(4,6)(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$

vi) B or C = $\mathbf{B} \cup \mathbf{C}$ = The elements which are in both B and C.

$= \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)$
$(3,4)(3,5)(3,6)(4,1)(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)\}$

vii) B and C = $\mathbf{B} \cap \mathbf{C}$ = The common elements is both B and C

$= \{(1,1)(1,2)(1,3)(1,4)(3,1)(3,2)\}$

viii) $\mathbf{A} \cap \mathbf{B}' \cap \mathbf{C}'$ = The common elements of A and not B and C.

$= \mathbf{A} \cap \mathbf{A} \cap \mathbf{C}' \left(\therefore \mathbf{B}' = \mathbf{A}\right)$ from (ii)

$C' = S - C = \{(1,5)(1,6)(2,4)(2,5)(2,6)(3,3)(3,4)(3,5)(3,6)(4,2)(4,3)(4,4)(4,5)(4,6)\}$
$(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$

$\therefore \mathbf{A} \cap \mathbf{B}' \cap \mathbf{C}' = \mathbf{A} \cap \mathbf{C}' = \{(2,4)(2,5)(2,6)(4,2)(4,3)(4,4)(4,5)(4,6)(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}$

Exercise – 14(b)

I . 1) A coin is tossed twice, what is the probability that atleast one tail occurs?

Sol: Let S be the sample space of the random experiment . Then

$$S=\{(H,H)(H,T)(T,H)(T,T)\} \Rightarrow n(S)=n=4$$

Let E be the event of getting at least one tail then

$$E=\{(H,T)(T,H)(T,T)\} \Rightarrow n(E)=m=3$$

$\therefore$ The probability of the event $E=P(E)=\dfrac{n(E)}{n(S)}=\dfrac{m}{n}=\dfrac{3}{4}$

2)If $\frac{2}{11}$ is the probability of an event A then what is the probability of the event 'not A'?

Sol:- Given that $P(\mathbf{A})=\dfrac{2}{11}$ $\Rightarrow$ P(not A) = P (A') = 1- P(A) = 1- $\dfrac{2}{11}=\dfrac{9}{11}$

3) A letter is chosen at radom from the letters word 'ASSASSINATION' . Find the probability that the letter is (i) a vowel (ii) a consonant

Sol: -Word given "ASSASSINATION.

Total. No. of letters = 13

Number of vowels 3(A)+2(I)+1(0) = 6

Number of consonants 4(S) +2(N)+1(T) = 7

(i)$\therefore$ Probability that the letter is a vowel $=\dfrac{n(v)}{n(s)}=\dfrac{6c_1}{13c_1}=\dfrac{6}{13}$

(ii) Probability that the letter is a consonant $=\dfrac{n(c)}{n(s)}=\dfrac{7c_1}{13c_1}=\dfrac{7}{13}$

4) Given $P(\mathbf{A})=\frac{3}{5}$, and $P(\mathbf{B})=\frac{1}{5}$. Find $P(\mathbf{A}\ or\ \mathbf{B})$ if A and B are mutually exclusive events .

Sol:- $\mathbf{P}(\mathbf{A}\ or\ \mathbf{B})=\mathbf{P}(\mathbf{A}\cup\mathbf{B})=\mathbf{P}(\mathbf{A})+\mathbf{P}(\mathbf{B})-\mathbf{P}(\mathbf{A}\cap\mathbf{B})$

$$=\frac{3}{5}+\frac{1}{5}=0\left(\therefore \mathbf{A}\cap\mathbf{B}=\phi \Rightarrow P(\mathbf{A}\cap\mathbf{B})=0\right)=\frac{4}{5}$$

II. 1) Which of the following can not be valid assignment probabilities for out comes of sample space $\mathbf{S}=\{w_1,w_2,w_3,w_4,w_5,w_6,w_7\}$

Assignment	w_1	w_1	w_3	w_4	w_5	w_6	w_7
(a)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(b)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(d)	-0.1	0.2	0.3	0.4	-0.2	0.1	0.3
(e)	$1/14$	$2/14$	$3/14$	$4/14$	$5/14$	$6/14$	$15/14$

Sol:- a) Sum of all probabilities = $0.1+0.01+0.05+0.03+0.01+0.2+0.6=1$.

Hence the assignment (a) is valid.

b) $\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=1$ Hence (b) is also valid.

c) $0.1+0.2+0.3+0.4+0.5+0.6+0.7=2.8\neq 1$ Hence the assignment (c) is not valid

d) As the probability of any event can't be negative , the assignment (d) is not valid.

e) $\frac{1}{14}+\frac{2}{14}+\frac{3}{14}+\frac{4}{14}+\frac{5}{14}+\frac{15}{14}=\frac{30}{14}>1$ and $w_7=\frac{14}{15}>1$ which is not possible . Hence

the assignment (e) is not a valid one.

2) A die in thrown, find the probability of the following events.

i) A prime number will appear

ii) A number greater than or equal to 3 will appear.

iii) A number less than or equal to one will appear.

iv) A number more than 6 will appear

v) A number less than 6 will appear.

Sol:- Let S be the sample space of the random experiment of throwing a die. Then

S = { 1,2,3,4,5,6} $\Rightarrow$ n (S) = 6.

i)Let E_1 be the event of getting a prime number $\Rightarrow E_1=\{2,3,5\}\Rightarrow n(E_1)=3$

$$\therefore\ P(E_1)=\frac{n(E_2)}{n(S)}=\frac{3}{6}=\frac{1}{2}$$

ii) Let E_2 be the event of getting a number great than of equal to 3

$E_2=\{3,4,5,6\}\Rightarrow n(E_2)=4$

$$\therefore\ P(E_2)=\frac{n(E_2)}{n(s)}=\frac{4}{6}=\frac{2}{3}$$

iii)Let E_3 be the event of getting a number less than equal to one

$E_3 = \{1\}\ n(E_3) = 1$

$\therefore\ P(E_3) = \dfrac{n(E_3)}{n(S)} = \dfrac{1}{6}$

iv)Let E_4 be the event of getting a number more than 6

$E_4 = \{\ \} = \phi \Rightarrow n(E_4) = 0$

$\Rightarrow P(E_4) = \dfrac{n(E_4)}{n(S)} = 0$

v) Let E_5 be the event of getting a number less than 6

$E_5 = \{1, 2, 3, 4, 5\} \Rightarrow n(E_5) = 5$

$\therefore P(E_5) = \dfrac{n(E_5)}{n(S)} = \dfrac{5}{6}$

3) A card is selected from a pack of 52 cards.

a) How many points are there in the sample space?

b) Calculate the probability that the card in an ace of spades.

c) Calculate the probability that the card is (i) an ace (ii) black card.

Sol:- a) A pack of cards contains 52 cards . Let S be the sample space

$\therefore$ The sample space $n(S) = 52c_1 = 52$.

b) Let E be the event of getting an ace of spade $n(E) = 1_{C_1} = 1$

$\therefore$ The probability of event getting an ace of spade = $= \dfrac{n(E)}{n(S)} = \dfrac{1}{52}$

c)(i) no. of aces in the pack of card = 4

$\therefore$ The probability that the card drawn is an ace $= \dfrac{4c_1}{52c_1} = \dfrac{4}{52} = \dfrac{1}{13}$

(ii) Number of black cards in the deck = 26

$\therefore$ The probability that selected card to be a black $= \dfrac{26c_1}{52c_1} = \dfrac{26}{52} = \dfrac{1}{2}$

4) A fair coin with 1 marked on one face and 6 in the other and a fair die are both tossed, find the probability that the sum of numbers that turn up is (i) 3 (ii) 12

Sol:- The total number of outcomes (possible) in a coin =2 and the total number of possible outcomes in a die =6.

$\therefore$ The told outcomes when the coin and die tossed together $=2\times6=12$

If S is the sample space $n(S)=12$

(i)Let E_1 the event that the sum of numbers is 3

$$E_1=\{(1,2)\}\Rightarrow n(E)=1$$

$$\therefore P(E_1)=\frac{n(E_1)}{n(S)}=\frac{1}{12}$$

(ii) Let E_2 be the event of the sum of the numbers on the coin and die to be 12

$$E_2=\{(6,6)\}\Rightarrow n(E_2)=1$$

$$\therefore P(E_2)=\frac{n(E_2)}{n(S)}=\frac{1}{12}$$

5) There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a women ?

Sol:- Let E be the required event that woman is selected.

$$n(E)=6c_1=6$$

Total number of persons in the council = 4+6 =10

Let the sample space be ' S' then $n(S)=10c_1=10$

$$\therefore P(E)=\frac{n(E)}{n(S)}=\frac{6}{10}=\frac{3}{5}$$

6) In a lottery, a person chooses six different natural numbers at random from 1 to 20. and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of wining the prize in the game (Hint : order of the numbers is not important)

Answer:- Number of ways of choosing 6 number from 20 (1 to 20) $20c_6$

If S is the sample space and E be the event of winning lottery

$n(S)=20c_6$ and $n(E)=1$

$$\therefore P(E)=\frac{n(E)}{n(S)}=\frac{1}{20c_6}=\frac{6\times5\times4\times3\times1\times141}{20.19.18.17.16.15.141}=\frac{1}{38,760}$$

7) Check whether the following probabilities P(A) and P(B) are consistently defined

i) P(A)=0.5 P(B)=0.7 $P(\mathbf{A}\cap\mathbf{B})=0.6$

ii)P (A) = 0.5 P(B)=0.4 $P(\mathbf{A}\cup\mathbf{B})$

Sol:- i) P(A) = 0.5 P(B) =0.7 $P(\mathbf{A}\cup\mathbf{B}) = 0.6 > P(\mathbf{A})$ this is not possible

Note that $\mathbf{P}(\mathbf{A}\cap\mathbf{B}) \le \mathbf{P}(\mathbf{A}), \le \mathbf{P}(\mathbf{B})$

So the given data is not consistent .

ii) P(A)= 0.5 P(B) =0.4 $\mathbf{P}(\mathbf{A}\cap\mathbf{B}) = 0.8$

$\mathbf{P}(\mathbf{A}\cup\mathbf{B}) = \mathbf{P}(\mathbf{A}) + \mathbf{P}(\mathbf{B}) - \mathbf{P}(\mathbf{A}\cap\mathbf{B}) \Rightarrow 0.8 = 0.5 + 0.4 - \mathbf{P}(\mathbf{A}\cap\mathbf{B})$

$\Rightarrow P(\mathbf{A}\cap\mathbf{B}) = 0.9 - 0.8 = 0.1$

$P(\mathbf{A}\cap\mathbf{B}) = 0.1 \le \mathbf{P}(\mathbf{A}) \,\&\, \mathbf{P}(\mathbf{B})$

$\Rightarrow P(A)$ and $P(B)$ are consistent .

8) Fill in the blanks in the following table

	P(A)	P(B)	$\mathrm{P}(\mathbf{A}\cap\mathbf{B})$	$\mathrm{P}(\mathbf{A}\cup\mathbf{B})$
i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	-
ii)	0.35	-	0.25	0.6
iii)	0.5	0.35	-	0.7

Sol:- i) $\mathbf{P}(\mathbf{A}\cup B) = \mathbf{P}(\mathbf{A}) + \mathbf{P}(\mathbf{B}) - \mathbf{P}(\mathbf{A}\cap\mathbf{B})$

$$= \frac{1}{3} + \frac{1}{5} - \frac{1}{15} \;= \frac{5+3-1}{15} - 7/15$$

ii) $\mathbf{P}(\mathbf{A}\cup\mathbf{B}) = \mathbf{P}(\mathbf{A}) + \mathbf{P}(\mathbf{B}) - \mathbf{P}(\mathbf{A}\cap\mathbf{B}) \Rightarrow \mathbf{P}(\mathbf{B}) = \mathbf{P}(\mathbf{A}\cup\mathbf{B}) - \mathbf{P}(\mathbf{A}) + \mathbf{P}(\mathbf{A}\cap\mathbf{B})$

$$= 0.25 - 0.25 + 0.6 \;= 0.5$$

iii) $\mathbf{P}(\mathbf{A}\cap\mathbf{B}) = \mathbf{P}(\mathrm{A}) + \mathbf{P}(\mathbf{B}) - \mathbf{P}(\mathbf{A}\cup\mathbf{B}) \;= 0.5 + 0.35 - 0.7 \;= 0.85 - 0.7 = 0.15$

9.) If E and F are events such that $P(E) = \frac{1}{4}, P(F) = \frac{1}{2}$ and P (E and F) =$\frac{1}{8}$, Find

(i) P (E or F) ii) P (not E and not F).

Sol:- i) $\mathbf{P}(\mathbf{E}) = \frac{1}{4}$ $\mathbf{P}(\mathbf{F}) = \frac{1}{2}$ $\mathbf{P}(\mathbf{E}\cap\mathbf{F}) = \frac{1}{8}$

$$\mathbf{P}(\mathbf{E}\cup\mathbf{F}) = P(\mathbf{E}) + P(\mathbf{F}) - \mathbf{P}(\mathbf{E}\cap\mathbf{F}) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}$$

ii) P (not E and not F) $= P(\mathbf{E}' \cap \mathbf{F}') = P(E\cup F)' = 1 - \mathbf{P}(\mathbf{E}\cup\mathbf{F}) = 1 - \frac{5}{8} = \frac{3}{8}$

10) Events E and F are such that P(not E or not F) = 0.25 . State whether E and F are mutually exclusive .

Sol:- P (not E or not F) =0.25 $\Rightarrow$ P $\left(E' \cup F'\right) = 0.25$ $\Rightarrow P(\mathbf{E} \cap \mathbf{F})' = 0.25$

$\Rightarrow 1 - P(\mathbf{E} \cap \mathbf{F}) = 0.25$ $\qquad \Rightarrow P(E \cap F) = 1 - 0.25 = 0.75$

$\therefore$ $P(E \cap F) \neq 0$ E and F are not mutually exclusive.

11) A and B are events such that $P(\mathbf{A}) = 0.42$, $P(B) = 0.48$ and $P(\mathbf{A}\ and\ B) = 0.16$. Determine (i) P (not A) ii) P (not B) iii) P (A or B)

Sol:- P(A) =0 0.42 $\qquad$ P(B)= 0.48 $\qquad$ $P(\mathbf{A} \cap \mathbf{B}) = 0.16$

(i) P(n of A) $= P\left(A^{|}\right) = 1 - P(\mathbf{A}) = 1 - 0.42 = 0.58$

(ii) P (n of B) $= P\left(B^{|}\right) = 1 - P(B) = 1 - 0.48 = 0.52$

(iii) P (A or B) $= \mathbf{P}(\mathbf{A} \cup \mathbf{B}) = P(\mathbf{A}) + P(B) - P(\mathbf{A} \cap \mathbf{B}) = 0.42 + 0.48 - 0.16 = 0.9 - 0.16 = 0.74$

III. 1) Three coins are tossed once. Find the probability of getting

i)3 heads ii)2 heads iii) at least 2 heads iv) at most 2 heads

v)no head vi)3 tails vii) exactly two tails viii) no tail ix) at most two tails

Sol:- Total out comes when three coins tossed once $n(S) = 2^3 = 8$

$$S = \{(H\ H\ H)(H\ H\ T)(T\ H\ H)(H\ T\ T)(T\ H\ T)(T\ T\ H)(T\ T\ T)\}$$

i)Let A bet the events of getting 3 heads

$$\therefore P(\mathbf{A}) = \frac{n(\mathbf{A})}{n(S)} = \frac{1}{8}$$

i) Let B be the event of getting 2 heads $\Rightarrow n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{8}$$

iii)Let C be the event of getting at least 2 heads $\Rightarrow n(C) = 4$

(i.e) getting 2 heads + getting 3 heads)

$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

iv)Let D be the event of getting almost2 heads $\Rightarrow n(D) = 7$

(i.e getting 2 heads+1 heads+0 heads = 3+3+1=7)

$$P(D)=\frac{n(D)}{n(S)}=\frac{7}{8}$$

v) Let E be event of getting no head $\Rightarrow n(E)=1$

(i e getting all tails = 1)

$$\therefore \mathbf{P}(E)=\frac{n(E)}{n(S)}=\frac{1}{8}$$

vi) Let F be the event of getting 3 tails $\Rightarrow n(F)=1$

$$\therefore P(F)=\frac{n(F)}{n(S)}=\frac{1}{8}$$

vii) Let G be the event of getting exactly two tails $\Rightarrow n(G)=3$

$$P(G)=\frac{n(G)}{n(S)}=\frac{3}{8}$$

viii) Let H be the event of getting no tails $\Rightarrow n(H)=1$

$$\therefore P(H)=\frac{n(H)}{n(S)}=\frac{1}{8}$$

ix) Let I be the event of getting at most two tails $\Rightarrow n(I)=7$

(i e getting 2 tails + 1 tails + o tails = 3+2+1=7)

$$P(I)=\frac{n(I)}{n(S)}=\frac{7}{8}$$

2) In class XI of a school 40% of the students study Mathematics and 30% study Biology 10% of the class study both Mathematics and Biology . If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology .

Sol:- Let the students study mathematics de noted by M and biology by B .

Given that $P(M)=40\%$, $P(B)=30\%$

$$P(M \text{ and } B)=P(M\cap B)=10\%$$

Now $P(M \text{ or } B)=P(M\cup B)=P(M)+P(B)-P(M\cap B)$

$$=\frac{40}{100}+\frac{30}{100}-\frac{16}{100}=\frac{60}{100}=60\%$$

3) In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing the both?

Sol:- Given that P(I) =0.8, P(II) = 0.7

P (at least one of I and II) = P(I $\cup$ II) =0.95

P (I$\cap$II)= ?

Now $P(I \cup II) = P(I) + P(II) - P(I \cap II)$

$0.95 = 0.8 + 0.7 - P(I \cap II) \Rightarrow P(I \cap II) = 1.5 - 0.95 \quad = 0.55$

4) The probability that a student will pass the find examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1 .It the probability of passing the English examination is 0.75 . What is the probability of passing the Hindi examination.

Sol:- Let E and H he the events of passing the find examination in English and Hindi respectively. Given that

$P(E' \cap H) = 0.5$

$P(E' \cap H') = 0.1$

$P(E) = 0.75$ Then

$P(H) = ?$

$P(E' \cap H') = P(E \cup H)' = 1 - P(E \cup H) \quad = 1 - P(E) - P(H) + P(E \cap H)$

$0.1 = 1 - 0.75 - P(H) + 0.5 \Rightarrow P(H) = 1 - 0.75 + 0.5 - 01 = 0.65$

5) In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one these students is selected at random, find the probability that

i) **The student opted for NCC or NSS**

ii) The student has opted neither NCS nor NSS

iii) The student has opted NSS but not NCC.

Sol:- Let A and B be the two events of students who opted NCC and NSS respectively given that $n(\mathbf{A}) = 30, \quad n(\mathbf{B}) = 32, n(\mathbf{A} \cap \mathbf{B}) = 24$

$n(S) = Total\ students = 60$

$\therefore P(A)=\frac{n(A)}{n(S)}=\frac{30}{60}$, $P(B)=\frac{n(B)}{n(S)}=\frac{32}{60}$, $P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{24}{60}$

i) Now the probability of the student opted for NCC or NSS $=P(\mathbf{A}\cup\mathbf{B})$

$$P(\mathbf{A}\cup\mathbf{B})=P(A)+P(B)-P(A\cap B)$$

$$=\frac{30}{60}+\frac{32}{60}-\frac{24}{60}=\frac{38}{60}=\frac{19}{30}$$

ii) The probability of the student who opted neither NCC nor NSS = $P\left(A^{'}\cap B^{'}\right)$

$$P\left(A^{'}\cap B^{'}\right)=P(A\cup B)$$

$$=1-P(A\cup B)=1-\frac{19}{30}=\frac{11}{30}$$

iii) Probability of the student who opted NSS but not NCC

$$=P(B)-P(A\cap B)=\frac{32}{60}-\frac{24}{60}=\frac{8}{60}=\frac{2}{15}$$

6) A fain coin is tossed four times, and a person win Rs1 for each head and lose Rs 1.50 for each tail that turns up . Form the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Sol:- Total number of out comes when a coin tossed four times $=2^4=16$

For these 16 cases sample space can be written as follows.

Sample Space		Amount
(H H H H)	-	$1+1+1+1=4.00$
(H H H T)	-	$1+1+1-1.50=1.50$
(H H T H)	-	$1+1-1.50+1=1.50$
(H T H H)	-	$1-1.50+1+1=1.50$
(T H H H)	-	$-1.50+1+1+1=1.50$
(H H T T)	-	$1+1-1.50-1.50=-1.00$
(H T H T)	-	$1-1.0+1-1.50=-1.00$
(T H H T)	-	$-1.50+1+1-1.50=-1.00$

(H T T H) - $1-1.50-1.50=-1.00$

(T H T H) - $-1.50+1-1.50+1=-1.00$

(T T H H) - $-1.50-1.50+1+1=-1.00$

(H T T T) - $-.50-1.50+1-1.50=-3.50$

(T H T T) - $-1.50+1-1.50-1.50=-3.50$

(T T H T) - $-1.50-1.50+1-1.50=-3.50$

(T T TH) - $-1.50-1.50-1.50+1=-3.80$

(T T T T) - $-1.50-1.50-1.50-1.50=-6.00$

The amounts 4.00,1.50,-1.00,-3.50,-6.00 occurs 1 time, 4 times, 6 times, 4 times and 1 time respectively 4.00- 1 time, 1.50 - 4 times -1.00- 6 times – 3.50 – 4 times – 600 -1 time

$\therefore$ 5 different types of amounts of money that we can have.

Probability of winning Rs 4.0 $=\frac{1}{16}$

Probability of wining Rs 1.50= $\frac{4}{16}=\frac{1}{4}$

Probability of wining Rs (-1.00) = $\frac{6}{16}=\frac{3}{8}$

(actually loosing)

Probability of wining Rs – 3.50 $=\frac{4}{16}=\frac{1}{4}$

Probability of wining Rs – 6.00 $=\frac{1}{16}$

Exercise 14 (c)

II. 1) A box contains 10 red marbles 20 blue marbles and 30 green marbles, 5 marbles are drawn from the box, what in the probability that

i)all will be blue ? ii)At least one will be green?

Sol: Red marbles = 10

Blue marbles = 20

Green marbles = 30

Total marbles = 60

Let S be the sample space of drawing 5 marbles from the box = $n(S)=60_{C_5}$

i)Let E be the event of getting all 5 marbles from the box = $n(E)=20c_5$

$$\therefore P(E)=\frac{n(E)}{n(S)}=\frac{20c_5}{60c_5}=\frac{20.19.18.17.18}{60.59.58.57.56}=\frac{34}{11977}$$

ii) Probability of selecting 5 marbles with at least one will be green

$=1-$ Probability of none of them green

$$=1-\frac{30_{C_5}}{60_{C_5}}$$

$$=1-\frac{351}{13452}=\frac{13101}{13452}.$$

2) 4 cards are drawn from a well- shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?

Sol:- Let S be the sample space of the random experiment. Then $n(S)=52_{c_4}$

Then E be the event of obtaining 3 diamonds and one spade.

$$n(E)=13_{C3}\times 13_{C_1}$$

$$\therefore P(E)=\frac{n(E)}{n(S)}=\frac{13_{C_3}.13_{C_1}}{52_{C_4}}$$

3) A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. if the die is rolled once, determine (i) $P(2)$ ii) $P(1\ or\ 3)$ iii) $P(not\ 3)$

Sol:- Let S be the sample space then $n(S)=6$

i)Let A be the event of getting 2 then $n(\mathbf{A})=3$

$$\therefore P(\mathbf{A})=\frac{n(A)}{n(S)}=\frac{3}{6}=\frac{1}{2}$$

ii) Let B be the event of getting a number 1 or 3 then $n(B)=2+1=3$

$\therefore$ The Probability of getting 1 or 3 $\therefore P(C)=\frac{n(C)}{n(S)}=\frac{5}{6}$

iii) Let 'C" be the event of " not getting 3" then $n(C)=5$

$$\therefore P(C)=\frac{n(C)}{n(S)}=\frac{5}{6}$$

4) In a certain lottery 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy

a) One ticket b) Two tickets c) Ten tickets

Sol:- Total tickets = 10,000 and Total prizes = 10

Total tickets with no prizes = 9990

a) The probability of not getting a prize when one ticket is bought

$$=\frac{9990_{C_1}}{10000_{C_1}}=\frac{9990}{10000}=\frac{999}{1000}$$

b) The probability of not getting a prize when two tickets are bought $=\frac{9990_{C_2}}{10000_{C2}}$

c) The probability of not getting a prize when you bought 10 tickets= $\frac{9990_{C_{10}}}{10000_{C_{10}}}$

5) A and B are two events $P(A)=0.54$ $P(B)=0.69$ and $P(A\cap B)=0.35$, then find

i) P(A $\cup$ B)ii) $P\left(\mathbf{A}^{'}\cap\mathbf{B}^{'}\right)$iii) $P\left(\mathbf{A}\cap\mathbf{B}^{'}\right)$iv) $P\left(\mathbf{B}\cap\mathbf{A}^{'}\right)$

Sol:- i) $P(A\cup B)=P(A)+P(3)-P(A\cap B)=0.54+0.69-0.35=0.88$

ii) $P\left(A^{'}\cap B^{'}\right)=P(A\cup B)^{'}$

$=1-P(A\cup B)$ $=1-0.88$ $=0.12$

iii) $P\left(A\cap B^{'}\right)=P(A)-P(A\cap B)$

$=0.54-0.35$ $=0.19$

iv) $P\left(B\cap A^{'}\right)=P(B)-P(A\cap B)$

$=0.69-0.35$ $=0.34$

6) Three letters are dictated to three persons and an envelope in addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Sol:- Support that L_1,L_2,L_3 be the letter and E_1,E_2,E_3 be their corresponding envelopes respectively.

The sample space $\underline{L_1\,E_1\quad L_2\,E_2\quad L_3\,E_3}$

$\underline{L_2\,E_2\quad L_1\,E_3\quad L_3\,E_1}$

$\underline{L_3\,E_3\quad L_1\,E_2\quad L_2\,E_1}$

$\underline{L_1\,E_1\quad L_2\,E_3\quad L_3\,E_2}$

$L_1 E_2 \quad L_2 E_3 \quad L_3 E_1$

$L_1 E_3 \quad L_2 E_1 \quad L_3 E_2$

$n(S) = 6$ Let 'A' be the event of inserting at least one letter is in its proper envelope

Than $n(A) = 4 \therefore P(A) = \dfrac{n(A)}{n(S)} = \dfrac{4}{6} = \dfrac{2}{3}$.

III. 1) Out of 100 students, two sections of 40 and 60 are formed . If you and your friend are among the 100 students, what is the probability, that (a) you both enter the same section (b) you both enter the different section ?

Sol:- Let section A consists 40, section B consists 60 students.

a)When you both enter the same section these are two cases

Case:-i) both of you enter in to section A.

Case :ii) both of you enter in to section B.

Case:- i) Suppose both of you enter in to section 'A'

No .of ways of selecting 40 students out of 100 = 100_{C40}

38 students (including both of you make 40) can be selected out of 98 students $= 98_{C_{38}}$

$\therefore P$ (both of you are in section A) = $\dfrac{98_{C_{38}}}{100_{C\,40}}$

$$= \frac{98!}{60!38!} \times \frac{40!60!}{100!}$$

$$= \frac{98!40.9.38!.60}{60!38!100.99.98!} = \frac{40.39}{100.99} = \frac{2.13}{5.33} = \frac{26}{165.}$$

Case ii) Both of you are in section 'B' then 60 students can be selected out of 100 = $100_{C_{60}}$

ways 58 (including both of you make 60) students can be selected from 98

students in $98_{C_{58}}$ ways.

$\therefore$ The probability that to enter both of you in Section B= P(B) $\dfrac{98_{C_{58}}}{100_{C_{60}}} = \dfrac{59}{165}$

$$\therefore P(A \cup B) = P(A) + P(B) = \frac{26}{165} + \frac{59}{165} = \frac{85}{165} = \frac{17}{33}$$

b) The probability that both of you enter the different sections

$= 1 -$ Probability of you both enter the some sections

$$=1-\frac{17}{33}=\frac{16}{33}$$

2) From the employees of a company, 5 persons are selected to represent them in the maintaining committee of the company . Particulars of 5 persona are is follows.

S NO	NAME	SEX	AGE IN YEARS
1	HARISH	M	30
2	ROHAN	M	33
3	SHEETHAL	F	46
4	ALIS	F	28
5	SALIM	M	41

A Person is selected at random from this group to act as spoke person.

What in the probability that the spoken person will be either male or over 35 year?

Sol:- given that Total No. of Persons = 5, Males = 3

No. of persons over 35 years of age = 2

Let A be the event in which the selected spokes person be a male .

and B be the event of selected spoken person be a person over 35 year of age

$$\therefore P(A)=\frac{3_{C_1}}{5_{C_1}}=\frac{3}{5}, \quad P(B)=\frac{2_{C_1}}{5_{C_1}}=\frac{2}{5}$$

And $P(A\cap B)=\frac{1_{C_1}}{5_{C_1}}=\frac{1}{5}$

$\therefore$ Required probability $P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{3}{5}+\frac{2}{5}-\frac{1}{5}=\frac{4}{5}$

3) If 4 – digit numbers greater than 5000 are randomly formed from the digits 0,1,3,5, and 7, what is the probability of forming a number divisible 5 when i) the digits are repeated ii) the repetition is not allowed?

Sol:- i) When the digits are repeated (repetition ins allowed)

X			

The thousands place should be filled with 5 or 7 so as to from 4- digit numbers greater than 5000.

$\therefore$ Number of ways filling 1000's place = 2

Number of ways of filling remains places = $5\times5\times5$

Number of 4 digit number greater then 5000 = $2\times5\times5\times5^{-}=250-1$

(Out of these one number 5,000 is to be deleted) = 249

X			

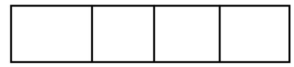
A number in divisible by 5 if it has 0 or 5 in unit place . 0,5

7,5

$\therefore$ Number of 4 digit numbers that are divisible by $5 = 2\times5\times5\times2-1=99$

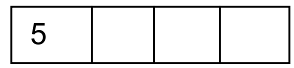
ii) When the repetition is not allowed

the thousands place can be filled (by (5 or 7) in 2 ways

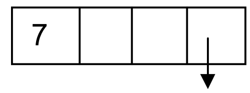


The remaining 3 places can be filled by the remaining 4 digits in $4P_3$ ways

$\therefore$ Number of 4 digit number $=2\times4P_3=2\times4\times3\times2=48$



Number of 4 digit numbers divisible by 5 starting with $5=1\times1\times3\times2=6$



Number of 4 digit numbers divisible by 5 starting with $5=1\times2\times3\times2=12$

$\therefore$ Number of 4 digit numbers greater than 5000 ad divisible by 5 $=6+12=18$

Now the probability of 4 digit number divisible by 5 when the digits are not repeated $=\frac{18}{48}=\frac{3}{8}$

4) The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suit case ?

Sol:- The number of ways of selecting 4 (no repetition) different digit (with 0 to 9) = 10_{C_4}

This each combination (of 4 digits) have 4! Arrangements

$\therefore$ we have $4!\times10_{C_4}$ numbers without repetitions out of which for one and only one arrangement with its right sequence to open

$\therefore$ The probability to open the suit case $=\frac{1}{4!10_{C_4}}=\frac{1}{5040}$

<u>Multiple Choice Questions(Key)</u>

1. $P(\mathbf{A}\cap\mathbf{B})=P(\mathbf{A})+P(B)-\mathbf{P}(\mathbf{A}\cup\mathbf{B})=1.5-P(\mathbf{A}\cup\mathbf{B}),\ 0\le P(\mathbf{A}\cup\mathbf{B})\le1\Rightarrow\quad P(\mathbf{A}\cup\mathbf{B})\ge0.5$

2. No. of days left in a leap year with Sunday = (Sat, <u>Sun</u>) (<u>Sun</u> , Mon)

$\therefore$ Probability that a leap year contains 53 Sundays $=\frac{2}{7}$

3. $n(S)=6^2=36,\ n(E)=6\ \therefore P(E)=\frac{1}{6}$

4. $n(S) = 6^2 = 36,\ n(E) = 6\ P(E) = \frac{1}{6}$

5. $P(\mathbf{A} \cap \mathbf{B}) = 0$ for mutually exclusive events which implies $P(\mathbf{A} \cup \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B})$

6. $E = \{2,3,5,7,11,13,17,19\} \Rightarrow P(E) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$

7. $P(\mathbf{A}' \cap \mathbf{B}') = 1 - P(\mathbf{A} \cup \mathbf{B}) = 1 - (0.5 + 0.3) = 0.2$

8. $P(\mathbf{A}') = \frac{26_{C_1}}{52_{C_1}} = \frac{26}{52} = \frac{1}{2}$

9. For impossible event E, P(E)= 0

For certain event E, P(E) = 1

For any event E, $P(E) + P(\overline{E}) = 1$

For any event E, $0 \le P(E) \le 1$

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1) 2	2) 2	3) 4	4) 1	5) 1	6) 3	7) 4	8) 3	9) 2