HAND BOOK

FOR

INTERMEDIATE FIRST YEAR

PHYSICS

BOARD OF INTERMEDIATE EDUCATION

HANDBOOK PREPARED BY

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UNIT -1

UNITS AND MEASUREMENTS

I. MULTIPLE CHOICE QUESTIONS:

1) Pascal is the unit of [2]

(1) power (2) pressure (3) potential energy (4) pressure gradient

Ans : $Pressure = \frac{Force}{Area} = \frac{F}{A} = \frac{N}{m^2}$ $or\ Pasal$

2) When the number 0.046508 is reduced to four significant figures then it becomes [1]

(1) 0.04651 (2) 4650 (3) 4.651 (4) 4.650

Ans : Preceding is raised by 1 if insignificant digit to be dropped is more than 5

3) Which of the following physical quantity has no dimensions? [3]

(1) Angular Velocity (2) Angular Acceleration (3) Angular Displacement (4) Stress

Ans : $Angular\ displacement = \frac{Arc}{Radius} = \frac{L}{L} = [M^0L^0T^0]$

4) Mention same dimensions in the dimensional formula of torque and angular momentum [3]

(1) mass, time (2) time, length (3) mass, length (4) mass

Ans : $Torque = Force\ \times distance = (MLT^{-2})L = [ML^2T^{-2}]$

$Angular\ momentum = Moment\ of\ inertia\ \times Angular\ velocity(ML^2)T^{-1} = [ML^2T^{-1}]$

5) The velocity of an object varies with time as $V = At^2 + Bt + C$ then the dimensions of C are [1]

(1) LT^{-1} (2) LT^{-2} (3) LT^{-3} (4) LT^{-4}

Ans : According to principle of homogeneity

V=C => C = $[LT^{-1}]$

6) Which of the following equation is dimensionally correct? [1]

(1) $\frac{1}{2}mv^2 = mgh$ (2) $V = ut + at$ (3) $s = ut + at^3$ (4) $t = s + uv$

Ans : $The\ dimesnions\ of\ LHS\ are: \frac{1}{2}mv^2 = (M)(LT^{-1})^2 = [ML^2T^{-2}]$

$The\ dimesnions\ of\ RHS\ are: mgh = (M)(LT^{-2})(L) = [ML^2T^{-2}]$

7) The velocity (V) of sound in air, pressure(P) and density (d) of air are related as $V \propto P^x d^y$. The value of x and y are [3]

(1) 1, 1/2 (2) -1/2, 1/2 (3)1/2, -1/2 (4)1/2, ½

Ans : $V \propto P^x d^y$

$$V = K\ P^x d^y$$
$$[LT^{-1}] = [ML^{-1}T^{-2}]^x [ML^{-3}]^y$$
$$LT^{-1} = M^x L^{-x} T^{-2x} M^y L^{-3y}$$

$$x + y = 0, -x - 3y = 1, -2x = -1$$
$$x = \frac{1}{2} \text{ and } y = -\frac{1}{2}$$

Note : In text book read density (4) as density (d)

8) The velocity of body is given by $V = At^2 + Bt + C$. If V and t are expressed in SI Units, then unit of A is **[3]**

(1) m/s (2) $\frac{m}{s^2}$ (3) $\frac{m}{s^3}$ (4) ms

Ans : According to principle of homogeneity

$$V = At^2 \Rightarrow A = \frac{V}{t^2} = \frac{LT^{-1}}{T^2} = LT^{-3} \Rightarrow A = m/s^3$$

9) The dimensional formula for product of two physical quantities P and Q is $[ML^2T^{-2}]$ and dimensional formula of $\frac{P}{Q}$ is $[MT^{-2}]$ then P is **[1]**

(1) Force (2) displacement (3) acceleration (4) velocity

Ans : Given $PQ = ML^2T^{-2}$ and $\frac{P}{Q} = MT^{-2}$

$$PQ \times \frac{P}{Q} = [ML^2T^{-2}][MT^{-2}]$$
$$P^2 = M^2L^2T^{-4}$$
$$P = \sqrt{M^2L^2T^{-4}} = MLT^{-2} = Force$$

10) E, M, L and G denote energy, mass, angular momentum and gravitational constant respectively. Then the dimensions of $EL^2M^{-5}G^{-2}$ is **[1]**

(1) angle (2) length (3) mass (4) time

Ans : $EL^2M^{-5}G^{-2} = [ML^2T^{-2}][ML^2T^{-1}]^2M^{-5}[M^{-1}L^3T^{-2}]^{-2}$

$$= [ML^2T^{-2}][M^2L^4T^{-2}]M^{-5}[M^2L^{-6}T^4]$$
$$= M^{1+2+2-5}L^{2+4-6}T^{-2-2+4}$$
$$= M^0L^0T^0 = Angle$$

II. FILL IN THE BLANKS:

1. **Plane angle is the ratio of length of arc to ________**
   Ans: Radius
2. **The unit of plane angle is ________**
   Ans: Radian
3. **The SI Unit of Luminous intensity of light is ________**
   Ans: Candela
4. **The dimensional formula for the coefficient of viscosity is ______**
   Ans: $[ML^{-1}T^{-1}]$

$$Coefficent\ of\ viscosity = \frac{Force}{Area \times velocity\ graident} = \frac{MLT^{-2}}{L^2 \times T^{-1}} = [ML^{-1}T^{-1}]$$

5. **The number of significant figures in $\mathbf{0.02308}$ is ________**
   Ans: 4
   If the number is less than one the zeroes on the right of the decimal point but left to the first non-zero digit are non-significant.
   0.02308, the first two zeroes are non-significant
6. **If F, M, T are the force, mass and time then $[FM^{-1}T^{2}]$denotes ________**
   Ans: Length
   $FM^{-1}T^{2} = [MLT^{-2}]M^{-1}T^{2} = M^{1-1}LT^{-2+2} = M^{0}L\,T^{0} = Length$
7. **A number $\mathbf{2.735}$ is rounded off to three significant figure is ________**
   Ans: 2.74
   If the insignificant digit 5 is preceded by an odd digit 3, then the preceding digit is raised by one.
8. **Plank's constant and _______ have same dimensions.**
   Ans: Angular momentum

$$Planks\ constant = \frac{Energy}{Frquency} = \frac{ML^{2}T^{-2}}{T^{-1}} = [ML^{2}T^{-1}]$$

$$Angular\ momentum = Moment\ of\ inertia\ \times Angular\ velocity = [ML^{2}]T^{-1} = [ML^{2}T^{-1}]$$

9. **The dimensional formula of angular impulse is ________**
   Ans: $[ML^{2}T^{-1}]$

$$Angular\ impulse = Torque\ \times time\ = [ML^{2}T^{-2}]T = ML^{2}T^{-1}$$

10. **A force is represented $by\ F = Ax^{2} + Bt^{2}$, where $x = distance$ and t= $time$ then the dimensions of A is ______**
    Ans: $ML^{-1}T^{-2}$
    According to principle of homogeneity

$$F = Ax^{2} \implies A = \frac{F}{x^{2}} = \frac{MLT^{-2}}{L^{2}} = [ML^{-1}T^{-2}]$$

III.<u>ONE WORD ANSWER QUESTIONS :</u>

1. **What is fundamental physical quantity?**

Ans: A physical quantity which is independent of any other physical quantity is called fundamental physical quantity

2. **What is derived physical quantity?**

Ans: The physical quantity which can be expressed in terms of the fundamental quantities are called derived physical quantities

3. **What is the SI unit of solid angle?**

Ans: SI Unit of solid angle is Steradian

4. **Write any one physical constant that is having dimensions.**

Ans: Universal gravitational constant, Plank constant, Boltzmann constant, Universal gas constant

5. **Which physical quantity has negative dimensions in mass?**

Ans: Universal gravitational constant

6. **Which physical quantity is represented by $\sqrt{gR}$, where R is radius of the Earth?**

Ans: Orbital velocity (or) velocity

7. **Which physical quantity is represented by PV, where P is pressure and V is volume?**

Ans: Work

8. **Name at least one physical quantity whose SI unit is Nm^{-2}**

Ans: Pressure, Stress, Modulus of Elasticity, Young's modulus, Modulus of rigidity, Bulk modulus

IV. **VERY SHORT ANSWER QUESTIONS:**

1. **What are significant figures and what do they represent when reporting the result of a measurement?**

Ans: **Significant figures:**

The digits of a number that are definitely known plus one more digit that is estimated are called significant digits or significant figures

Ex: Time period of a simple pendulum is 1.62,

The digits 1 and 6 are reliable while the digit 2 is uncertain, The measured value has three significant figures.

2. **Distinguish between fundamental units and derived units.**

Ans: **Fundamental Units:** Units of fundamental quantities are called fundamental units Fundamental units can neither be derived from one another nor can they be resolved into other units.

Derived Units: Units of derived quantities are called derived units

3. **What are the two fundamental physical quantities used to measure angle?**

Ans: 1) Plane Angle 2) Solid Angle

4. **Why do we have different units for the same physical quantity?**

Ans: We have different systems like F.P.S system, C.G.S system, M.K.S system and S.I system, hence we have different units for the same physical quantity

(or)

The order of magnitude differs significantly depending on the context, so different units are used for the same physical quantity.

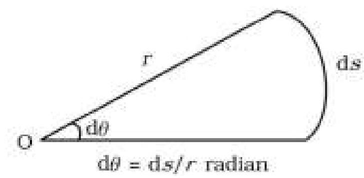
Ex : Distance : (i) Interatomic distance : Angstrom Units (ii) Interstellar distances : light year (iii) Daily distance measurements **:** m and km

5. Define plane angle.

Ans: Plane angle is the ratio of length of arc to radius

Plane angle = length of arc / radius

$$d\theta = \frac{ds}{r}\ radian$$



6. What is meant by dimension of a physical quantity?

Ans: The dimensions of a physical quantity are defined as the powers to which fundamental quantities are raised to represent it.

$Force = [MLT^{-2}]$, The Force has 1 dimension in mass M, 1dimension in length L, and -2 dimensions in Time (T).

7. What is meant by dimensional formula?

Ans: The physical quantity is expressed with proper dimensions in fundamental quantities then that expression is called its dimensional formula.

Ex: Dimension formula of Volume is[$M^0L^{-3}T^0$]

Dimension formula of Density is $[M^1L^3T^0]$

8. Write the physical quantities that are having Jm^{-1} and Jm^{-2} as their units.

Ans: Force has unit (Jm^{-1}) => $ML^2T^{-2} \times L^{-1} = [MLT^{-2}]$

Surface tension has unit (Jm^{-2}) => $ML^2T^{-2} \times L^{-2} = [MT^{-2}]$

V. <u>SHORT ANSWER QUESTIONS:</u>

1. In a system of units, the unit of force is $100N$, unit of length is $10m$ and the unit of time is $100s$.What is the unit of mass in this system?

Ans: $Force\ (F) = 100N$

$$Length\ (L) = 10m\ \ Time\ (T) = 100s\ \ mass\ (m) = \frac{F}{a} = \frac{F}{LT^{-2}}$$

$$m = \frac{FT^2}{L} = \frac{100\times(100)^2}{10}$$

$$m = 10^5 kg$$

2. **$1\ calorie = 4.2J$ where $1\ J = 1\ kgm^2s^{-2}$. Suppose we employ a system of units in which the unit of mass is $\propto kg$, the unit of length is βm, and the unit of time is γs. Show that a calorie has a magnitude $4.2 \propto^{-1} \beta^{-2}\gamma^2$ in the terms of new units.**

Ans: $1\ calorie = 4.2J$, where $1J = 1kg.m^2.s^{-2}$

SI	New System
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 1\ kg$	$M_2 = \alpha\ kg$
$L_1 = 1\ m$	$L_2 = \beta\ m$
$T_1 = 1\ s$	$T_2 = \gamma\ s$

$$Dimensional\ formula\ of\ Energy = ML^2\ T^{-2}$$

$$n_1U_1 = n_2U_2$$

$$\text{Now}\quad n_2 = 4.2\left(\frac{1kg}{\alpha kg}\right)\left(\frac{1m}{\beta m}\right)^2\left(\frac{1s}{\gamma s}\right)^{-2}$$

$$n_2 = 4.2\ \alpha^{-1}\beta^{-2}\gamma^2$$

3. **The velocity of a body is given by $V = At^2 + Bt + C$. If V and t are expressed in SI units, what are the units and dimensions of A, B and C?**

Ans:

Given $V = At^2 + Bt + C$

According to Principle of homogeneity

1. $V = At^2 \Rightarrow A = \frac{V}{t^2} = \frac{LT^{-1}}{T^2} = LT^{-3} \Rightarrow A = ms^{-3}$
2. $V = Bt \Rightarrow B = \frac{V}{t} = \frac{LT^{-1}}{T} = LT^{-2} \Rightarrow B = ms^{-2}$
3. $V = C \Rightarrow C = LT^{-1} \Rightarrow C = ms^{-1}$

4. What are fundamental quantities? State their units in SI system.

Ans:

S.No	Fundamental Quantity	SI Unit	Symbol
1	Length	meter	m
2	Mass	Kilogram	kg
3	Time	Second	s
4	Electric Current	Ampere	A
5	Thermodynamic Temperature	Kelvin	K
6	Amount of substance	mole	mol
7	Luminous intensity	Candela	cd

5. Explain the uses of Dimensional methods with examples.

Ans 1. To convert one system of units to another:

$1\,J = n\,ergs,$

Joule is the unit of energy,

Dimensional formula of Energy = $M L^2 T^{-2}$

$$n = \frac{1J}{ergs} \Rightarrow 1J = n(ergs)$$

We have $n_2 [M_2^a L_2^b T_2^c] = n_1 [M_1^a L_1^b T_1^c]$ ($\therefore$ a = 1, b = 2, c = -2) ($n_2 = n$, $n_1 = 1$)

$$n = \left(\frac{M_1}{M_2}\right)\left(\frac{L_1}{L_2}\right)^2\left(\frac{T_1}{T_2}\right)^{-2}$$

As units of M, L and T are kg, m and s in SI and g, cm, s in CGS system.

$$n = \left(\frac{1kg}{1g}\right)\left(\frac{1m}{1cm}\right)^2\left(\frac{1s}{1s}\right)^{-2}$$

$$n = \left(\frac{1000g}{1g}\right)\left(\frac{100cm}{1cm}\right)^2\left(\frac{1s}{1s}\right)^{-2}$$

$$n = 1000 \times 100 \times 100 = 10^7$$

$$1\,J = 10^7\ ergs$$

2. To check the correctness of an equation:

Let us consider $\frac{1}{2}\ mv^2\ =\ mgh$

The dimensions of LHS are $(M)(LT^{-1})^2 = ML^2\ T^{-2}$

The dimensions of RHS are $(M)(LT^{-2})(L) = ML^2\ T^{-2}$

The dimensions of LHS and RHS are the same and hence the equation is dimensionally correct.

3. To derive the relationship between different physical quantities

The dependence of time period T on the quantities l, g and m as a product may be written as

$$T\ =\ K\ l^x g^y m^z$$

By dimensions on both sides we have

$(M^0 L^0\ T^1)\ \ =\ \ K\ (L)^x (LT^{-2})^y (M)^z$

$(M^0 L^0\ T^1)\ \ =\ \ K\ L^{x+y} T^{-2y} M^z$

$x + y = 0, -2y = 1, z = 0$

So, $x = \frac{1}{2}, y\ =\ \frac{-1}{2}\ and\ z = 0$, then

$T\ =\ K\ l^{1/2} g^{-1/2} m^0\ =\ K\ l^{1/2} g^{-1/2}$

$T\ =\ K\sqrt{\frac{l}{g}}$, actually K=$2\pi$. Hence $T\ =\ 2\pi\sqrt{\frac{l}{g}}$

6. What are the rules for arithmetic operations with significant figures?

Ans: 1. In multiplication or division, the final result should retain as many significant figures as there are in the original numbers with the least significant figures

Example: $Density\ =\ \frac{4.237g}{2.51cm^3} = 1.688\ =\ 1.69\ g/cm^3$

2. In addition or subtraction, the final result should retain as many decimal places are there in the number with the least decimal places

Example: The sum of numbers 436.32g, 227.2g and 0.301g by arithmetic addition is 663.821g. Final result should be rounded off to 663.8g

7. What is the rounding off a number and what are the rules to be followed in it?

Ans: 1. Preceding digit is raised by 1 if the insignificant digit to be dropped is more than 5, and is left unchanged if latter is less than 5.

Example: A number 2.746 rounded off to three significant figures is 2.75, while the number 2.743 would become 2.74

2. If the Preceding digit is even, the insignificant digit is simply dropped and If it is odd, Preceding digit is raised by one

Example: The number 2.745 rounded off to three significant figures is 2.74, while the number 2.735 would also become 2.74

VI. PROBLEMS:

1. If the velocity of light c, plank's constant h and the gravitational constant G are taken as fundamental quantities, then express length, mass and time in terms of dimensions of these quantities (Ans- $\sqrt{hc/G}$, $\sqrt{Gh/c^3}$, $\sqrt{Gh/c^5}$.

Ans: Dimensional Formula for

$$c = L\,T^{-1} \quad h = M\,L^2\,T^{-1} \quad G = M^{-1}L^3\,T^{-2} \quad i)\ Let\ M = c^x h^y G^z$$

$$(M^1 L^0\,T^0) = (L\,T^{-1})^x (M\,L^2\,T^{-1})^y (M^{-1}L^3\,T^{-2})^z$$

$$(M^1 L^0\,T^0) = M^{y-z} L^{x+2y+3z} T^{-x-y-2z}$$

Comparing powers,

$$x + 2y + 3z = 0, -x - y - 2z = 0, y - z = 1$$

Solving we get, $x = \frac{1}{2}, y = \frac{1}{2}\ and\ z = \frac{-1}{2}$,

$$M = \sqrt{\frac{hc}{G}} = c^{1/2} h^{1/2} G^{-1/2}$$

(ii) Let $L = c^x h^y G^z$

$$(M^0 L^1\,T^0) = (L\,T^{-1})^x (M\,L^2\,T^{-1})^y (M^{-1}L^3\,T^{-2})^z$$

$$(M^0 L^1\,T^0) = M^{y-z} L^{x+2y+3z} T^{-x-y-2z}$$

Comparing powers,

$y - z = 0, x + 2y + 3z = 1, -x - y - 2z = 0$

Solving we get, $x = \frac{-3}{2}, y = \frac{1}{2}\ and\ z = \frac{1}{2}$,

$$L = \sqrt{\frac{hG}{c^3}} = c^{-3/2} h^{1/2} G^{1/2}$$

(iii.) Let $T = c^x h^y G^z$

$(M^0 L^0\,T^1) = (L\,T^{-1})^x (M\,L^2\,T^{-1})^y (M^{-1}L^3\,T^{-2})^z$

$(M^0 L^0\,T^1) = M^{y-z} L^{x+2y+3z} T^{-x-y-2z}$

Comparing powers,

$y - z = 0, x + 2y + 3z = 0, -x - y - 2z = 1$

Solving we get, $x = \frac{-5}{2}, y = \frac{1}{2}\ and\ z = \frac{1}{2}$,

$$T = \sqrt{\frac{hG}{c^5}} = c^{-5/2}h^{1/2}G^{1/2}$$

2. An artificial satellite is revolving around a planet of mass M, radius R in a circular orbit of radius r, using dimensional analysis show that the period of satellite $T = \frac{K}{R}\sqrt{\frac{r^3}{g}}$ where K is dimensionless constant and g is acceleration due to gravity.

Ans: According to Kepler's Third Law $T^2 \alpha r^3$

T is a function of g and R

Let $T = Kr^{3/2}g^xR^y$

Corresponding dimensional formulas are

$(M^0L^0\,T^1) = K\,L^{3/2}(L\,T^{-2})^xL^y$

$(M^0L^0\,T^1) = K\ L^{3/2+x+y}T^{-2x}$

Comparing powers, $3/2 + x + y = 0, -2x = 1$

Solving we get, $x = \frac{-1}{2}, y = -1,$

$$T = Kr^{3/2}g^{-1/2}R^{-1} \qquad T = \frac{K}{R}\sqrt{\frac{r^3}{g}}$$

3. State the number of significant figures in the following

(a) 6729 (b) 0.024 (c) 0.08240 (d) 6.032 (e) 4.57×10^8

Ans : a. 6729 → No. of significant figures = 4. Since all non zero digits are significant

b. 0.024 → No. of significant figues = 2. Since the zero after decimal point and before 2 is not significant

c. 0.08240 → No. of significant figures = 4. Since the zero before 8 and after decimal is not significant but zero after 4 is significant

d. 6.032 → No. of significant figures = 4. Since all zeroes occurring between two non zero digits are significant

e. 4.57×10^8 → No. of significant figures = 3. Since non zero digits before and after decimal are significant

4. **A stick has a length of $12.132cm$ and another has a length of $12.4cm$. If the two sticks are placed end to end, what is the total length? If the two sticks are placed side by side, what is the difference in their lengths?**

Ans:

a. Let the lengths of the rods be $l_1 = 12.132cm$ and $l_2 = 12.4cm$

l_2 has one decimal place, l_1 has to be rounded to have only two decimal places

$l = l_1 + l_2$ = 12.13+12.4 = 24.53,

This is to be rounded off to have one decimal place only, the result is **24.5cm**

b. $l_1 = 12.132cm$ and $l_2 = 12.4cm$

$l_1$is rounded only to two decimal places

$l = l_2 - l_1$ = 12.4-12.13 = 0.27,

This is to be rounded off to have one decimal place only, the result is **0.3cm**

5. **Each side of a cube is measured to be $7.203m$. What is (a) the total surface area (b)the volume of the cube to appropriate significant figures. (Ans- a) 311.3m^2 b) 373.7 m^3)**

Ans: Length of side = 7.023m

1. Total surface area = $6a^2$

= $6 \times (7.203)^2$

= 311.29

This should be rounded off to four significant figures as 7.203 has 4 significant figures.

The result is $311.3m^2$

2. Volume of a cube is $a^3 = (7.203)^3$ = 373.71

This should be rounded off to four significant figures as 7.203 has 4 significant figures. The result is $373.7m^3$

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UNIT – 2

MOTION IN A STRAIGHT LINE

I. MULTIPLE CHOICE QUESTIONS

1. The velocity of a body depends on time according to the equation V=10+ $2t^2$. The body is undergoing **(3)**

(1) Uniform acceleration (2) uniform retardation
(3) non-uniform acceleration (4) zero acceleration

Ans : Given V = 10 + $2t^2$ $\therefore \quad a = \frac{dV}{dt} = 4t$

$\because$ a is positive and varies with time (Non-Negative), then the body is undergoing non-uniform acceleration.

2. Which of the following is true? **(4)**

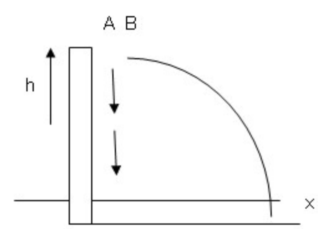
(1) Speed is always equal to magnitude of velocity.
(2) Speed is always greater than the magnitude of velocity.
(3) Velocity is always greater than speed.
(4) Speed in general is either equal to or greater than the magnitude of velocity.

Ans : Speed in general is either equal to or greater than the magnitude of Velocity.

3. A bullet is dropped from a certain height, at then same time another bullet is fired horizontally from the same height both bullets will hit the ground **(2)**

(1) one after the other (2) simultaneously (3) depends on the observer
(4) depends on the velocity of the fired bullet.

Ans :
Simultaneously since both bullets experience the same gravitational acceleration
$\because$ Horizontal motion of bullet doesn't effect its vertical motion.



4. If a car is moving with a uniform velocity of 60 km/h. Then its acceleration is **(1)**

(1) 0 km/ h^2 (2) 10 km/ h^2 (3) 20 km/ h^2 (4) 30 km/ h^2

Ans : $0 km/h^2$, $\because$ For uniform velocity, the acceleration is Zero.

5. A ball is thrown vertically upward with an initial velocity of 50 m/s. Its maximum height is (take g=10 m/ s^2) **(1)**

(1) 125 m (2) 250 m (3) 500 m (4) 1000 m

Ans : We know

$$H_{max} = \frac{u^2}{2g} \qquad \Rightarrow H_{max} = \frac{50 \times 50}{2 \times 10} = 125m$$

6. **A particle moves in a straight line with an initial velocity of 10 m/s and a constant acceleration of 2 m/s^2. Its displacement after 5 seconds will be:** **(2)**

(1) 50 m (2) 75 m (3) 150 m (4) 200 m

Ans : We know

$$s = ut + \frac{1}{2}at^2$$
$$= 10 \times 5 + \frac{1}{2} \times 2 \times 5^2$$
$$= 50 + 25$$
$$= 75m$$

7. **A body starts from rest and moves with a uniform acceleration of 2 m/s^2. Its velocity after 10 seconds will be** **(4)**

(1) 5 m/s (2) 10 m/s (3) 15 m/s (4) 20 m/s

Ans : We know v = u + at

Given u = 0 m/s

v = at

v = 2 x 10

v = 20 m/s

8. **For the motion with uniform velocity, the slope of the velocity-time graph is equal to**

(1) 1 m/s (2) zero (3) initial velocity (4) final velocity **(2)**

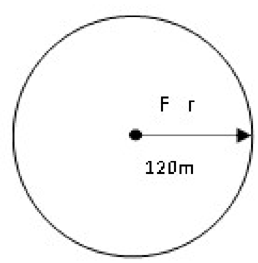
Ans : For uniform motion, velocity –time graph is a horizontal line.

9. **If a cyclist takes two minutes to complete half revolution on a circular path of 120 m radius, what is the average speed?** **(3)**

(1) 1 m/s (2) 2 m/s (3) 3.14 m/s (4) 4.14 m/s

Ans :

Average Speed $= \frac{Total\, dis\tan ce}{Total\, time\, taken}$



Distance travelled, d=half of perimeter.

$$= \frac{2\pi r}{2} = \pi r$$

Time =2 min = 2×60 =120sec.

$$V_{avg} = \frac{\pi r}{120} = \frac{3.14 \times 120}{120} = 3.14 m/s.$$

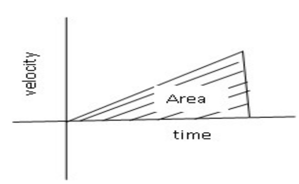
10. **In a particular time, the net displacement travelled by the body is equal to** **(3)**

(1) the area which v- t graph encloses with displacement axis

(2) the area which x- t graph encloses with the time axis

(3) the area which v- t graph encloses with time axis

(4) the area which a – t graph encloses with time axis



Ans : The total distance travelled by the body is equal to the area which v- t graph encloses with time axis.

Note : 4th option in text book read as mentioned in hand book.

11. A particle moves along a straight line. At a time t, the distance x of the particle from the origin is given by x=20 + 12t- t^3, where x is in meters and t in sec, the time taken for the particle for coming to rest **(2)**

(1) 1 sec (2) 2 sec (3) 4 sec (4) 6 sec

Ans : Given $x = 20 + 12t - t^3$

Velocity, $\frac{dx}{dt} = 12 - 3t^2$

Time taken for the particle for coming to rest.

$\because$ V=0

$\therefore 12 - 3t^2 = 0 => 3t^2 = 12$

$=> t^2 = 4$

$=> t = 2Sec$

12. A car travels half to the distance with speed V and the remaining distance with speed 2V. Its average speed is **(1)**

(1) $\frac{4V}{3}$ (2) $\frac{3V}{4}$ (3) $\frac{V}{3}$ (4) $\frac{2V}{3}$

Ans : Average Speed $= \frac{2V_1V_2}{V_1+V_2}$

$= \frac{2(V)(2V)}{V+2V}$

$= \frac{4V^2}{3V}$

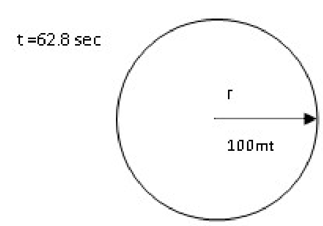
$= \frac{4V}{3}$

13. A car runs at constant speed on a circular track of radius 100m, taking 62.8 seconds for every circular lap. The average velocity and average speed of each circular lap respectively is **(3)**

(1) 10 m/s, 0 m/s (2) 0 m/s, 0 m/s (3) 0 m/s, 10 m/s (4) 10 m/s, 10 m/s

Ans : Displacement after one circular lap=0 m

Average Velocity= $\frac{total\ displacement}{total\ time\ taken} = \frac{0}{62.8} = 0m/s$



Distance covered in one lap = circumference

=2πr=200π m

Average Speed $= \frac{total\ distance}{total\ time\ taken}$

$= \frac{200\pi}{62.8} = \frac{200 \times 3.14}{62.8}$

$= \frac{628}{62.8} = 10m/s$

14. The motion of the particle is given by the equation S= (2t^3+5t^2+16t+8)m. The value of acceleration of the particle at t=1 sec is **(2)**

(1) 20 m/s^2 (2) 22 m/s^2 (3) 24 m/s^2 (4) 26 m/s^2

Ans. Given $s = 2t^3 + 5t^2 + 16t + 8$

$$V = \frac{ds}{dt} = 6t^2 + 10t + 16$$

$$a = \frac{dV}{dt} = 12t + 10$$

At t = 1 Sec , a = 12 (1) + 10

∴ a = 22m/s^2

15. The ratio of distances travelled by a freely falling body in the 1st, 2nd, 3rd, 4th second

(1) 1:2:3:4 (2) 1:4:9:16 (3) 1:3:5:7 (4) 1:1:1:1 **(3)**

Ans : For a freely falling body initial velocity u = 0, a = g

$$s_n = u + \frac{a}{2}(2n-1),$$

$$s_n = \frac{g}{2}(2n-1)$$

$$\Rightarrow S_n \alpha (2n-1)$$

Here n = 1 , 2, 3, 4

$S_1 : S_2 : S_3 : S_4$ = 1: 3: 5: 7

16.A stone falls freely under gravity. It covers distances $h_{1,}h_2$ and h_3 in the first 5 seconds, the next 5 seconds and the next 5 seconds respectively. The relation between $h_{1,}h_2$ and h_3 is **(4)**

(1) $h_2 = 3h_1$ and $h_3 = 3h_2$ (2) $h_1 = h_2 = h_3$ (3) $h_1 = 2h_2 = 3h_3$ (4) $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$

Ans : Given, a stone falls feeling under gravity it covers distances h_1, h_2, h_3.........

We know $S = \frac{1}{2}gt^2$

$$S = S_2 - S_1 = \frac{1}{2}g\left(t_2^2 - t_1^2\right)$$

$$= S\alpha\left(t_2^2 - t_1^2\right)$$

$$h_1 : h_2 : h_3 = (5^2 - 0^2) : (10^2 - 5^2) : (15^2 - 10^2)$$

$$= 25 : 75 : 125 = 1 : 3 : 5$$

$$\frac{h_1}{h_2} = \frac{1}{3} \qquad \Rightarrow h_1 = \frac{h_2}{3} \rightarrow (1)$$

$$\frac{h_2}{h_3} = \frac{3}{5} \qquad \Rightarrow h_2 = \frac{3h_3}{5} \rightarrow (2)$$

$$\therefore h_1 = \frac{1}{3}\left(\frac{3h_3}{5}\right) = \frac{h_3}{5} \rightarrow (3)$$

From (1) & (3) $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$

II. FILL IN THE BLANKS

1. _______________ deals with the study of motion of objects without taking into account, the factors which cause the motion.
Ans : Kinematics
2. The shortest path travelled by a moving object in a given time from initial to final position is called _______________.
Ans : Displacement
3. The distance covered by a body along a semicircular path of radius r is __________
Ans : πr
4. The slope of velocity –time graph gives _____________
Ans :. Acceleration
5. The slope of displacement-time graph gives _____________
Ans : Velocity
6. The area under acceleration-time graph gives _____________
Ans : Change in Velocity
7. The acceleration of a body moving with uniform velocity is __________________
Ans : Zero
8. The velocity of a body at any instant of time is called _______________
Ans : Instantaneous Velocity

9. The value of an acceleration of a freely falling body is _____________

Ans :. a=g=9.8 m/s^2

10. The ratio of distances travelled by a freely falling body from the rest in the first three seconds is _______________.

Ans : 1:3:5

$$S_n = u + \frac{a}{2}(2n-1)$$

$$u = 0 \,\&\, a = g$$

$$S_n = \frac{g}{2}(2n-1) \Rightarrow S_n \alpha (2n-1)$$

Here n = 1, 2, 3
$S_1 : S_2 : S_3 = 1 : 3 : 5$.

II. VERY SHORT ANSWER QUESTIONS

1.What is uniform motion?
Ans : If a body moves equal distances in equal intervals of time, then it is called uniform motion.
2. What is acceleration?
Ans : Acceleration is the rate of change of velocity with time.

$$a = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta V}{\Delta t}$$

3. What is average speed?

Ans : It is the total distance travelled by the object divided by the total time taken to travel the distance.

$$\text{Average Speed } = \frac{\text{Total path length}}{\text{Total time}}$$

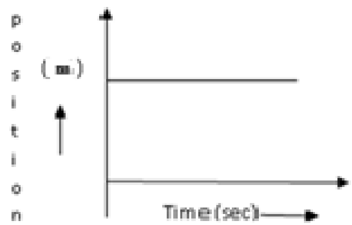
4. What is instantaneous velocity?

Ans : The velocity of an object at that instant of time is called instantaneous velocity.

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

5. Draw position-time graph for an object at rest.

Ans : Position – time graph for an object at rest.



6. What represents the area under the curve of velocity-time graph?

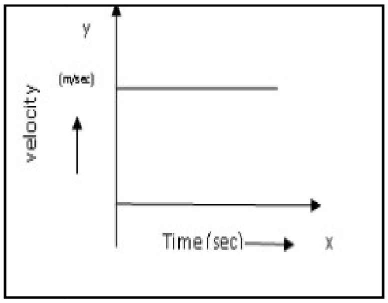
Ans : The area under v-t curve represents the displacement since $\vartheta = \frac{d}{t}$, $d = \vartheta t$

7. Mention one similarity between speed and velocity.

Ans : Speed and velocity are measured in same units [m/sec] (or) They both measure the rate of change of position.

8. Draw velocity-time graph for a particle moving with uniform velocity.

Ans : Velocity-time graph for uniform motion



9.What is the time taken by the body, when it is freely dropped from a height "h"?

Ans : Under free fall of an object.

$$S = ut + \frac{1}{2}at^2, \qquad u = 0, a = g, s = h$$

$$h = +\frac{1}{2}gt^2, \qquad t = \sqrt{\frac{2h}{g}}$$

IV. VERY SHORT ANSWER QUESTIONS

1. The states of motion and rest are relative. Explain.

Ans : **Rest:** If the position of a body does not change with respect to surroundings, it is said to be at rest.

Motion: If the position of a body changes with respect to surroundings, it is said to be in motion.

By the definitions rest and motion are relative with respect to surroundings.

2.Is it necessary that the direction of acceleration and velocity of a body should be the same? Give an example.

Ans : No, the velocity and acceleration of a body are not always in the same direction.

Ex:- When brakes are applied, the velocity of body before coming to rest is opposite to retarding acceleration.

3. Is it possible for a body to have acceleration but zero velocity? Give an example.

Ans : In case of vertically projected body, at maximum height its velocity is zero. But acceleration g (Vertically downward), which is not zero.

4. Which is greater, a velocity of 72 kmph or 25 m/s?

Ans : 72 km/hr $= 72 \times \frac{5}{18} = 20 m/\sec$ $\left(\because 1 \text{ km/hr} = \frac{5}{18}\ m/\sec\right)$

Hence 25 m/Sec is greater than 20 m/sec.

5. Give an example of one dimensional motion where a particle moving along a straight line comes to rest and moves backward.

Ans : Oscillation of simple harmonic oscillator is the example for the particle comes to rest periodically.

6. A vehicle travels half the distance with a speed V_1 and other half with speed V_2. What is the average speed?

Ans : Average Speed $= \frac{total\ distance}{total\ time\ taken}$

$$= \frac{\frac{L}{2} + \frac{L}{2}}{\frac{L}{2V_1} + \frac{L}{2V_2}} = \frac{2V_1V_2}{V_1 + V_2}$$

7. A body covers a distance of 5m along a semicircular path from its one end to another end. What is the ratio of its distance covered to its displacement?

Ans : If r is the radius of semicircular path, then

Distance of semicircular path = πr

Given πr=5 => $r = \frac{5}{\pi}$

Displacement of semicircular path = 2r

$$= 2\frac{5}{\pi} = \frac{10}{\pi}$$

$$\therefore \frac{Distance}{Displacement} = \frac{5}{\frac{10}{\pi}} = \frac{\pi}{2} = \frac{22}{7 \times 2} = \frac{11}{7} = 11:7$$

8. Displacement travelled by a body is given by S=2t+5 t^2 m. What is the acceleration of that body?

Ans : Given, $s = 2t + 5t^2$

$$V = \frac{ds}{dt} = 10t + 2$$

$$\therefore \text{acceleration, } a = \frac{dv}{dt} = 10 m/sec^2$$

9. An object falling through a fluid is observed to have an acceleration given by a=g-bv, where g is the gravitational acceleration and b is constant. After a long time it is observed to fall with a constant velocity. What would be the value of this constant velocity?

Ans : Given

$$a = g - bv \qquad [\therefore a = \frac{dv}{dt}]$$

$$\frac{dv}{dt} = g - bv$$

$$0 = g - bv \ (V = \text{constant})$$

$$bV = g \Rightarrow V = \frac{g}{b}$$

10.Under what circumstances is the relationship s = vt valid?

Ans : When a body moves with uniform velocity and along a straight line, then s=vt valid.

V. SHORT ANSWER QUESTIONS

1. **Can the equations of kinematics be used when the acceleration varies with time? If not, what form would these equations take?**

Ans : The standard equations of kinematics are derived under the assumption that the acceleration is constant.

They are

$$V = u + at$$

$$S = ut + \frac{1}{2}at^2$$

$$V^2 - u^2 = 2as$$

When the acceleration varies with time, these equations cannot be used directly.

Given the acceleration varies with time, we express it as a(t).

1) To find the velocity v(t), integrate the acceleration function with respect to time.

$V_t = \int a(t)dt + V_0$ Where V_0 is the initial velocity.

2) To find the position x(t), integrate the velocity function with respect to time.

$x_t = \int V(t)dt + x_0$ Where x_0 is the initial position.

2.A particle moves in a straight line with uniform acceleration. Its velocity at time t=0 is V_1 and

at time t = t is V_2. The average velocity of the particle in this time interval is $\left(\frac{V_1+V_2}{2}\right)$.

Is this correct? Substantiate your answer.

Ans : Acceleration of a particle is given by, a= $\frac{\Delta V}{\Delta t}$

Given, At t=0, u = V_1

At t = t, V = V_2

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{v_2 - v_1}{t - 0} = \frac{v_2 - v_1}{t}$$

When the particle is in uniform velocity, the displacement of particle in time 't' is

$$s = ut + \frac{1}{2}at^2$$

$$s = v_1\, t + \frac{1}{2}\left(\frac{v_2 - v_1}{t}\right)t^2$$

$$= \left[v_1 + \left(\frac{v_2 - v_1}{2}\right)\right] t$$

$$s = \left(\frac{v_2 - v_1}{2}\right) t$$

Average Velocity, $V_{avg} = \frac{Displacement}{Time\ Interval}$

$$= \frac{\left(\frac{v_2 - v_1}{2}\right) t}{t}$$

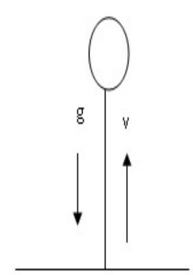
$$V_{avg} = \left(\frac{V_1 + V_2}{2}\right)$$

3. Can the velocity of an object be in a direction other than the direction of acceleration of the object? If so, give an example.

Ans : Yes, the velocity of an object be in a direction other than the direction of acceleration of the object.

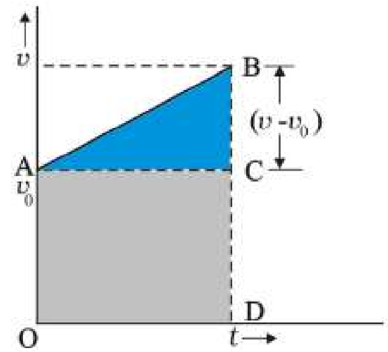
Ex:- If an object is projected vertically upwards,

The Velocity is directed vertically upwards but acceleration due to gravity directed vertically downwards.



4. Obtain the equation of motion S= ut + $\frac{1}{2}at^2$ for constant acceleration using graphical method.

Ans : Consider a body moving with uniform acceleration 'a'. Let initial velocity of the body is υ_0 and it gains a velocity 'υ' after 't' seconds.



The graph plotted between velocity and time is a straight line as shown in the diagram.

The Area under this Curve is :

Area between instants 0 and t

= Area of triangle ABC + Area of rectangle OACD

$= \frac{1}{2}\ (v - v_0)\ t + v_0\ t$------------(1)

As we know, the area under v-t curve represents the displacement. Therefore, the displacement S of the object is

$S = \frac{1}{2}\ (v - v_0)\ t + v_0 f$ ------------(2)

But in uniform acceleration motion,

$v - v_0 = at$ -------------(3)

By substituting equation (3) in (2)

$S = v_0\ t + \frac{1}{2}\ at^2$

5. Explain the terms the average velocity and instantaneous velocity. When are they equal?

Ans : **<u>Average Velocity:</u>** The average velocity is defined as the change in position or displacement (Δx) divided by the time intervals (Δt) in which the displacement occurs.

$$V_{average} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Where x_2 and x_1 are the positions of the object at time t_2 and t_1 respectively.

<u>Instantaneous Velocity</u>: The velocity at an instant is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small.

$$V_{instaneous} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

In uniform motion, average velocity and instantaneous velocities are equal.

VI. PROBLEMS :

1. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1}. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1}. What is the (a) magnitude of average velocity and (b) average speed of the man over the time interval 0 to 50 min.

Ans : Time taken by man to go from his home to market,

$$t_1 = \frac{distance}{speed} = \frac{2.5}{5} = \frac{1}{2} hr$$

Time taken by man to go from market to his home,

$$t_2 = \frac{2.5}{7.5} = \frac{1}{3} hr$$

Total time taken $= t_1 + t_2 = \frac{1}{2} + \frac{1}{3} = \frac{5}{6} hr$

=50 min.

In time interval 0 to 50 min.

Total distance travelled =2.5+2.5 = 5km.

Total displacement = 0km

(1) Average Velocity $= \frac{displacement}{time} = 0 km/hr$

(2) Average speed $= \frac{distance}{time} = \frac{5}{\frac{5}{6}} = 6 km/hr$

2. A car travels the first third of a distance with a speed of 10 kmph, the second third at 20 kmph and the last third at 60 kmph. What is its mean speed over the entire distance?

Ans : Given $v_1 = 10 kmph, v_2 = 20 kmph, v_3 = 60 kmph$

Mean speed $= \frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1}$

$$= \frac{3\times10\times20\times60}{200+1200+600} = \frac{30\times1200}{2000} \quad = 18kmph \quad = 18\times\frac{5}{18} = 5 m/\sec$$

3. A bullet moving with a speed of 150 m s^{-1} strikes a tree and penetrates 3.5 cm before stopping. What is the magnitude of its retardation in the tree and the time taken for it to stop after striking the tree?

Ans : Given, Velocity of bullet, u=150 m/s

Final Velocity, v=0

Distance travelled, S=3.5cm

$= 3.5\times10^{-2} m$

Acceleration, $a = \frac{v^2-u^2}{2s}$

$$= \frac{0^2 - 150^2}{2\times3.5\times10^{-2}} \quad = \frac{-22500}{7\times10^{-2}}$$

$$= -3.214\times10^{5}\, m/\sec^{2}$$ (Negative sign for retardation)

Time taken to stop, $t = \frac{v-u}{a}$

$$= \frac{0-150}{-3.214\times10^{5}}$$

$$= \frac{150}{3.214\times10^{5}}$$

$$= 4.67\times10^{-4}\, s$$

4.A motorist drives north for 30 min at 85 km/h and then stops for 15 min. He continues travelling north and covers 130 km in 2 hours. What is his total displacement and average velocity?

Ans : 1) In first part:

Given, V_1 = 85km/hr, t_3 = 30 min = $\frac{30}{60}$ hr

$S_1 = v_1 t_1 = 85 \times \frac{30}{60} = 42.5$ km

2) In Second part:

$s_2 = 0,\ t_2 = 15\min$

3) In third part:

$s_3 = 130km,\ t_3 = 120\min = 2hr$

Total displacement

$$s = s_1 + s_2 + s_3 = 42.5 + 0 + 130 = 172.5km$$

Total time travelled

$$t = t_1 + t_2 + t_3 = 30 + 15 + 120$$

=165 min

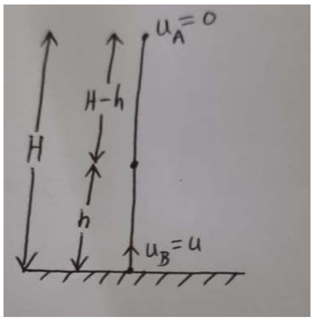
=2 hr 45 min

$$= 2\frac{3}{4}hrs = \frac{11}{4}hrs$$

Average velocity $= \dfrac{Total\ displacement}{Total\ time\ travelled}$ $= \dfrac{172.5}{\frac{11}{4}} = 62.72\, km/hr$

5. A ball is dropped from the top of a building and at the same time and identical ball B is thrown vertically upward from the ground. When the balls collide the speed of A is twice that of B. At what fraction of the height of the building did the collision occur?

Ans : Given $V_A = 2V_B$



For ball A,

$$H - h = \frac{1}{2}gt^{2} \text{----------------(1)}$$

For ball B,

$$h = ut - \frac{1}{2}gt^{2} \text{----------------(2)}$$

When the ball dropped, V = u + at

$$V_a = gt \text{------------------------(3)}$$

When the ball thrown up, v = u - at

$$v_B = u - gt \text{---------}(4)$$

$$V_A = 2V_B$$

$$gt = 2(u - gt)$$

$$gt = 2u - 2gt$$

$$u = \frac{3gt}{2} \text{-------------}(5)$$

Add (1) & (2) ,

$$H = ut$$

$$H = \frac{3gt^2}{2}$$

$$\therefore \frac{h}{H} = \frac{ut - \frac{1}{2}gt^2}{\frac{3}{2}gt^2}$$

$$\Rightarrow \frac{h}{H} = \frac{\frac{3gt^2}{2} - \frac{1}{2}gt^2}{\frac{3}{2}gt^2} = \frac{2}{3}$$

$\therefore$ Fraction of height of collision $= \frac{2}{3}$

6. Drops of water fall at regular intervals from the roof of a building of height 16m. The first drop strikes the ground at the same moment as the fifth drop leaves the roof. Find the distances between successive drops.

Ans : Time taken by 1st drop

$$s = ut + \frac{1}{2}at^2$$

$$16 = \frac{1}{2}at^2$$

$$t^2 = \frac{16 \times 2}{g}$$

$$t = 4\sqrt{\frac{2}{g}}$$

Number of intervals = 4

Time taken between each interval $= \dfrac{4\sqrt{\frac{2}{g}}}{4} \qquad = \sqrt{\frac{2}{g}}$

Distance between successive drops:

$$s_1 - s_2 = 16 - \frac{1}{2} g\left(3\sqrt{\frac{2}{g}}\right)^2$$

$$= 16 - \frac{1}{2}.g.9\frac{2}{g} = 16 - 9 = 7m$$

$$s_2 - s_3 = 9 - \frac{1}{2} g\left(2\sqrt{\frac{2}{g}}\right)^2$$

$$= 9 - \frac{1}{2}.g.4\frac{2}{g} = 9 - 4 = 5m$$

$$s_3 - s_4 = 4 - \frac{1}{2} g\left(\sqrt{\frac{2}{g}}\right)^2$$

$$= 4 - \frac{1}{2}.g.\frac{2}{g} = 4 - 1 = 3m$$

$$s_4 - s_5 = 1 - 0 = 1mt$$

7. A food packet is dropped from an aeroplane, moving with a speed of 360 kmph in a horizontal direction , from a height of 500m. Find (i) its time of descent(ii) the horizontal distance between the point at which the food packet reaches the ground and the point above which it was dropped.

Ans : Given velocity of plane, v=360kmph

$$= 360 \times \frac{5}{18} = 100 m/\sec$$

Height of above ground, h=500m

Consider g=$10 m/\sec^2$

Time of distant $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 500}{10}} = \sqrt{100} = 10\sec$

Horizontal distance between point of dropping and point where it reaches the ground =Range(R)

$R = u \times t = 100 \times 10 = 1000m$.

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UNIT – 3

MOTION IN A PLANE

I. MULTIPLE CHOICE QUESTIONS

1. A particle moving with uniform speed in a circular path maintains **(4)**

(1) Constant velocity (2) constant acceleration

(3) Constant velocity but varying acceleration (4) varying velocity and varying acceleration

2. The projection or component of the vector 3 $\bar{i}$ + 4 $\bar{k}$ on y-axis is **(4)**

1)7 2) 5 3)4 4)0

Ans : y- component is zero in the given vector.

3. The angle made by the vector $\hat{A} = \hat{i} + \hat{j}$ with x-axis is **(3)**

1) 22.5^0 2) 30^0 3) 45^0 4) 90^0

Ans : Given $A_x = 1, A_y = 1,\ \text{Tan}\,\theta = \left|\frac{A_y}{A_x}\right| = \frac{1}{1} = 1 \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45^0$

4. A body is projected into air from the ground at an angle 90^0 to the horizontal. On reaching the highest point above the ground **(1)**

1) Its velocity is zero, acceleration is not zero
2) Its acceleration is zero, velocity is not zero
3) Both the velocity and acceleration are not equal to zero
4) Both the velocity and acceleration are equal to zero

Ans : For a vertically thrown up body at the highest point, its velocity is zero, acceleration is not zero.

5. Two balls are projected from the same point in directions 30^0 and 60^0 with respect to the horizontal. What is the ratio of their initial velocities if they attain the same maximum height?

1) $\sqrt{3}:1$ (2) $1:\sqrt{3}$ 3) 1:1 (4) 3:1 **(1)**

Ans :

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$\Rightarrow u^2 \alpha \frac{1}{\sin^2\theta}$$

$$\therefore \frac{u_1}{u_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{\sin 60^0}{\sin 30^0} = \sqrt{3}:1$$

6. An aeroplane is dropping a bomb while moving horizontally at a constant speed. When air resistance is taken into consideration, then the bomb **(2)**

1) falls along side the plane 2) falls on earth behind the plane

3) falls on earth ahead of the plane 4) falls on earth exactly below the plane.

Ans : Due to air resistance the horizontal velocity of the bomb changes. It falls behind the plane.

7. Which of the following is the vector quantity? **(3)**

1) Distance 2) Speed 3) Acceleration 4) Temperature

8. Which of the following is a scalar quantity? **(2)**

1) Momentum 2) Mass 3) Velocity 4) Force

9. Which of the following is not a vector quantity? **(4)**

1) Weight 2) Torque 3) Momentum 4) Potential energy

10. Which of the following physical quantity can be negative? **(3)**

1) Speed 2) Mass 3) Velocity 4) Distance

11. Vector addition obeys **(3)**

1) Commutative law only 2) associative law only

3) both commutative and associative laws 4) neither commutative nor associative laws

12. The relation between the time of flight of projection T_f and the time to reach the maximum height t_m is **(2)**

1) $T_f = t_m$ 2) $T_f = 2t_m$ 3) $T_f = \frac{t_m}{2}$ 4) $T_f = \sqrt{2}t_m$

Ans : Time of flight (T_f) = 2 x Time of Ascent (t_m)

13. Which of the following is not a projectile motion ? **(4)**

1) A stone is thrown horizontally from a building
2) A bullet fired from a gun at angle 45^0 w.r.t. ground
3) The jump of a frog
4) A car moving in a straight line

14. A missile is fired for a maximum range with an initial velocity of 30m/s. If g = 10 m/s^2 , then the range of the missile will be **(2)**

1) 100m 2) 90m 3) 80m 4) 70m

Ans : $R_{\max} = \frac{u^2}{g} = \frac{30\times30}{10} = 90m$

15) The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectile is **(3)**

1) $\theta = 45^0$ 2) $\theta = \tan^{-1}\left(\frac{1}{4}\right)$ 3) $\theta = \tan^{-1}(4)$ 4) $\theta = \tan^{-1}(2)$

Ans : Given

$$H_{\max} = R$$

$$\frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin 2\theta}{g}$$

$$\frac{u^2 \sin^2\theta}{2} = u^2 .2 \sin\theta . \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 4 \Rightarrow \tan\theta = 4$$

$$\theta = \tan^{-1}(4)$$

II. FILL IN THE BLANKS

1. The dimensional formula for centripetal acceleration in a circular motion is ____________

Ans : $M^0L^1T^{-2}$

2. If θ is the angle of projection, g is the acceleration due to gravity, then maximum height of projectile is ____________

Ans : $H_{\max} = \dfrac{u^2\sin^2\theta}{2g}$

3. The equation for the trajectory of a projectile motion is ___________

Ans : $y = ax - bx^2$

4. The path of the projectile with respect to ground is ____________

Ans : Parabola

5. The resultant of multiplying the scalar with a vector is a _________

Ans : Vector

6. The time required by a projectile to reach its highest point in terms of *u* and *g* is ______

Ans : $t_a = \dfrac{u\sin\theta}{g}$

7. The position of a particle is r =3.0t*i* +2.0t^2*j*+5.0*k*, where t is in seconds, the coefficients have the proper units of r to be in metres, then V (t)= ______________

Ans : $\boldsymbol{V(t) = \frac{dr}{dt} = 3i + 4tj}$

8 . The angle between two vectors A and B with magnitudes $\sqrt{3}$ and 4 respectively, and

$\overline{A}.\overline{B} = 2\sqrt{3}$ is _____________

Ans : $\cos\theta = \dfrac{\overline{A}.\overline{B}}{|\overline{A}||\overline{B}|} = \dfrac{2\sqrt{3}}{\sqrt{3}\times 4} = \dfrac{1}{2}$

$$\cos(\theta) = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

III. ONE WORD ANSWER QUESTIONS :

1. When are the two vectors said to be equal vectors?

Ans : Two vectors $\overline{A}$ and $\overline{B}$ are said to be equal if they have the same magnitude and the same direction**.**

2. Can a vector of zero magnitude have non-zero components?

Ans : No, a vector of magnitude zero should have components of zero magnitude\

3. When will the magnitude of sum of two vectors be maximum and minimum?

Ans : The sum of two vectors is maximum when they are in the same direction and is minimum when they act in opposite directions.

4. At what angle of projection of body with respect to ground, its range is maximum?

Ans :. Range is maximum at $\theta = 45^0$ $\left(\therefore R = \dfrac{u^2 \sin 2\theta}{g}\right)$

5. What is the acceleration of a projectile at the top of its trajectory?

Ans : At the top of the trajectory, acceleration of projectile is g, in downwards.

6. What is the direction of velocity at any point on the path of a circularly moving object?

Ans : The direction of velocity at any point on the path of a circularly moving object is always tangent to the circle.

7. What is the angle between the direction of velocity and acceleration at the highest point of a projectile path?

Ans :. $\theta = 90^0$

IV. VERY SHORT ANSWER QUESTIONS

1. What are scalars and vectors?

Ans : Scalars: The quantities which have only magnitude and no direction are called scalar quantities.

E.g: Mass, Distance, Time, etc

Vectors: The quantities which have both magnitude and direction and obeys triangle law of vector addition or parallelogram law of vector addition are called vector quantities.

Eg: Displacement, Velocity, Force etc.

2. State Triangle Law of vector addition.

Ans : If the magnitude and direction of two vectors are represented by two sides of a triangle taken in order, then the third side of the triangle taken in reverse order will give the resultant both in magnitude and direction.

3. When two right angled vectors of magnitude 7 units and 24 units combine, what is the magnitude of their resultant?

Ans : Given $\left|\overline{A}\right|$ = 7 units, $\left|\overline{B}\right|$ = 24 units, $\theta = 90^0$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2}$$

$$R = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$$

4. If $\overline{P} = 2\hat{i} + 4\hat{j} + 14\hat{k}$ and $\overline{Q} = 4\hat{i} + 4\hat{j} + 10\hat{k}$ find the magnitude of $\overline{P} + \overline{Q}$

Ans : Given $\overline{P} = 2\overline{i} + 4\overline{j} + 14\overline{k}$ and $\overline{Q} = 4\overline{i} + 4\overline{j} + 10\overline{k}$

$$\overline{P} + \overline{Q} = 6\overline{i} + 8\overline{j} + 24\overline{k}$$

Magnitude of $\overline{P}+\overline{Q}$,

$$\left|P+Q\right|=\sqrt{6^2+8^2+24^2}=\sqrt{36+64+576}=\sqrt{676}=26\,\text{units}$$

5. Can two vectors of unequal magnitude add up to give the zero vector? Can three unequal vectors add up to give the zero vector?

Ans : No two unequal vectors can never give zero vector by addition. But three unequal vectors when added may give zero vectors.

6. If vertical component of a vector is equal to its horizontal component, what is the angle made by the vector with X-axis?

Ans : Horizontal component of vector $\overline{a}$ = vertical component of vector $\overline{a}$

$$a_x=a_y$$

$$a\cos\theta=a\sin\theta\Rightarrow\tan\theta=1\Rightarrow\theta=45^0$$

7. Two forces of magnitude 3 units and 5 units act at 60^0 with each other. What is the magnitude of their resultant?

Ans : Given $\left|\overline{A}\right|=3,\left|\overline{B}\right|=5,\theta=60^0$

Magnitude of Resultant, R = $\sqrt{A^2+B^2+2AB\cos\theta}$ = $\sqrt{3^2+5^2+2\times3\times5\times\cos60}$

= $\sqrt{9+25+15}$ = $\sqrt{49}$ = 7 units

8. Why does the vertical component of velocity for a projectile change with time, where the horizontal component of the velocity remains constant in parabolic path?

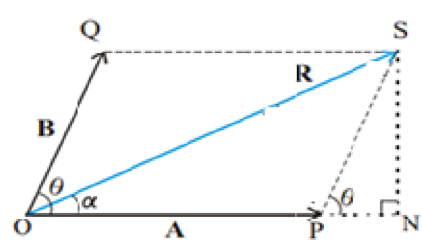
Ans : The vertical component of velocity for a projectile changes with time due to the constant downward force of gravity, where as the horizontal component of velocity remains constant in the absence of external force like air resistance.

V. SHORT ANSWER QUESTIONS

1.Using parallelogram law of vectors derive an expression for the magnitude and direction of the resultant vector.

Ans : Let $\overline{A}$ and $\overline{B}$ be two adjacent vectors, θ be angle between them. From the diagram OPSQ is parallelogram. Extend the line OP and draw a normal N from S. The diagonal OS is the resultant $\overline{R}$ both in direction and magnitude.

Magnitude of $\overline{R}$:



From the figure, triangle OSN

$$OS^2=ON^2+SN^2$$

$$OS^2=\left(OP+PN\right)^2+SN^2\left(\therefore ON=OP+PN\right)$$

From triangle PNS,

$$\cos\theta=\frac{PN}{PS}\Rightarrow PN=PS\cos\theta=B\cos\theta$$

And $\sin\theta=\dfrac{SN}{PS}\Rightarrow SN=PS\sin\theta=B\sin\theta$

$$OS^2 = OP^2 + PN^2 + 2OP.PN + SN^2$$
$$R^2 = \left(A + B\cos\theta\right)^2 + \left(B\sin\theta\right)^2$$
$$= A^2 + 2AB\cos\theta + B^2\cos^2\theta + B^2\sin^2\theta$$
$$R^2 = A^2 + 2AB\cos\theta + B^2$$
$$\therefore R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Direction of $\overline{R}$:

Let the resultant $\overline{R}$ is making an angle $\propto$ with $\overline{A}$

From triangle OSN, $\tan\alpha = \dfrac{SN}{ON} = \dfrac{SN}{OP+PN} = \dfrac{B\sin\theta}{A+B\cos\theta}$

$$\alpha = \tan^{-1}\left[\frac{B\,\text{Sin}\,\theta}{A+B\cos\theta}\right]$$

2. Define unit vector, null vector and position vector.

Ans : Unit vector is a vector of unit magnitude.

Unit vector is along the direction of vector $\overline{A}$

Unit vector of $\overline{A}$ is $\hat{A} = \dfrac{\overline{A}}{|A|}$

Null vector : A vector having zero magnitude is called a null vector or zero vector.

Position vector: The position of a point can be described by a position vector, by joining the point with the origin of reference frame.

If P(x,y,z) be the coordinates and O be the origin, then position vector

$$\overline{OP} = \overline{A} = x\overline{i} + y\overline{j} + z\overline{k}$$

Magnitude of $\overline{A} = \left|\overline{A}\right| = \sqrt{x^2 + y^2 + z^2}$

3. If $\left|\overline{a}+\overline{b}\right| = \left|\overline{a}-\overline{b}\right|$, prove that the angle between $\overline{a}$ and $\overline{b}$ is 90^0?

Ans : Given $\left|\overline{a}+\overline{b}\right| = \left|\overline{a}-\overline{b}\right|$

$$\sqrt{a^2 + b^2 + 2ab\cos\theta} = \sqrt{a^2 + b^2 - 2ab\cos\theta}$$

Squaring on both sides

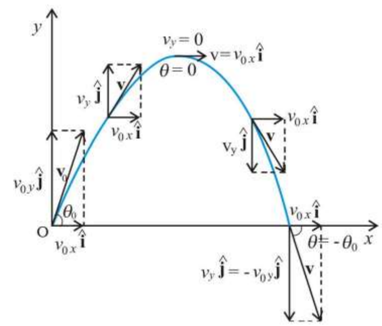
$$a^2 + b^2 + 2ab\cos\theta = a^2 + b^2 - 2ab\cos\theta$$

$$4ab\cos\theta = 0$$

$$\cos\theta = 0 \Rightarrow \cos\theta = \cos 90^0 \qquad \therefore \theta = 90^0$$

4. Show that the trajectory of an object thrown at certain angle with the horizontal is a parabola?

Ans : An object that projected making an angle θ with the horizontal (other than 90^0) is called projectile. The path followed by it is called trajectory.



Let a body is projected from point 'O', with velocity u at an angle. θ with horizontal. The velocity u can be resolved into two rectangular components u_x and u_y along X-axis and Y-axis.

$u_x = u\cos\theta$ and $u_y = u\sin\theta$

After time t, horizontal distance travelled,

$x = u\cos\theta\, t$ ------ (1)

After a time 't' sec,

Vertical displacement, $s = ut + \frac{1}{2}at^2$

Y= $u\sin\theta t$ - $\frac{1}{2}gt^2$

From (1), $t = \frac{x}{u\cos\theta}$

$$y = \left(\frac{u\sin\theta}{u\cos\theta}\right)x - \frac{1}{2}(g)\left(\frac{x}{u\cos\theta}\right)^2$$

$$= (\tan\theta)x - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

Let $\tan\theta$=A and $\frac{g}{2u^2\cos^2\theta} = B$

Then $y = Ax - Bx^2$ (parabola equation)

Hence the path of a projectile is a parabola

5. Show that the maximum height and range of a projectile are $u^2\,\mathrm{Sin}^2\theta/2g$ and $u^2\,\mathrm{Sin}\,2\theta/g$ respectively, where the terms have their regular meanings.

Ans : Let a body is projected with an initial velocity 'u' and with an angle θ to the horizontal.

Along x-axis, $u_x = u\cos\theta$

Along y-axis, $u_y = u\sin\theta$

Maximum Height (H_{max}):

The Vertical distance covered by the projectile until its vertical component becomes Zero.

From Equation, $v^2 - u^2 = 2as$

At maximum height, $v = 0$

$u = u_y = u\sin\theta$

$a = -g$

$s = h_{max}$

so $0^2 - u^2\sin^2\theta = 2(-g)h_{max}$

$-u^2\sin^2\theta = -2gh_{max}$

$$h_{max} = \frac{u^2 sin^2\theta}{2g}$$

Horizontal Range (R): It is the distance covered by projectile along the horizontal between the point of projection to the point on ground where the projectile returns again

Hence Range of Projectile (R) = horizontal component of velocity x Time of flight

$$R = u\cos\theta \times T$$

$$= u\cos\theta \times \frac{2u sin\theta}{g} \qquad \left(\because T = \frac{2u\sin\theta}{g}\right)$$

$$= \frac{u^2\left(2\sin\theta\cos\theta\right)}{g}$$

$$R = \frac{u^2\sin 2\theta}{g} \qquad (\because 2\sin\theta\cos\theta = \sin 2\theta)$$

6. Show that the maximum height reached by a projectile launched at an angle of 45^0 is one quarter of its range.

Ans. In Projectiles,

$$\text{Range (R)} = \frac{u^2 sin2\theta}{g}$$

$$\text{Maximum height (H)} \quad h_{max} = \frac{u^2 sin^2\theta}{2g}$$

When θ = 45⁰

$$R_{max=} \frac{u^2 sin2(45)}{g} = \frac{u^2\sin 90}{g}$$

$$R_{max} = \frac{u^2}{g}$$

$$H_{max} = \frac{u^2\sin^2 45}{2g} = \frac{u^2\left(\frac{1}{\sqrt{2}}\right)^2}{2g} = \frac{u^2}{4g} \qquad \therefore h_{max} = \frac{1}{4}\left(\frac{u^2}{g}\right) \qquad h_{max} = \frac{1}{4} R_{max}$$

7. If the horizontal ranges described by two projectiles, projected at angles $\left(45^0+\alpha\right)$ and $\left(45^0-\alpha\right)$ from the same point and same velocity, then the ratio of horizontal ranges will be?

Ans. (I) When the angle of projection, $\theta = 45^0 + \alpha$, the horizontal range is

$$R = \frac{u^2 sin2(45+\alpha)}{g}$$

$$= \frac{u^2 sin(90+2\alpha)}{g}$$

$$R = \frac{u^2\cos 2\alpha}{g}$$

(II) when $\theta = 45^0 - \alpha$, the horizontal range is

$$R^1 = \frac{u^2\sin 2(45-\alpha)}{g}$$

$$R = \frac{u^2 sin(90-2\alpha)}{g}$$

$$R^1 = \frac{u^2 \cos 2\alpha}{g} \qquad \therefore R^1 = R$$

8. If θ is the angle of projection, R is the range, h is the maximum height, T is the time of flight, then show that (a) $\tan\theta = \frac{4h}{R}$ and (b) $h = \frac{gT^2}{8}$

Ans : If θ is angle of projection, then

$$h = \frac{u^2 sin^2\theta}{2g} \quad ....(1)$$

$$R = \frac{u^2 \sin 2\theta}{g} \quad --------(2)$$

$$T = \frac{2u\sin\theta}{g} \quad ....-- (3)$$

(a) If Equation (1) is divided by (2) then

$$\frac{h}{R} = \frac{\frac{u^2 sin^2\theta}{2g}}{\frac{u^2 \sin 2\theta}{g}} = \frac{sin^2\theta}{2.2\sin\theta\cos\theta}$$

$$= \frac{\tan\theta}{4} \qquad \therefore \tan\theta = \frac{4h}{R}$$

(b) From Equation (1)

$$h = \frac{u^2 sin^2\theta}{2g}$$

Divide and multiply with 4g

$$= \frac{u^2 sin^2\theta}{2g} \text{ x } \frac{4g}{4g}$$

$$= \frac{g}{8}\left[\frac{2u\sin\theta}{g}\right]^2$$

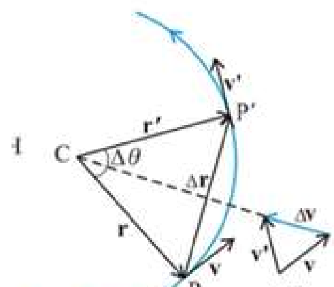
$$h = \frac{gT^2}{8}$$

9. Define centripetal acceleration? Derive an expression for the centripetal acceleration of a particle moving with uniform speed v along a circular path of radius r.

Ans. **<u>Centripetal acceleration</u>:** - The acceleration of an object moving with speed ϑ in a circle of radius R has a magnitude $\frac{\vartheta^2}{R}$ and is always directed towards the centre. This is called centripetal acceleration.

Derivation :-

When a particle moves on a circular path with uniform speed, then its velocity remains changes at each point of the path due to changing the direction of velocity. In this situation the direction of acceleration always remains towards the centre of the path.



(b2)

Suppose a particle is moving on circular path of radius r with uniform speed v. In time interval Δt, it reaches from point P to P^1. Velocity of the particle at these points is $\overline{\vartheta}$ and $\overline{\vartheta^1}$ and the distance along circular path is are Δs. Since the speed is constant.

Average acceleration $|a| = Lt_{\Delta t \to 0} \frac{|\Delta\vartheta|}{\Delta t}$ ....(1)

From diagram

$$\frac{|\Delta\vartheta|}{\vartheta} = \frac{|\Delta r|}{r}$$

$$|\Delta\vartheta| = \vartheta\frac{|\Delta r|}{r} \text{...........(2)}$$

Sub equation (2) in equation(1)

$$|a| = Lt_{\Delta t \to 0} \frac{\vartheta|\Delta r|}{r\Delta t}$$

$$|a| = \frac{\vartheta}{r} Lt_{\Delta t \to 0} \frac{|\Delta r|}{\Delta t}$$

$$|a| = \frac{\vartheta^2}{r}\left(\because Lt_{\Delta t \to 0} \frac{|\Delta r|}{\Delta t} = \vartheta\right)$$

VI. PROBLEMS

1. A projectile is fired at an angle of 60^0 to the horizontal with an initial velocity of 800 m/s:

i. Find the time of flight of the projectile before it hits the ground.
ii. Find the distance it travels before it hits the ground (range)
iii. Find the time of flight for the projectile to reach its maximum height.

Ans. i) $T = \frac{2u\sin\theta}{g}$, $T = \frac{2\times 800\times \sin 60^0}{9.8}$

$$= \frac{800\times\sqrt{3}}{9.8} = 141.4 \text{ sec}$$

$$ii) R = \frac{u^2 2\sin\theta\cos\theta}{g}$$

$$= \frac{(800)^2 \times 2 \times \sin 60^0 \times \cos 60^0}{9.8} = 56.4 \text{ km}$$

iii) $t_f = \frac{u\sin\theta}{g} = \frac{800 \times \sin 60^0}{9.8}$ = 70.7 (sec) or $t_f = \frac{T}{2} = \frac{141.4}{2} = 70.7\,\text{sec}$

2. A particle is projected from the ground with some initial velocity making an angle of 45^0 with the horizontal. It reaches a height of 7.5m above the ground while it travels a horizontal distance of 10m from the point of projection. Find the initial speed of projection, $\left(g = 10m/s^2\right)$

Ans. $y = (\tan\theta)x - \frac{gx^2}{2u^2\cos^2\theta}$

$7.5 = \tan 45^0 \times 10 - \frac{10 \times 10^2}{2u^2 \times \cos^2\left(45^0\right)}$

Initial speed u = 20 m/sec

3. For a particle projected slantwise from the ground, the magnitude of its position vector with respect to the point of projection, when it is at the highest point of the path is found to be $\sqrt{2}$ times the maximum height reached by it. Show that the angle of projection is $\text{Tan}^{-1}(2)$

Ans. Given that h is max height and r is radius

H → h

$r = \sqrt{2}H$

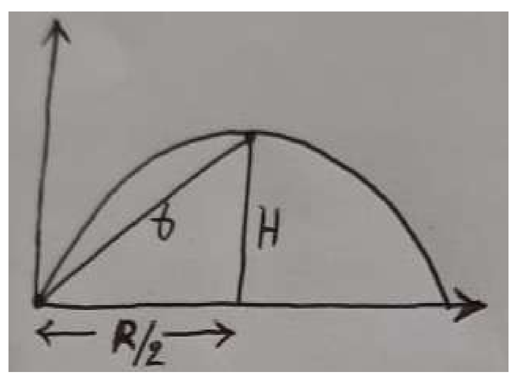
$\Rightarrow r^2 = 2H^2$

But $r^2 = H^2 + \frac{R^2}{4} \Rightarrow 2H^2 = H^2 + \frac{R^2}{4}$

$H^2 = \frac{R^2}{4} \Rightarrow H = \frac{R}{2}$

$\frac{u^2\sin^2\theta}{2g} = \frac{1}{2}\left[\frac{u^2 2\sin\theta\cos\theta}{g}\right]$

Tan θ = 2 → Tan^{-1} (2)



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UNIT – 4

LAWS OF MOTION

I. MULTIPLE CHOICE QUESTIONS:

1) When a bus starts suddenly the passengers are pushed back. This is an example for **[1]**

(1) Newton's First Law (2) Newton's Second Law (3) Newton's Third Law

(4) Newton's law of gravitation

Ans : The lower part of his body comes into motion along with the bus whereas the upper part of his body tends to remain at rest due to inertia of rest

2. Which of the following is an example of centripetal force? **[3]**

(1) Pushing door (2) Mud flying off tire (3) planets orbiting around the sun

(4) churning of butter in buttermilk

Ans : The gravitational force between the sun and planets acts as centripetal force, keeping them in their orbits

3. A non-zero net force exerted on a body does not change its **[4]**

(1) Direction of motion (2) momentum (3) acceleration (4) inertia

4) Rate of change of momentum is proportional to **[3]**

(1) Acceleration (2) velocity (3) force (4) displacement

5) The net force (F)experienced by a body of mass m whose displacement is **[2]**

$y = ut + \frac{1}{2}gt^2$ is

(1) $F = ma$ (2) $F = mg$ (3) $F = 0$ (4) $F \neq 0$

Ans : $y = ut + \frac{1}{2}gt^2$

$$\frac{dy}{dt} = u + \frac{1}{2}g.2t$$

$$\frac{d^2y}{dt^2} = a = 0 + g$$

$$F = ma = mg$$

6) The linear momentum of a particle as a function of time t is given by P = $a + bt$. What is the force acting on the particle?

[2]

(1) a (2) b (3) a / b (4) b/a

Ans $P = a + bt$; $F = \frac{dP}{dt} = b$

7) The inertia of a moving object depends on **[4]**

(1) momentum of the object

(2) speed of the object

(3) shape of the object

(4) mass of the object

Ans : The inertia of a moving object is directly related to its mass. More is the mass of the object and more will be the inertia. Inertia does not depend upon the speed, shape or momentum.

8) A force accelerates a lighter vehicle more easily than a heavier vehicle. Guess the behind this? [2]

(1) Newton's first law of motion (2) Newton's Second Law of motion

(3) Newton's Third Law of motion (4) Newton's Law of gravitation

Ans: According to Newton's Second Law of motion, Acceleration is inversely proportional to its mass($F = ma$). This means that if the same force is applied to two objects with different masses, the object with less mass will experience a greater acceleration.

9) Calculate the time needed for a net force of $5N$ to change the velocity of a $10\ kg$ mass by $2\ m/s$ **[3]**

(1) $2sec$ (2) $6sec$ (3) $4sec$ (4) $3sec$

Ans : $F = ma = m\left(\frac{v-u}{t}\right)$

$$5 = 10\left(\frac{2-0}{t}\right)$$

$$\therefore t = 4sec$$

II. FILL IN THE BLANKS:

1) Friction opposes the ______ between the surfaces in contact with each other.

Ans: Relative motion

Friction acts a force that resists the movement or tendency of movement of one surface against another when they are touching

2) ______ is required to keep a body in accelerated motion.

Ans: (External force) non zero net force

According to Newton's Second Law of motion, a net external force is necessary to change the state of motion of an object.

3) Kinetic friction produces ______ energy

Ans: Heat

Friction produces heat because it causes the molecules on rubbing surfaces to move faster and have more energy

4) Powder is sprinkled on the carom board to ______ friction between the striker and the board surface

Ans: Reduce

The powder fills in small gaps on the board, creating a smoother surface that allows the striker and coins to glide more easily

5) To every action there is always an equal and ______ reaction.

Ans: opposite

Newton's third law of motion states that to every action there is an equal and opposite reaction

6) Impulse is the product of force and time which equals ______

Ans: Change in momentum

Impulse = Force × time = Change in momentum

7) ______ gives the measure of inertia.

Ans: Mass

Inertia is a body's resistance to changes in its state of motion. A larger mass means greater inertia requiring more force to accelerate it

8) The area under F-t graph represents--------------

Ans : Impulse $J = \int F dt = F \Delta t$

∫Fdt = The area under the Force- time graph

9) The Newton's Second Law is consistent with the first law only if ------------------

Ans : F=0

The Newton's Second Law is consistent with First Law. F=0 implies a=0

10) Sliding friction is ______ than limiting value the static friction.

Ans : Less

_ When an object is already moving the force of friction is less than the force required to initiate movement from a stationary position

III. ONE WORD ANSWER QUESTIONS:

1) State Newton's first Law.

Ans: Every body continues to be in its state of rest or of uniform motion in a straight line unless compelled by an external force to change that state.

2) State Newton's third Law.

Ans: To every action, there is always an equal and opposite reaction

3) Write the mathematical equation of Newton's Second Law.

Ans : $F = \frac{dP}{dt} = ma$

4) State the SI unit of Force.

Ans : Newton

5) Define centripetal force.

Ans: Centripetal force is the force acting on an object in curvilinear motion directed towards the axis of rotation or center of curvature.

6) According to Newton's Third Law every force is accompanied by an equal and opposite force. How can a movement ever take place?

Ans: Because both action and reaction are taking place on different bodies

7) Why does a heavy refile not recoil as strongly as light rifle using the same cartridges?

Ans:

1. Recoil of the gun $\upsilon = \frac{mu}{M} \Rightarrow \upsilon \alpha \frac{1}{M}$

2. Due to heavy mass of the rifle, the recoil is less

IV. VERY SHORT ANSWER TYPE QUESTIONS

1) What is inertia? What gives the measure of inertia?

Ans: The resistance of the body to change its state of rest or state of uniform motion in a straight line is called inertia of the body Mass gives the measure of inertia

2) When a bullet is fired from a gun, the gun gives a kick in the backward direction. Explain.

Ans : From the law of conservation of momentum, total momentum before and after firing is same and is equal to zero. So as the bullet gets forward momentum, the gun will get backward momentum. Then the gun gives a kick in the backward direction.

Due to Law of Conservation of Momentum,Recoil of the gun $V = \frac{mu}{M}$

Where M= mass of the gun, m= mass of the bullet and u = velocity of the bullet

3) If a bomb at rest explodes into two pieces, the pieces must travel in opposite directions. Explain.

Ans: According to Law of Conservation of momentum

$Mu = m_1V_1 + m_2V_2$

initially the bomb is at rest, hence u=0

$$m_1V_1 + m_2V_2 = 0$$

$$m_1\mathrm{V} = -m_2V_2$$

Negative sign indicates that the pieces must travel in opposite direction

4) Define force. What are the basic forces in nature?

Ans: **Force:** The force on which changes or tends to change the state of rest or motion of a body

Basic forces

1. Gravitational force
2. Electromagnetic force
3. Nuclear force
4. Weak Nuclear force

5) State the SI units and dimensions of Linear Momentum.

Ans: 1. Dimensional Formula of Linear momentum is MLT^{-1}

2. SI unit is kg m s^{-1}(or) N s

6) State the SI units and dimensions of impulse

Ans: 1) Dimensional formula of Impulse = MLT^{-1}

2) SI Unit is Kg m s^{-1} (or) N s

7) Can the coefficient of friction be greater than one?

Ans : Yes, coefficient of friction may be greater than one. In some particular cases it is possible they are:

1) Due to increase in the inner molecular attractive forces between surfaces when the contact surfaces are highly polished.
2) When the contact surfaces of the bodies are interlocking, the coefficient of friction may be greater than one.

8) Why does the car with a flattened tyre stop sooner than the one with inflated tyres?

Ans: Flattened tyre deforms more than the inflattened tyre

Due to greater deformation of the tyre, rolling friction is large, hence it stops soon

9) A horse has to pull harder during the start of the motion than later. Explain.

Ans: Static friction is greater than kinetic friction. Before starting the motion a horse acted upon by static friction and later it is acted upon by kinetic friction. So the horse has to pull the cart harder at the beginning to overcome static friction.

10) What happens to the coefficient of friction if the weight of the body is doubled?

Ans: 1) If weight of a body is doubled, coefficient of friction does not change

2) Coefficient of friction is independent of normal reaction

11. Write the expressions for maximum possible speed of a car moving on (a) level road (b) banked road.

Ans: a) Maximum possible speed of a car moving on a level road is $V_{max} = \sqrt{\mu_s Rg}$

b) Maximum possible speed of a car on a banked road is $V_{max} = \sqrt{Rg\ tan\theta}$

V. SHORT ANSWER TYPE QUESTIONS

1) **A stone of mass $0.1kg$ is thrown vertically upwards. Give the magnitude and direction of the net force on the stone (a) during its upward motion, (b) during its downward motion, (c) at the highest point, where it momentarily comes to rest.**

Ans: Given that mass of stone $m = 0.1kg,\ g = 9.8ms^{-2}$

a) During upward motion:

Magnitude of net force on the stone. $F = |-mg| = 0.1 \times 9.8 = 0.98N$

Direction of net force is in upward direction

b) During downward motion:

Magnitude of net force on the stone $F = mg = 0.1 \times 9.8 = 0.98N$

Direction of net force is in downward direction

c) At the highest point:

Magnitude of net force on the stone $F = mg = 0.1 \times 9.8 = 0.98N$

At highest point of stone, direction is indeterminate

2) **Define the terms momentum and impulse. State and explain the Law of Conservation of linear momentum. Give examples.**

Ans: **Momentum:** The product of mass and velocity of a body is called momentum

$$P = mV$$

Impulse: The product of force and time that produces finite change in momentum of the body is called impulse.

$$\text{Impulse} = \text{Force} \times \text{time} = F \times t = mv - m\mu$$

<u>Law of conservation of linear momentum</u>: The total momentum of an isolated system of interacting particles remains constant if there is no resultant external force acting on it.

<u>Explanation</u>: momentum of a system before collision = momentum of system after collision

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Examples: 1) motion of a rocket, 2) Bullet-gun motion

3) **Why are shock absorbers used in motorcycles and cars?**

Ans : 1) When a scooter or a car moves on a rough road it receives an impulse to the jerkey motion

2) In case the shock absorbers are used in the vehicle, the time of impact increases and decreases the implusive force

3) Due to increased value of the time of impact the force of impact is reduced

4) So it saves the vehicle and its occupants from experiencing reverse jerks

4) **Explain the terms limiting friction, dynamic friction and rolling friction.**

Ans: <u>Limiting friction:</u> The maximum value of static friction is called limiting friction

<u>Dynamic friction:</u> The resistance encountered by a sliding body on a surface is called dynamic friction

<u>Rolling friction:</u> The resistance encountered by a rolling body on a surface is called rolling friction

5) **Explain the advantages and disadvantages of friction.**

Ans:

<u>Advantages of friction:</u>

1) Safe walking on the floor, motion of vehicles etc. are possible due to friction

2) Nails, screws are driven into walls or wooden surfaces due to friction

3) Friction helps the fingers hold things or objects like pen, pencil and water tumbler etc.

4) A match stick is lighted due to friction

5) A mechanical power transmission of belt drive is possible due to friction

<u>Disadvantages of friction:</u>

1) Due to friction, there is large amount of power loss in machines and engines

2) Due to friction, wear and tear of the machine increases and reducing their life.

3) Due to friction, some energy gets converted into heat which goes as waste

6) **Mention the methods used to decrease friction.**

Ans:

<u>1) Polishing:</u> By polishing the surfaces of contact, friction can be reduced

2) <u>Ball bearings:</u> The rolling friction is less than the sliding friction, hence free wheels of a cycle, motor car, dynamos etc. are provided with ball bearings to reduce friction.

3) Lubricants: The lubricants form a thin layer between surfaces of contact. It reduces the friction in light vehicles or machines. Oils like 'three in one' are used as lubricants. In heavy machines grease is used.

4) Streamlining: Automobiles and aeroplanes are streamlined to reduce the friction due to air.

7) State the Laws of rolling friction.

Ans:

Laws of rolling friction:

1) The smaller the area of contact, the lesser will be the rolling friction

2) The larger the radius of rolling body, the lesser will be the rolling friction

3) The rolling friction is directly proportional to the normal reaction

$$f_r \,\alpha\, N \implies f_r = \mu_r N$$

8) Derive an expression for the maximum possible speed of a car moving on a banked road?

Ans: Let the coefficient of friction is μ and banking angle is θ.

For a car taking a turn on a banked road with friction, the forces acting on the car are :

1) Weight - mg 2) Normal reaction - N 3) frictional force – f 4) Centripetal force - $\frac{mV^2}{R}$

There is no acceleration along the vertical direction, the net force along this direction muse be zero

$$N\cos\theta = mg + f\sin\theta \qquad \_\_\_\_\_\_\_\_1$$

The centripetal force is provided by the horizontal components of N and f.

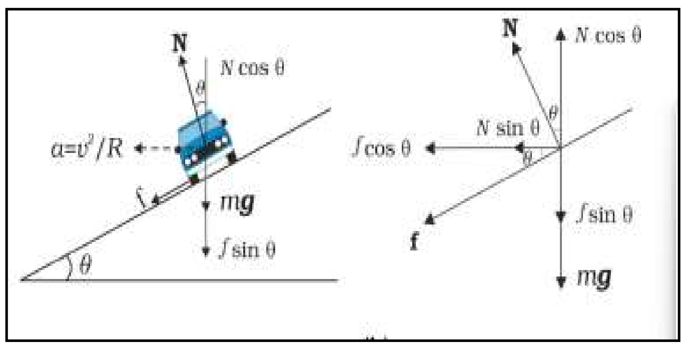
$$N\sin\theta + f\cos\theta = \frac{mV^2}{R} \qquad \_\_\_\_\_\_\_\_2$$

But $f \le \mu_s N$

Thus, to obtain V_{max}, we put $f = \mu_s N$

$$N\cos\theta = mg + \mu_s N\sin\theta \qquad \_\_\_\_\_\_\_\_3$$

$$N\sin\theta + \mu_s N\cos\theta = \frac{mV^2}{R} \qquad \_\_\_\_\_\_\_\_4$$



$$N = \left(\frac{mg}{\cos\theta - \mu_s \sin\theta}\right)$$

Substituting value of N in Eq. 4

$$\left(\frac{mg}{\cos\theta - \mu_s \sin\theta}\right)(\sin\theta + \mu_s \cos\theta) = \frac{mV_{max}^{\ \ 2}}{R}$$

$V_{max^2} = \text{Rg}\left[\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right]$

Maximum possible speed of car on banked road if $\mu_s = 0$

$$\therefore V_{max^2} = \text{Rg} \tan\theta$$

$$V_{max} = \sqrt{Rg tan\theta}$$

VI. LONG ANSWER TYPE QUESTIONS

1) **a. State Newton's second Law of motion. Hence derive the equation of motion $F = ma$ from it.**

b. A body is moving along a circular path such that its speed always remains constant. Should there be force acting on the body?

Ans:

(a) Newtons Second Law of Motion:

The rate of change of momentum of a body is directly proportional to the external forces applied and takes place in the same direction

Derivation of F=ma:

1) Let a body of mass m moving with velocity v under the action of an external force F in the direction of velocity
2) Momentum of the body $p = m$v
3) According to Newton's second law of motion

$$\frac{dp}{dt} \propto F$$

$$F = K\frac{dp}{dt}$$

From equations 1 and 2

$F = K\,\frac{\text{d (mv)}}{\text{dt}} = Km\,\frac{\text{dv}}{\text{dt}}$

$the\ rate\ of\ change\ of\ velocity = \frac{\text{dv}}{\text{dt}} = a$

Hence $F = Kma$

Unit force:

The force which when acting on a body of mass of 1kg produces in it an acceleration of $1ms^{-2}$

From eq3., If $F = 1, m = 1\ and\ a = 1$. We get K=1

$Hence\ F = \frac{\text{dP}}{\text{dt}} = ma$

(b) Suppose a body is moving along a circular path, through its speed always remains constant, Its velocity changes at every point and resultant force acts on the body.

VII. PROBLEMS

1) **A force of $2i + j - k$ N acts on a body which is initially at rest. At the end of 20 $seconds$ the velocity of the body is $4i + 2j - 2k\ m/s$. What is the mass of the body ?**

Ans: Given

$\vec{F} = 2i + j - k, N$

$V = 4i + 2j - 2km/s$

u = 0 and t = 20 sec

F = ma

$$F = m\left(\frac{v-u}{t}\right)$$

$$m = \frac{F.t}{v-u}$$

$$m = \frac{(2i+j-k).20}{(4i+2j-2k)-0}$$

$$m = \frac{(2i+j-k).20}{2.(2i+j-k)}$$

mass (m) = $10kg$

2) **A ball of mass m is thrown vertically upward from the ground and reaches a height h before momentarily coming to rest. If g is acceleration due to gravity. What is the impulse received by the ball due to gravity force during its flight (neglect air resistance)?**

Ans: mass = m

initial velocity of a ball to reach height h is $u = \sqrt{2gh}$

On return journey:

velocity of a ball to reach the ground $v = -\sqrt{2gh}$

Impulse $J = m\,(u - v)$

$$J = m\left(\sqrt{2gh} - \left(-\sqrt{2gh}\right)\right)$$

$$J = m\left(\sqrt{2gh} + \sqrt{2gh}\right)$$

$$J = 2m\sqrt{2gh}$$

$$J = \sqrt{8m^2gh}$$

3) A container of mass $200kg$ rests on the back of an open truck. If the truck accelerates at $1.5m/s^2$,What is the minimum coefficient of static friction between the container and the bed of the truck required to prevent the container from sliding off the back of the truck?

Ans: $m = 200kg,\ a = 1.5ms^{-2}\ and\ g = 9.8ms^{-2}$

$$ma = \mu_s mg$$

$$\mu_s = \frac{a}{g} = \frac{1.5}{9.8} = 0.153$$

4) A bomb initially at rest at a height of $40m$ above the ground suddenly explodes into two identical fragments. One of them starts moving vertically down-wards with an initial speed of $10m/s$.If acceleration due to gravity is $10m/s^2$. What is the separation between the fragments $2s$ after the explosion.

Ans: Bomb explodes into two fragments 1 and 2

For first fragment:

$$u_1 = 10\ m/s$$
$$t = 2\ s$$
$$g = 10\ ms^{-2}$$

Now displacement of 1st fragment

$$S_1 = u_1 t + \frac{1}{2} gt^2$$
$$S_1 = 10 \times 2 + \frac{1}{2} \times 10 \times 2^2$$
$$S_1 = 20 + 20 = 40m$$

For second fragment :

$$u_2 = -u_1 = 10\ m/s$$
$$t = 2\ s$$
$$g = 10\ ms^{-2}$$

Now displacement of 2nd fragment

$$S_2 = u_2 t + \frac{1}{2} gt^2$$
$$S_2 = -10 \times 2 + \frac{1}{2} \times 10 \times 2^2$$
$$S_2 = -20 + 20 = 0m$$

The separation of two fragments $S_1 + S_2 = 40m + 0m = 40m$

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UNIT – 5

WORK, ENERGY AND POWER

I. MULTIPLE CHOICE QUESTIONS

1. A man lifted a box of 50 N weight steadily from a ground to a height of 8 m. Then the work done by him on the box is **(4)**

(1) 25 J (2) 100 J (3) 200 J (4) 400 J

Ans: Given weight = mg = 50 N.

Height = h = 8m

Work done w = FS = mg × h = 50 × 8 = 400 J.

2. Two masses 2 kg and 8 kg are moving with equal kinetic energies. The ratio of the magnitudes of their liner momenta is **(3)**

(1) 1:4 (2) 4:1 (3) 1:2 (4) 2:1

Ans: We have $\frac{P1}{P2} = \sqrt{\frac{m1}{m2}}$ (we have k = $\frac{P^2}{2m}$, k is same then P α $\sqrt{m}$)

$\frac{P1}{P2} = \sqrt{\frac{2}{8}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$ $P_1 : P_2$ = 1: 2

3. Internal forces can change **(2)**

(1) the linear momentum, not kinetic energy.

(2) the kinetic energy, not the linear momentum.

(3) linear momentum as well as kinetic energy.

(4) neither the linear momentum nor the kinetic energy.

Ans. internal forces can change the kinetic energy of a system, but not its linear momentum.

4. The kinetic energy of a body is 4 times its momentum. Then the speed of the body is **(1)**

(1) 8 m/s (2) 4 m/s (3) 2 m/s (4) 10 m/s

Ans. Given kinetic energy = 4 (momentum)

K = 4P

$\frac{1}{2} m\vartheta^2 = 4(m\vartheta)$ $\frac{\vartheta}{2} = 4$ ϑ = 8 m/s

5. A man increases the speed of his car from 20 m/s to 40 m/s on a smooth road. The ratio of final kinetic energy to the initial kinetic energy is **(2)**

(1) 1:4 (2) 4:1 (3) 1:2 (4) 2:1

Ans. Given ϑ_1 = 20 m/s, ϑ_2 = 40 m/s

Ratio of Final K.E to initial K.E = $\frac{\frac{1}{2}m\vartheta_2^2}{\frac{1}{2}m\vartheta_1^2}$

$$= \frac{\vartheta_2^2}{\vartheta_1^2} = \frac{(40)^2}{(20)^2} = \frac{16}{4} = \frac{4}{1} \qquad = 4:1$$

5. The work energy theorem states that the change in kinetic energy of a body is equal to the network done by **(4)**

(1) all conservative and non conservative forces only

(2) all external and internal forces only

(3) all pseudo forces only

(4) all types of forces

Ans. From work energy theorem the net work done by all types of forces.

7. A body is falling freely under the action of gravity alone in vacuum. Which of the following quantities remain constant during the fall? **(3)**

(1) Kinetic energy

(2) Potential energy

(3) Total mechanical energy

(4) Total linear momentum

Ans. When a body falls freely under the action of gravity alone in a vacuum, its total mechanical energy remains constant. Here the decrease in potential energy is equal to gain in kinetic energy.

8. The work done by weight lifter holding a 150 kg mass steadily on his shoulder in a time of 30 second is **(3)**

(1) 450 J (2)180 J (3)0 J (4) 50 J

Ans. Given m = 150 kg, t = 30 sec

Work done W = FS

In this case displacement S = 0

$$W = F \times 0 = 0$$

Work done is ZERO

9. A person fires a bullet of mass 50 grams with a speed of 200 m/s on a soft plywood of thickness 2 cm. The bullet emerges with 10% of its initial kinetic energy. Then its final kinetic energy is **(4)**

(1) 500 J (2) 1000 J (3) 2000 J (4) 100 J

Ans. Given mass m = 50 gm $= 50 \times 10^{-3}$ kg

Speed ϑ = 200 m/s

Kinetic energy K.E $= \frac{1}{2} m\vartheta^2$

$$= \frac{1}{2} \times 50 \times 10^{-3} \times (200)^2$$

$$= \frac{1}{2} \times 50 \times 10^{-3} \times 4^2 \times 10^4$$

Final K.E = 10% initial K.E

= 10% × 1000 = 100 J

10. A rain drop of mass 1 gm is falling from a height of 1 km. It hits the ground with a speed of 50 m/s. The work done by the gravitational force is (Take acceleration due to gravity g = 10 m/s^2) (2)

(1) 5 J (2) 10 J (3) 15 J (4) 20 J

Ans. Given m = 1gm = 1×10^{-3} kg, g = 10 m/s^2

Height h = 1 Km =10^3 m

work done = potential energy

W = mgh = $1 \times 10^{-3} \times 10 \times 10^3$

= 10 J

11. The elastic potential energy of a spring of force constant K for an extension x is (2)

(1) kx (2) 1/2 kx^2 (3) 1/2 Kx (4) 1/2 K^2x^2

Ans. Elastic potential energy = $\frac{1}{2}$ Kx^2

When K = force constant, x = extension.

12. A bob of mass M is suspended by light string of length L. The speed of the bob at the lowest point of vertical circular path is (4)

(1) $\sqrt{gL}$ (2) $\sqrt{2gL}$ (3) $\sqrt{3gL}$ (4) $\sqrt{5gL}$

Ans. The speed of the bob at local point of vertical circular path is $\sqrt{5gL}$.

II. FILL IN THE BLANKS

1. The work done by a conservative force in moving a body along a closed path is_________

Ans : zero.

In closed path the work done is zero.

2. Kinetic energy is lost in case of _____________ collisions.

Ans : . Inelastic

Inelastic collisions K.E is not conserved.

3. The rate at which work is done is called _________________

Ans : Power

The rate at which work is done is called power

4. 5 kwh = _________ joules.

Ans. 18×10^6 J.

5kwh = $5 \times 3.6 \times 10^6$ J

= 18×10^6 J.

5. The power utilized when work of 1000 J is done in 2 seconds is ________________

Ans. 500 Watts

Power P = $\frac{W}{t} = \frac{1000}{2} = 500$ watts

6. The compressed or elongated spring possesses ____________ energy.

Ans. Potential energy.

Compressed or elongated spring possesses potential energy

7.A force f= 5î+3j+2k N is applied over a particle which displaces it from its origin to the point r = (3i - 2j) m. The work done on the particle is _______

Ans. 9 J

Work done W =F.S

Hence F = (5i +3j +2k) N

S = (3i – 2j) m

W = (5i + 3j + 2k). (3i – 2j)

= 15 – 6 = 9 J

8. Sailing ship employ ___________ energy of the wind.

Ans. Kinetic

Sailing ship employ kinetic energy of the wind.

9. The area under force-displacement curve is ________

Ans. Work done

The area under force – displacement curve gives work done.

10. When two equal masses undergo oblique elastic collision with one of them at rest, after the collision they will move at _______________ to each other

Ans. Right angles.

When two equal masses undergo oblique elastic collision with one of them at rest, after the collision they will move at right angles to each other

11. 1 horse power = _____________ watts.

Ans. 746

1 horse power = 746 watt.

III. ONE WORD ANSWER QUESTIONS

1. When a body of mass M is lifted up through a height h, what is the work done by gravitational force?

Ans. Work done by gravitational force is Mgh where g is acceleration due to gravity.

2. Which physical quantity is conserved in all kinds of collisions?

Ans. Linear momentum (or) total energy is conserved in all type of collisions.

3. When two bodies undergo perfectly inelastic collision, what is their relative velocity after collision?

Ans. In a perfectly inelastic collision the two bodies stick together after collision so their relative velocity after collision is zero.

4. A man carrying a bag of sand walks on a horizontal road with uniform velocity. What is the work done by him if the road is smooth?

Ans. Work done w = FScosθ = 0.

Because force required to hold sand bag and his displacement are particular to each other.

5. If two vectors 2î+2j+k and 3î-6j+nk are perpendicular to each other then what is the value of n?

Ans. If two vectors perpendicular to each other their dot product is zero.

(2j + 2j + k) . (3i – 6j + nk) = 0

6 – 12 + n = 0

- 6 + n = 0 , n = 6

6. A motor boat is going in a river with a velocity $\vartheta = 7\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ m/s. If the resisting force due to stream is F = $9\hat{\imath}+3\hat{\jmath}-3\hat{k}$ N. What is the power of the boat?

Ans. P = $\bar{F}.\bar{\vartheta}$

P = $(9\hat{\imath} + 3\hat{\jmath} - 3\hat{k})$. $(7\hat{\imath} + 2\hat{\jmath} - 5\hat{k})$

P = 63 +6 +15

P = 84 Watts.

7. Find the angle between F = $(\hat{\imath} + 2\,\hat{\jmath} - \hat{k})$ N and displacement d = $(-\,\hat{\imath} + \hat{\jmath} - 2\hat{k})$ m.

Ans. F.d = Fdcosθ

$(\hat{\imath} + 2\hat{\jmath}$ - k) . $(-\hat{\imath} + \hat{\jmath}$- 2k) = -1 + 2 +2 = $\sqrt{6}\sqrt{6}\,cos\theta$

-1+2+2 = 6 $cos\theta$

3 = 6cosθ

Cos θ = $\frac{3}{6} = \frac{1}{2}$ θ = 60°

8. A body is dragged along a horizontal floor by a rope. The rope makes an angle of 60° with the horizontal. If the tension in the rope is 100 N and the body is moved through a distance of 10 m, then what is the work done?

Ans. Given tension F = mg = 100 N

Distance S = 10m

Angle θ = 60°

Work done W = FScosθ

W = 100 × 10 × cos 60°

= 1000 × $\frac{1}{2}$ = 500 J

9. What is the dimensional formula for force constant or spring constant?

Ans. We have F = - Kx

Spring constant K = $\frac{F}{x} = \frac{MLT^{-2}}{L}$ = [MT^{-2}]

Dimensional formula of force constant is [MT^{-2}]

IV. VERY SHORT ANSWER QUESTIONS

1. State the conditions under which a force does not work.

Ans. Conditions for a force does not work are

1. The displacement is zero.
2. The angle between force and displacement is zero.

2. Define work, power and energy. State their SI units.

Ans. **Work:-** The work done by the force is defined to be product of component of the force in the direction of displacement and the magnitude of displacement

Work (W) = (Fcosθ) d = F.d

S.I Unit is joules

Energy: - The capacity to do work is called energy

S.I Unit is joules

Power :- The time rate at which work is done is called power.

Power P = $\frac{W}{t} = \frac{work\ done}{time}$

S.I Units is watts (W)

3. State the relation between the kinetic energy and momentum of a body.

Ans. A body of mass m is moving with velocity ϑ, its kinetic energy is KE = $\frac{1}{2}m\vartheta^2$ and its momentum is P = m ϑ

$$KE = \frac{1}{2}m\vartheta^2 = \frac{1}{2}\frac{m\vartheta^2}{m} \times m$$

$$= \frac{1}{2}\frac{m\vartheta^2}{m} = \frac{p^2}{2m}$$

4. State the sign of work done by a force in the following

(a) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket

Ans. In this case force and displacement are in same direction θ = 0

The work done is positive W = FScosθ = FS

(b) work done by gravitational force in the above case

Ans. In this case force and displacement are in opposite direction and θ = 180°

The work done is negative W = FScos180° = - FS

5. State the sign of work done by a force in the following

(a) Work done by friction on a body sliding down an inclined plane

Ans. As frictional force and displacement are in opposite directions, work done is negative

(b) Work done by gravitational force in the above case.

Ans. As gravitational force and displacement of the body are in the same direction, work done is positive.

6. State the sign of work done by a force in the following

(a) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity

Ans. As applied force and displacement are in the same direction, work done is positive.

(b) Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

Ans. As force and displacement are in opposite direction work done is negative.

7. State if each of the following statements is true or false. Give reasons for your answer

(a) Total energy of a system is always conserved, no matter what internal and external forces on the body are present

Ans. True - Total energy is always conserved.

(b) The work done by earth's gravitational force in keeping the moon in its orbit for its one revolution is zero.

Ans. True – The work done by a conservative force for a closed path is zero. Gravitational force is conservative force.

V. SHORT ANSWER QUESTIONS

1. What is potential energy? Derive an expression for the gravitational potential energy.

Ans. Potential energy: - Potential energy is the stored energy by virtue of the position or configuration of a body.

Ex: - 1. A stretched bow-string possess potential energy

2. The fault lines in the earth's crust are like compressed springs and possess potential energy.

Expression for Gravitational Potential energy

The gravitational force on a body of mass m near the earth's surface is mg.Where g is acceleration due to gravity

Let the body raised up to a height h. Here h is very small compared to the radius of the earth, then we can ignore the variation of g near the earth surface.

The work done by the external agency against gravitational force is.

$$W = F \times s = - mg \times h = - mgh$$

The negative sign indicates that the gravitational force is downward.

The negative of work done by the gravitational force in raising the object to a height h is equal to the gravitational potential energy denoted by **V**(h)

$$V(h) = - (- mgh) = mgh$$

$$\text{Gravitational Potential energy } V(h) = mgh$$

2. A lorry and a car moving with the same momentum are brought to rest by the application of brakes, which provide equal retarding forces. Which of them will come to rest in shorter time? Which will come to rest in less distance?

Ans. Given momentum P = mϑ is same for both lorry and car. Here force applied by brakes is also same for lorry and car.

According to work – energy theorem

Work done to stop the body = Kinetic energy stored

$$W = F.\,S = \frac{1}{2}\,m\vartheta^2 = \frac{p^2}{2m}$$

$$\therefore \quad S \,\alpha\, \frac{1}{m}$$

Since stopping distance is inversely proportional to mass lighter mass vehicle car will travel longer distance when momentum force are same So lorry travels less distance than car before it is stopped.

The retarding force acting on the vehicle is $F = \frac{Change\ in\ momentum}{time}$

Since force and momentum are same for both car and lorry t is independent of mass, so both lorry and car comes to rest in same time.

3. Distinguish between conservative and non-conservative forces with one example each.

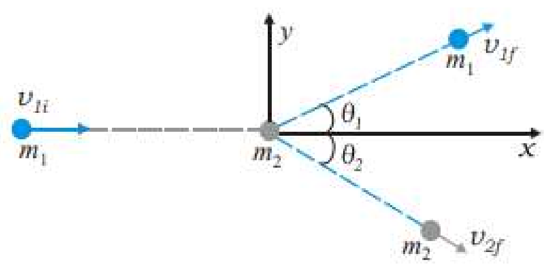
Ans. **<u>Conservation Forces: -</u>** If the work done by the force in a closed path is zero, then the force is called conservative force. Work done by the conservative force depends only on the end points and does not depend on the path followed.

Ex:- Gravitational force, Elastic spring forces.

<u>Non Conservative Force :-</u> If the work done by the force in a closed path is not zero then the force is called non conservative force. Work done by the conservative force depends on the path followed between two end points.

Ex: Frictional force, Viscous force.

4. Show that two equal masses undergo oblique elastic collision will move at right angles after collision, if the second body initially at rest.



Consider two bodies of equal mass m. The first particle is moving with ϑ_{1i} velocity collides with second particle which is at stationary as shown in figure.

The initial and final velocities of m_1 mass are ϑ_{1i} and ϑ_{1f} respectively.

The initial and final velocity of m_2 mass are ϑ_{2i} and ϑ_{2f} respectively.

According to law of conservation of linear momentum

total linear momentum before = total linear momentum after collision

$$m_1 \vartheta_{1i} + m_2 \vartheta_{2i} = m_1 \vartheta_{1f} + m_2 \vartheta_{2f} \longrightarrow 1$$

Along X – direction $\vartheta_{2i} = 0$ eqn **1** can be written as

$$m_1 \vartheta_{1i} = m_1 \vartheta_{1f} \cos\theta_1 + m_2 \vartheta_{2f} \cos\theta_2$$

If $m_1 = m_2$ $\vartheta_{1i} = \vartheta_{1f} \cos\theta_1 + \vartheta_{2f} \cos\theta_2$

Square on both sides $\vartheta_{1i}^2 = \vartheta_{1f}^2 \cos^2\theta_1 + \vartheta_{2f}^2 \cos^2\theta_2 + 2\vartheta_{1f}\vartheta_{2f}\cos\theta_1\cos\theta_2 \longrightarrow 2$

Along Y- direction $\vartheta_{1i} = \vartheta_{2i} = 0$

$$0 = m_1 \vartheta_{1f} \sin\theta_1 - m_2\vartheta_{2f} \sin\theta_2$$

If $m_1 = m_2 = m$, $0 = \vartheta_{1f} \sin\theta_1 - \vartheta_{2f} \sin\theta_2$

Squaring on both sides $0 = \vartheta_{1f}^2 sin^2\theta_1 + \vartheta_{2f}^2 sin^2\theta_2 - 2\vartheta_{1f}\vartheta_{2f}sin\theta_1 sin\theta_2 \longrightarrow 3$

Adding eqn **2** and **3**

$$\vartheta_{1i}^2 + 0 = \vartheta_{1i}^2 = \vartheta_{1f}^2\, cos^2\theta_1 + \vartheta_{2f}^2\, cos^2\theta_2 + 2\vartheta_{1f}\vartheta_{2f}cos\theta_1 cos\theta_2 + \vartheta_{1f}^2 sin^2\theta_1 + \vartheta_{2f}^2 sin^2\theta_2 - 2\vartheta_{1f}\vartheta_{2f}sin\theta_1 sin\theta_2$$

$$\vartheta_{1i}^2 = \vartheta_{1f}^2\, (cos^2\theta_1 + sin^2\theta_1) + \vartheta_{2f}^2\, (cos^2\theta_1 + sin^2\theta_1) + 2\vartheta_{1f}\vartheta_{2f}(cos\theta_1 cos\theta_2 - sin\theta_1 sin\theta_2)$$

$$\vartheta_{1i}^2 = \vartheta_{1f}^2 + \vartheta_{2f}^2 + 2\vartheta_{1f}\vartheta_{2f}(cos\ \theta_1 + \theta_2) \longrightarrow 4$$

From the law of conservation of kinetic energy $\frac{1}{2}\, m_1\vartheta_{1i}^2 = \frac{1}{2}\, m_1\vartheta_{1f}^2 + \frac{1}{2}\, m_{12}\vartheta_{2f}^2$

If $m_1 = m_2 = m$

$$\vartheta_{1i}^2 = \vartheta_{1f}^2 + \vartheta_{2f}^2 \longrightarrow 5$$

Substitute eqn **5** in **4**

$$\vartheta_{1i}^2 = \vartheta_{1i}^2 + 2\vartheta_{1f}\vartheta_{2f}(cos\ \theta_1 + \theta_2)$$

$$2\vartheta_{1f}\vartheta_{2f}(cos\ \theta_1 + \theta_2) = 0$$

$$cos\ (\theta_1 + \theta_2) = 0 \qquad \therefore (\theta_1 + \theta_2) = 90^\circ$$

Two equal masses undergo oblique collision will move at right angles after collision.

VI. LONG ANSWER QUESTIONS

1. Define work and kinetic energy. State and prove work energy theorem for constant force.

Ans. **Work:** The work done by the force is defined to be the product of component of force in the direction of the displacement and the magnitude of this displacement.

$$\therefore \text{Work done } W = (F \cos\theta)\, d = F.d.$$

Here F is the force acting on an object of mass m and d is displacement.

Kinetic energy: The kinetic energy of an object is a measure of the work an object can do by the virtue of its motion. If an object of mass m has velocity ϑ, its kinetic energy is given by.

$$K = \frac{1}{2}\, m\vartheta^2$$

Work – Energy Theorem:-

Statement: - The change in kinetic energy of a particle is equal to the work done on it by the net force.

Proof – If **u** and $\boldsymbol{\vartheta}$ are the initial and final speed of a body then the relation for rectilinear motion under constant acceleration **a** is given as

$$\vartheta^2 - u^2 = 2as$$

Multiplying both sides by m/2

$$\frac{m}{2}(\vartheta^2 - u^2) = \frac{m}{2} \times 2as$$

$$\frac{1}{2}m\vartheta^2 - \frac{1}{2}mu^2 = mas$$

From Newton's law of motion F = ma

$$\frac{1}{2}m\vartheta^2 - \frac{1}{2}mu^2 = F.s$$

Here $\frac{1}{2}m\vartheta^2$ is the final kinetic energy K_f and $\frac{1}{2}mu^2$ is the initial kinetic energy K_i and F.s is equal to work done **W**.

$$K_f - K_i = W$$

Hence proved

2. Define elastic and inelastic collisions. Derive expressions for the final velocities of bodies in one dimensional elastic collision, when one body collides with other body which is initially at rest.

Ans. **Collisions:** A collision is a strong physical interaction between two or more bodies which causes a sudden change in their momenta.

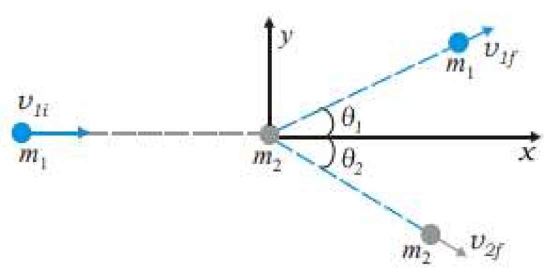
Elastic collisions: The collision in which both momentum and kinetic energy are conserved is known as elastic collision.

Ex:- collision between fundamental particles.

Inelastic collisions: The collision in which only momentum is conserved but not kinetic energy is known as inelastic collision.

Ex. Collision between a bullet and its target.

One dimensional elastic collisions: If the velocities of colliding bodies before and after collisions are confined to the same straight line then such collisions are called one dimensional collisions



Consider two particles of masses $\mathbf{m_1}$ and $\mathbf{m_2}$ .The particle m_1 is moves with speed $\boldsymbol{\vartheta_{1i}}$ collies with the stationary particle $\mathbf{m_2}$ as shown in figure after collisions $\mathbf{m_1}$ and $\mathbf{m_2}$ move in different directions.

Consider one dimensional elastic collision, in this case $\theta_1 = \theta_2 = 0$.

From the law of conservation momentum

$m_1 \vartheta_{1i} + m_2 \vartheta_{2i} = m_1 \vartheta_{1f} + m_2 \vartheta_{2f}$

m_2 is initially stationary so $\vartheta_{2i} = 0$

$m_1 \vartheta_{1i} = m_1 \vartheta_{1f} + m_2 \vartheta_{2f}$

$m_1 (\vartheta_{1i} - \vartheta_{1f}) = m_2 \vartheta_{2f}$ ⟶ 1

From the law of conservation of kinetic energy

$\frac{1}{2} m_1 \vartheta_{1i}^2 = \frac{1}{2} m_1 \vartheta_{1f}^2 + \frac{1}{2} m_2 \vartheta_{2f}^2$ $(\because \vartheta_{2i} = 0)$

$m_1 \vartheta_{1i}^2 = m_1 \vartheta_{1f}^2 + m_2 \vartheta_{2f}^2$

$m_1 (\vartheta_{1i}^2 - \vartheta_{1f}^2) = m_2 \vartheta_{2f}^2$ ⟶ 2

$$\frac{eqn1}{eqn2} \Rightarrow \frac{m_1(\vartheta_{1i}^2 - \vartheta_{1f}^2)}{m_1(\vartheta_{1i} - \vartheta_{1i})} = \frac{m_2 \vartheta_{2f}^2}{m_2 \vartheta_{2f}}$$

$\vartheta_{1i} + \vartheta_{1f} = \vartheta_{2f}$ ⟶ 3

Substitute eqn **3** in eqn **1**

$m_1(\vartheta_{1i} - \vartheta_{1f}) = m_2(\vartheta_{1i} - \vartheta_{1f})$

$$m_1 \vartheta_{1i} - m_1 \vartheta_{1f} = m_2 \vartheta_{1i} - m_2 \vartheta_{1f}$$

$$m_1 \vartheta_{1i} - m_2 \vartheta_{1f} = \vartheta_{1f} (m_1 + m_2)$$

$\vartheta_{1i} (m_1 + m_2) = \vartheta_{1f} (m_1 + m_2)$

$\therefore \vartheta_{1f} = \frac{m_1 - m_2}{m_1 + m_2} \vartheta_{1f}$ ⟶ 4

From eqn **3** $\vartheta_{1f} = \vartheta_{2f} - \vartheta_{1i}$

Substitute the above in eqn **1** we get $m_2(\vartheta_{1i} - (\vartheta_{2f} - \vartheta_{1i}) = m_2 \vartheta_{2f}$

$$m_1 \vartheta_{1i} - m_1 \vartheta_{2f} + m_1 \vartheta_{1i} - m_2 \vartheta_{2f}$$

$$m_1 \vartheta_{1i} + m_1 \vartheta_{1f} = (m_1 + m_2) \vartheta_{2f}$$

$$2 m_1 \vartheta_{1i} = (m_1 + m_2) \vartheta_{2f}$$

$$\vartheta_{2f} = \frac{2 m_1 \vartheta_{1i}}{m_1 + m_2} \longrightarrow 5$$

Equations **4, 5** are the expressions for the final velocities.

3. State and prove law of conservation of mechanical energy in case of a freely falling body.

Statement:- The total mechanical energy of a system is conserved if the forces, doing work on it are conservative.

A body undergoes displacement Δx under the action of a conservative force F.

From work energy theorem

$$\Delta K = F(x).\Delta x$$

If force is conservative, the potential energy

$$\Delta K = F(x).\Delta x$$

$$\Delta K + V(x) = 0$$

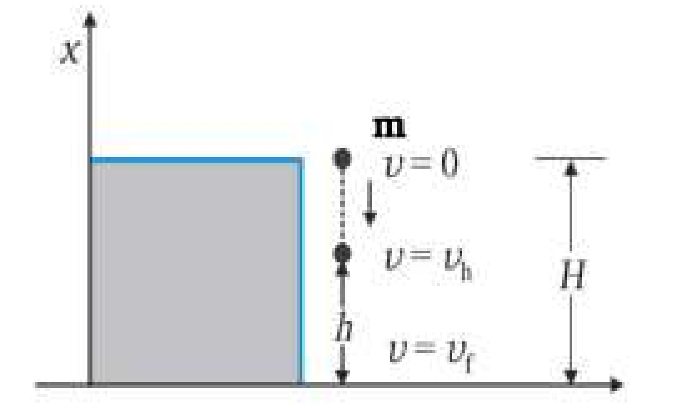
$$\Delta(K + V) = 0$$

Here K+V is the Sum of kinetic and potential energies of the body is a constant

The kinetic energy K and Potential energy V(x) may vary from point to point but the sum is a constant.

Proof:

Suppose a ball of man **m** being dropped from a cliff of height **H** . The total mechanical energy of the ball at zero level, at height **h** and at height **H** are E_0 , E_h,E_H respectively.



E_H = mgH ⟶ 1

E_h = mgh + $\frac{1}{2}m\vartheta_h^2$ ⟶ 2

E_0 = $\frac{1}{2}m\vartheta_f^2$ ⟶ 3

The constant force is a special case of partially dependent force F(x).

Hence the mechanical energy is conserved

∴ $E_H = E_0$ (conservative force depends on end points)

$\cancel{m}gh = \frac{1}{2}\cancel{m}\vartheta_f^2$

$\vartheta_f^2 = 2gH$

$\vartheta_f = \sqrt{2gh}$

Further $E_H = E_0$ (conservative force depends on end point) mgh = $mgh + \frac{1}{2}m\vartheta_h^2$

2mg (H-h) = $m\vartheta_h^2$

$\vartheta_h^2 = 2g\ (H - h)$

At a height H, the energy is purely potential it is partially converted into kinetic at height h and is fully kinetic this illustrates the conservation of mechanical energy.

<u>Alternative method:</u>

<u>At height H:-</u> The ball is dropped from height H. Then velocity **ϑ = Zero**.

K.E = 0

Potential energy P.E = mgH

Total energy at H is E_H = mgH ⟶ (1)

At height h :- The ball reached at a height H after travelling (H-h) distance.

Here K.E = $\frac{1}{2}m\vartheta_h^2$

Here ϑ_h^2 = 0 + 2g (H-h) = mg (H-h)

ϑ_h^2 = 2g (H-h)

K.E = $\frac{1}{2}m\ (2g\ (H - h) = mg\ (H - h)$

P.E is = mgh

Total energy at h is E_h = mgH ⟶ (2)

At ground level :- The ball reached the ground after travelling **H** distance

Here K.E = $\frac{1}{2}m\vartheta_f^2$

∴ $\vartheta_f^2 = 0^2 + 2gh$ =2gH

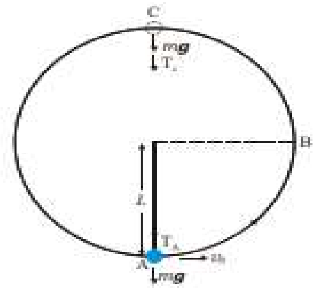
K.E = $\frac{1}{2}\ m\ \times 2gh = mgH$

At ground (h= o) P.E = mg (0) = 0

∴ Total energy at ground E_0 = mgH ⟶ (3)

From **1,2,3** the total energy is constant so law of conservation of mechanical energy is proved in case of a freely falling body.

4.An object is tied to a string and rotated in a vertical circle; obtain expressions for speeds of the object at the highest and lowest points when the string becomes slack only at the highest point.



Ans. Let us consider a bob of mass **m** is suspended **by** a light string of length **L**
Suppose ϑ_o is the horizontal velocity at the lowest point A, such that the bob completes a semi – circular trajectory in the vertical plane with the string becoming slack only on reaching the highest point 'C'

At the lowest point A two external forces acting on the bob. They are gravitational forces and tension (T) in the String we take the potential energy of the system to be zero at the lowest point A.

The total mechanical energy E of the system is conserved.

At a total mechanical energy $E = \frac{1}{2} m\vartheta_0^2 + 0 \longrightarrow$ 1

From figure: The resultant force at A provides necessary centripetal force

At lowest point **A**, $T_A - mg = \frac{m\vartheta_0^2}{L}$ (from Newton's second law)

Here $\mathbf{T_A}$ is the tension in the spring at point A.

At the highest point C, only when tension at C(T_C) becomes zero the string becomes slack. At point C Total energy $E = \frac{1}{2} m\vartheta_c^2 + mg\,(h)$ (Here h = 2L)

$$E = \frac{1}{2} m\vartheta_c^2 + 2mgL \longrightarrow \quad 2$$

From Newton's second law at point **C**

$$T_c + mg = \frac{m\vartheta_C^2}{L}$$

$$\cancel{m}g = \frac{\cancel{m}\vartheta_C^2}{L} \quad (\because T_C = 0)$$

$$\vartheta_C^2 = gL$$

$$\boldsymbol{\vartheta_c = \sqrt{gL}} \longrightarrow \quad 3$$

This is an expression for speed of the bob at the highest point. Comparing equations **1, 2**

$$\frac{1}{2}m\vartheta_0^2 = \frac{1}{2}m\vartheta_C^2 + 2mgL$$

$$\vartheta_0^2 = \vartheta_c^2 + 4gL$$

$$\vartheta_0^2 = gL + 4gL$$

$$\therefore \boldsymbol{\vartheta_0} = \sqrt{\mathbf{5}g\mathbf{L}} \longrightarrow 4$$

This is an expression for the speed of the object at the lowest point.

VII. PROBLEMS

1. A machine gun fires 360 bullets per minute and each bullet travels with a velocity of 600 m/s. If the mass of each bullet is 5 g, find the power of the machine gun.

Ans:

Velocity of each bullet ϑ = 600 m/s

Mass of each bullet m = 5 mg = $5\times\ 10^{-3}\ kg$

No .of bullets (n) = 360

Time (t) = 1min = 60 sec

Power of machine gun = work done / time

Here work done = K.E of n bullets

$$W = n \times \frac{1}{2}m\vartheta^2$$

$$W = \frac{1}{2}mn\vartheta^2$$

$$\therefore \text{Power } P = \frac{1}{2} \times \frac{mn\vartheta^2}{t}$$

$$= \frac{1}{2} \times \frac{5\times10^{-3}\times360\times(600)^2}{60}$$

Power = 5400 watts =5.4 KW.

2. Find the useful power used in pumping 3425 m^3 of water per hour from a well 8 m deep to the surface, supposing 40% of the horse power during pumping is wasted. What is the horse power of the engine?

Ans. Given Volume of the water V = 3425m^3

Density of watts (d) = 1000 kg/m^3

Mass = volume × density

= 3425 × 1000 = 3425 × 10^3 kg

g = 9.8 m/s^2

height h = 8m

time t = 1hr =3600 sec

40% of horse power during pumping is wasted means.

Useful horse power of engine = $\frac{100}{60}$ × power of engine = $\frac{100}{60}\left(\frac{mgh}{t}\right)$

$$=\frac{100}{60}\times\frac{3425\times10^{3}\times9.8\times8}{3600}$$

$$=\frac{100}{\cancel{60}}\times\frac{3425\times9.8\times8}{36}\times\frac{\cancel{10^{3}}}{\cancel{10^{2}}}$$

$$=\frac{3425\times9.8\times8\times10^{2}}{36\times6}$$

$$= 1243.148\times10^{2}$$

$$=124.31\times10^{3}\text{ watt}$$

$$=\frac{124.31}{746}\times10^{3}$$

$$= 0.1666\times10^{3} \quad = 1.66\times10^{2}\text{ hp}$$

3. A pump is required to lift 600 kg of water per minute from a well 25 m deep and to eject it with a speed of 50 m/s. Calculate the power required for the above task.

Take g=10 m/s^2.

Ans : Mass of water m = 600 kg

Time t = 1 minute = 60 sec

Height h = 25m

Speed ϑ = 50 m/s

Power (p) = $\frac{work\ done}{time}$

$$=\frac{K.E+P.E}{time}$$

$$=\frac{\frac{1}{2}m\vartheta^{2}+mgh}{t}$$

$$=\frac{\frac{1}{2}\times600\times(50)^{2}+600\times10\times25}{60}$$

$$=\frac{300\times2500\times25\times6000}{60}$$

$$=\frac{75000+15000}{60}$$

$$=\frac{90000-}{\cancel{60}}$$

P = 15000 watts

P = 15 KW

4. A block of mass 5 kg initially at rest at the origin is acted on by a force along the X-positive direction represented by F = (20 + 5x)N. Calculate the work done by the force during the displacement of the block from x = 0 to x=4 m.

Ans : Given m = 5 kg and F = (20 + 5x) N

We have w = $\int_0^4 F\,dx$

$= \int_0^4 (20 + 5x)dx$

$= 20\,\lfloor x \rfloor_0^4 + 5\left[\frac{x^2}{2}\right]_0^4$

$= 20 \times 4 + 5\left[\frac{4^2}{2} - 0\right]$

$= 80 + 5\left[\frac{16}{2}\right]$

= 80 + 40 = 120 J

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UNIT – 6

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

I. MULTIPLE CHOICE QUESTIONS

1. The vector sum of linear momentum of a system of particles about the centre of mass is
(1) Zero (2) infinity (3) negative (4) either positive or negative **(1)**
Ans : Since the total momentum of the system is the sum of the individual momenta, and the centre of mass is defined in a way that balances these momenta, the net momentum about the center of mass must be zero.

2. A person can easily balance on a moving bicycle but cannot balance on a stationary one due to **(2)**
(1) law of conservation of mass
(2) law of conservation of angular momentum
(3)law of conservation of linear momentum
(4) law of conservation of energy
Ans : The spinning wheels of a moving bicycle possess angular momentum, which resists any changes in their direction of rotation.

3. All the particles of a body are situated at a distance R from the origin. The distance of the center of mass of the body from the origin is **(2)**
(1) = R (2) ≤ R 3) > R (4) ≥ R
Ans : If all the particles of a body are situated at a distance R from the origin, they effectively lie on a sphere or circle (depending on the dimension) with a radius of R. The center of mass of this system is a weighted average of the positions of all the particles, and it will always be located within or on the sphere. Therefore, the distance of the center of mass from the origin cannot be greater than R.

4. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which one of the following will not be affected. **(1)**
(1)Angular momentum (2) moment of inertia (3) angular velocity (4) rotational kinetic energy
Ans : Taking the same mass of sphere, if radius is increased, then moment of inertiaand angular velocity will change but according to law of conservation of momentum, angular momentum will not change.

5. The measure of the force that can cause an object to rotate about an axis is **(1)**
(1)Torque (2) angular momentum (3) impulse (4) moment of inertia
Ans : The measure of the force that can cause an object to rotate about an axis is torque. It is a rotational force that produces a turning or twisting effect.

6. In the following physical quantities, which is not a vector is **(4)**
(1)Angular velocity (2) Torque (3)Angular momentum (4) work

Ans : Angular velocity, torque, and angular momentum are vector quantities. They have both magnitude and direction.

7. The time rate of change of angular momentum is called **(1)**
(1) Torque (2) V/r (3) power (4) linear momentum

Ans : The time rate of change of angular momentum is called torque. This is analogous to how force is the time rate of change of linear momentum.

8. A particle is moving in a circle of radius “r” which a constant speed “V”. Its angular velocity is (2)

(1) Vr (2) V/r (3) Zero (4) Vr^2

Ans : Angular velocity is the rate of change of angular displacement with respect to time. It's related to linear velocity (V) and the radius (r) of the circle by the following formula: $\omega = V / r$.

9. Moment of momentum is called (2)

(1) Linear momentum (2) angular momentum (3) moment of inertia (4) torque

Ans : The "moment of momentum" is also called angular momentum. It's the rotational equivalent of linear momentum and is a measure of an object's tendency to continue rotating.

II. FILL IN THE BLANKS

1. **The centre of mass of a triangular lamina is located at centroid.**
2. **A vector with zero magnitude is called null vector.**
3. **The time rate of change of angular velocity is called angular acceleration.**
4. **The ratio between torque about a fixed axis and angular acceleration is moment of inertia.**
5. **Expression for rotational kinetic energy in terms of moment of inertia (I) and angular velocity (ω) is $\frac{1}{2} I\omega^2$.**
6. **A body with perfectly definite and unchanging shape is called rigid body.**
7. **If i and j are the unit vectors along X and Y axes respectively, then $\hat{j} \times \hat{i} = -\hat{k}$.**
8. **Moment of force is called torque or couple.**
9. **Pair of equal and opposite forces with different lines of action is called couple or torque.**
10. **The moment of inertia of a thin circular ring having radius “R” and mass “M” about its diameter is $MR^2/2$.**

III. ONE WORD ANSWER QUESTIONS

1. Define axis of rotation.

Ans: Axis of rotation is an imaginary line about which a given body or system is rotating.

2. What is the rotational analogue of force?

Ans : Torque

3. Define torque.

Ans : It is the force that tends to cause rotation.

4. What is the analogue of mass in rotational motion ?

Ans : Moment of Inertia

5. What is the SI unit of moment of inertia ?

Ans : $kg\text{-}m^2$

6. What is the use of a flywheel in vehicles ?

Ans : It resists the sudden increase or decrease of the speed of the vehicle.

7. What is the expression for the work in rotational motion about a fixed axis ?

Ans : $w = \tau\theta$

IV. VERY SHORT ANSWER QUESTIONS

1. Is it necessary that a mass should be present at the center of mass of any system. Give an example.

Ans : No. Any mass need not be present at the center of mass of a system.

2. Two rigid bodies have same moment of inertia about their axes of symmetry. Of the two, which body will have greater kinetic energy.

Ans : $K = \frac{1}{2}I\omega^2 = \frac{L^2}{2I} \Rightarrow K \alpha \frac{1}{I}$

∴The rigid body having less moment of inertia will have greater kinetic energy.

3. Why are spokes provided in a bicycle wheel ?

Ans : The spokes of cycle wheel increase its moment of inertia. The greater the moment of inertia, the greater is the opposition to any change in uniform rotational motion. As a result the cycle runs smoother and speeder.

4. We cannot open or close the door by applying force at the hinges. Why ?

Ans : When the force is applied at the hinges, the line of action of the force passes through the axis of rotation i.e, r = 0, so we cannot open or close the door by pushing or pulling it at the hinges.

5. Why do we prefer a spanner of longer arm as compared to the spanner of shorter arm ?

Ans : A spanner with longer arm provides more torque compared to a spanner with shorter arm. Hence longer arm spanner is preferred.

6. By spinning eggs on a table top, how will you distinguish in hard boiled egg from a raw egg ?

Ans : To distinguish between a hard boiled egg and a raw egg, we spin each on a table top. The egg which spins at a slower rate shall be raw. This is because in a raw egg, liquid matter inside tries to get away from its axis of rotation. Therefore, its moment of inertia increases. As $\tau = \mathbf{I}\,\boldsymbol{\alpha} = \textbf{constant}$, therefore α decreases i.e. raw egg will spin with smaller angular acceleration.

7. Why should a helicopter necessarily have two propellers?

Ans : If there were only one propeller in the helicopter then, due to law of conservation of angular momentum, the helicopter itself would have turned in the opposite direction. Hence, if should have two propellers.

8. If the polar ice caps of the earth were to melt, what would be the effect of the length of the day be ?

Ans : Earth rotates about its polar axis. When ice of polar caps of earth melts, mass concentrated near the axis or rotation spreads out. Therefore, moment of Inertia increases. As no external torque acts, $L = I \times \omega = I.\frac{2\pi}{T}$= constant. With increase of I, T will increase i.e. length of the day will increase.

9. Why is it easier to balance a bicycle in motion ?

Ans : When a bicycle is in motion, it is easy to balance because the principle of conservation of angular momentum is involved.

V. <u>SHORT ANSWER QUESTIONS</u>

1. Distinguish between centre of mass and centre of gravity.

Ans : **<u>Centre of mass :</u>**

1. This is the point at which entire mass of the body is supposed to be concentrated.
2. It is independent of the acceleration due to gravity.

3. It may or may not lie within the body.
4. Centre of mass of the body is defined to describe the nature of motion of a body as a whole.

Centre of gravity :

1. This is the point at which entire weight of the body is supposed to be concentrated.
2. It depends on the acceleration due to gravity.
3. It always lies within the body.
4. It is defined to know the amount of stability of the body.

2. Show that the system of particles moving under the influence of an external forces moves as if the force is applied at its centre of mass.

Ans : Consider a system of particles of masses. $m_1, m_2 .... m_n$ moves with velocity vectors $\overrightarrow{\vartheta_1}, \overrightarrow{\vartheta_2}, \overrightarrow{\vartheta_3} ....., \overrightarrow{\vartheta_n}$

$\therefore$ *Velocity of centre of mass*

$$\overline{\vartheta}CM = \frac{1}{M}\left\{m_1\overline{\vartheta_1} + m_2\overline{\vartheta_2} + ...m_n\overline{\vartheta_n}\right\}$$

$$\because m_1 + m_2 + ... + m_n = M$$

We know

$$\therefore \overline{a} = \frac{d}{dt}\left(\overline{\vartheta}\right) \quad \therefore \overline{a} = \frac{d}{dt}\left(\overline{\vartheta}\right)$$

$\therefore$ Acceleration of centre of mass

$$a_{CM} = \frac{d}{dt}\left(\overline{\vartheta}_{CM}\right)$$

$$\therefore a_{CM} = \frac{1}{M}\{m_1\frac{d}{dt}\overrightarrow{\vartheta_1} + m_2\frac{d}{dt}\overrightarrow{\vartheta_2} + ... + m_n\frac{d}{dt}\overrightarrow{\vartheta_n}\}$$

But Force(F)=ma, so total force on the body is

$$F = Ma_{CM} = m_1a_1 + m_2a_2 + m_3a_3 + ... + m_na_n$$

or Total Force F = $ma_{CM} = F_1 + F_2 + F_3 + ... + F_n$

Hence, total force on the body is the sum of forces on individual particles and it is equals to force on centre of mass of the body.

3. Define vector product. Explain the properties of a vector product with two examples.

Ans : The vector product of two vectors is given by

$$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}|\,\sin\theta\hat{n}$$

Here, $|\vec{A}|$, $|\vec{B}|$ are the magnitudes of the given vectors, 'θ' is the angle between the corresponding vectors and '$\hat{n}$' is the unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

The following are the properties of the vector product of two vectors:-

The Vector Product does not obey commutative law i.e., $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

1. The Vector Product is distributive. This denotes that if there exist two vectors, $\vec{a}$ and $\vec{b}$, then the vector product of both the vector is distributive in nature i.e
   $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
2. The Vector Product of unit vector itself is 0. i.e $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$
3. The Vector Product of the vector itself is 0 i.e. $\vec{a} \times \vec{a} = 0$.

Examples:

1. Torque, $\vec{\tau} = \vec{r} \times \vec{F}$
2. Angular momentum, $\vec{L} = \vec{r} \times \vec{p}$

4. Define angular velocity (ω). Derive v = rω.

Ans : **Angular velocity (ω):**The rate of change of angular displacement of a body is called angular velocity, i.e., $\omega = \frac{d\theta}{dt}$

Derivation of v = rω : Let a particle p be displaced by ds in time interval dt on the circumference of circle of radius r through an angle dθ.

Angular velocity $\omega = \frac{d\theta}{dt}$

but Linear velocity $\vartheta = \frac{dS}{dt}$

We know that angle = $\frac{arc}{radius}$

$$d\theta = \frac{ds}{r}$$

$$ds = rd\theta$$

$$\vartheta = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$$

5. Define angular acceleration and torque. Establish a relation between angular acceleration and torque.

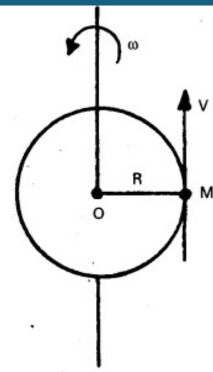
Ans : Angular acceleration : The rate of change of angular velocity is called angular acceleration

i.e., $\alpha = \frac{d\omega}{dt}$

Torque : The rate of change of angular momentum is called torque or the moment of Force is called torque.

$$\tau = \frac{dL}{dt}$$

Relation between angular acceleration and Torque: Consider a rigid body of mass 'M' rotating in a circular path of radius 'R' with angular velocity 'ω' about fixed axis.



By definition,

$$\tau = \frac{dL}{dt} = \frac{d(I\omega)}{dt}$$

Where I =moment of inertia of a body

$$\tau = I\left(\frac{d\omega}{dt}\right)$$

[∵ I = constant]

But $\frac{d\omega}{dt} = \alpha$

$$\therefore \ \tau = l\alpha$$

6.Write the equations of motion for a particle rotating about a fixed axis.

Ans : Equations of motion for a particle rotating about a fixed axis:

1. $\omega_f = \omega_i + \alpha t$

2. $\theta = (\omega_f + \omega_i)t/2$

3. $\theta = \omega_i t + \frac{1}{2}\alpha t^2$

4. $\omega_f^{\ 2} - \omega_i^2 = 2\alpha\theta$

Where ω_i = Initial angular velocity

ω_f = Final angular velocity

α = Angular acceleration

θ = Angular displacement

t = time

VI. LONG ANSWER QUESTIONS

1. **State and prove the principle of conservation of angular momentum. Explain the principle of conservation of angular momentum with examples**.

Ans : Angular momentum of a body remains constant when the external torque is zero.

$L = I\,\omega$ = constant

or $I_1\,\omega_1 = I_2\,\omega_2$.

Here I represents the moment of inertia and ω the angular velocity.

Proof: By definition, the rate of change of angular momentum is called torque.

i.e.,

$$\tau = \frac{dL}{dt}$$

If $\tau = 0 \Rightarrow \frac{dL}{dt} = 0$

or L = constant

$\Rightarrow L_1 = L_2$

$\therefore I_1\,\omega_1 = I_2\,\omega_2$

Examples:

(i) A diver jumping into the pool of water, make use of the law of conservation of angular momentum. When the diver jumps in the air, he folds his hands and legs inwards so that his moment of inertia decreases. Consequently, his angular speed increases and he spins faster.

(ii) A girl rotating in a swivel chair decreases or increases her speed by stretching her arms or bringing her arms closer to the body by using the law of conservation of angular momentum.

2. (a) Using the expressions for Power and Kinetic Energy of rotational motion, derive the relation $\tau = I\alpha$.

(b)The moment of inertia of a body about a given axis is 1.2kg-m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1500J, for how much duration an angular acceleration of 25 rad/s^2 must be applied about that axis ?

Ans : (a)The power is given by $P = \tau\,\omega$ ----(1)

Rotational Kinetic Energy, K.E. = $1/2\, I\omega^2$ -------(2)

The power of rotational motion is equal to time rate of work done during the rotational motion. Since the work done is stored in the form of kinetic energy,

P = d(K.E. of rotational motion)/dt

Using the equations (1) and (2), we have, $\tau\,\omega = \frac{d}{dt}\left[\frac{1}{2} I\omega_2\right]$

$\frac{d}{dt}\left[\frac{1}{2}I\omega_2\right]$ $\tau\,\omega = \frac{1}{2} \cdot 2.\, I\,\omega\frac{d\omega}{dt}$

$\tau = I\frac{d\omega}{dt}$

We know that rate of change of angular velocity is called angular acceleration (α)

$\therefore \tau = I\alpha$

(b) $I = 1.2 kgm^2$, $\omega_0=0$,

$\alpha = 25 rad\, s^{-2}$, $t=?$

Additional rotational K.E.=1500J

$\frac{1}{2}I\omega^2 - \frac{1}{2}I\,\omega_0^2 = 1500$

As $\omega_0=0$, $\frac{1}{2}I\,\omega^2=1500$

$$\omega^2 = \frac{3000}{I}$$

$$\omega = \sqrt{\frac{3000}{1.2}}$$

$= 50\ rad\ s^{-1}$

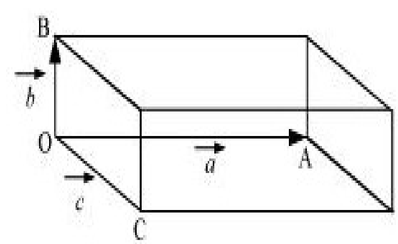
From $\omega = \omega_0 + \alpha t$

$50 = 0+25t$,

$t= 50/25 = 2s$

VII. PROBLEMS

1. **Show that a.(b x c) is equal in magnitude to the volume of the parallel piped formed on the three vectors a, b and c**.



Ans:- Volume of the given parallelepiped = abc

$\overrightarrow{OC} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$

Let $\hat{n}$ be a unit vector perpendicular to both b and c.

Hence, $\hat{n}$ and a have the same direction.

$$\therefore \vec{b}\times\vec{c} = bc\,\mathrm{Sin}\,\theta\hat{n}$$

$= bc\hat{n}$

$\bar{a}.\left(\bar{b}.\bar{c}\right) = \bar{a}.(bc\hat{n})$

$= abc\cos\theta$

$= abc\cos 0$

$= abc$

= Volume of the parallelopiped.

2. A rope of negligible mass is wound round a hollow cylinder of mass of 3kg and radius 40 cms . What is the acceleration of the cylinder if the rope is pulled with a force of 30N ? What is the linear acceleration of the rope ? Assume that there is no slipping.

Ans : Here M = 3kg, r= 40 cm = 0.4 m

Moment of inertia of the hollow cylinder about its axis

$I = MR^2 = 3(0.4)^2 = 0.48\ kgm^2$

Force applied, F = 30 N

$\therefore$ Torque, $\tau = \bar{r} \times \bar{F} = 30 \times 0.4 = 12N$ - m.

If α is the angular acceleration, produced, then from $\tau = I\alpha$

$$\alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25 rads^{-2} \qquad \alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25 rads^{-2}$$

Linear acceleration, $a = r\alpha = 0.4 \times 25 = 10\ m/s^2$

3. A coin is kept a distance of 10 cm from the centre of a circular turntable. If the coefficient of static friction between the table and the coin is 0.8 find the frequency of rotation of the disc at which the coin will just begin to slip.

Ans: Here, radius, r = 10cm = 0.1 m; $\mu_s = 0.8$

$$F = \mu mg$$

$$mr\omega^2 = \mu mg$$

$$r\omega^2 = \mu g$$

$$\omega = \frac{\mu g}{r} = \sqrt{\frac{0.8\times 9.8}{0.1}} = \sqrt{8\times 9.8}$$

$$= 8.854\ rad/s$$

$$\omega = 2\pi n$$

$$\therefore\ frequency\ n = \frac{\omega}{2\pi} = \frac{8.854}{2\times 3.14} = 1.409 rps$$

$$n = 1.409\times 60 = 84.54\ rpm$$

4. **Particles of masses 1g, 2g, 3g…. 100g are kept at the marks 1 cm. 2cm, 3cm ….100cm respectively on 2 meter scale. Find the moment of inertia of the system of particles about a perpendicular bisector of the meter scale.**

Ans: By adding all the masses, we get M = 5050g

$$\therefore S_n = \frac{n(n+1)}{2}$$

$$M = \frac{100 \times 101}{2} = 5050$$

= 5.050 kg, = 5.1 kg, and L = 1 m

M.I. of the meter scale $= \frac{mL^2}{12} = \frac{5.1 \times 1^2}{12}$

= 0.425kg m^2

= 0.43kg m^2

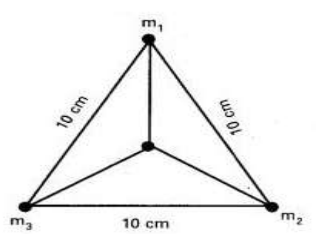
= 0.43 kg m^2

5. **Three particles each of mass 100g are placed at the vertices of an equilateral triangle of side length 10 cm. Find the moment of inertia of the system about an axis passing through the centroid of the triangle and perpendicular to its plane.**

Ans : M = 100 g = 100 x 10^{-3}kg

Side a = 10 cm

Moment of mertia, I = $3Mr^2$



$$= 3 \times 100 \times 10^{-3} \times \left[\frac{10}{\sqrt{3}} \times 10^{-2}\right]^2$$

$$= \frac{3 \times 10^{-1} \times 10^2 \times 10^{-4}}{3} = 10^{-3}\,kgm^2.$$

6. The moment of inertia of a fly wheel making 300 revolutions per minute is 0.3 kgm^2. Find the torque required to bring it to rest in 20s.

Ans : Here I = 0.3kg m^2 ; $\frac{N}{t} = \frac{300\ revolutions}{1\ minute}$

$$\omega_i = \frac{2\pi N}{t} = 2\pi \times \frac{300}{60} = 10\pi \text{ rad/s}$$

$$t = 20\,S; \omega_f = 0; \tau = ?$$

$$\tau = I\alpha = I\left(\frac{\omega_f - \omega}{t}\right)$$

$$\therefore \tau = 0.3\left(\frac{0 - 10\pi}{20}\right) = -\frac{0.3\pi}{2} = -0.471Nm.$$

7. When 100 J of work is done on a fly wheel, its angular velocity is increased from 60rpm to 180rpm. What is the moment of inertial of the wheel ?

Ans : Here, Initial frequency

$$n_1 = \frac{60}{60} = 1Hz$$

Initial angular velocity

$$\omega_1 = 2\pi\, n_1 = 2\pi \text{ rad/sec}$$

Final frequency $n_2 = \frac{180}{60} = 3Hz$

Final angular velocity

$$\omega_2 = 2\pi\, n_2 = 2\pi \times 3 = 6\pi \text{ rad/sec}$$

Work done = 100 J from work – energy theorem,

Work done = change in K.E.

$$W = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2$$

$100=\frac{1}{2}I[(6\pi)^2-(2\pi)^2]$

$100=\frac{1}{2}I(32\pi)^2$

$I=\frac{200}{32\pi^2}=0.634kg\text{-}m^2$

$I=0.634kg\text{-}m^2$

8. Four spheres, each diameter 2a and mass 'm' are placed with their centres on the four corners of a square of the side b. Calculate the moment of inertia of the system about any side of the square.

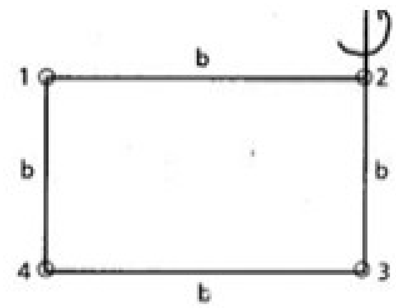
Ans : Here $I_1=\frac{2}{5}ma^2+mb^2$

$I_2=\frac{2}{5}ma^2+mb^2$

$I_3= I_4=\frac{2}{5}ma^2$

$I=I_1+I_2+I_3+I_4$

$I=\frac{8}{5}ma^2+ 2mb^2$



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UNIT-7
GRAVITATION

I. MULTIPLE CHOICE QUESTIONS

1. A planet moves around the sun in an elliptical orbit at a given point P, it is closest form the sun at a distance d_1 and has a speed V_1. At another point Q. when it is farthest from the sun at a distance d_2, its speed V_2 will be [3]

(1) $\frac{d_1^2 V_1}{d_2^2}$ (2) $\frac{d_2 V_1}{d_1}$ (3) $\frac{d_1 V_1}{d_2}$ (4) $\frac{d_2^2 V_2}{d_1^2}$

Note : In text book read V_2 as V_1 in all options.

Ans: Angular Momentum is constant

mvr = constant

m → constant

$V_1 r_1 = V_2 r_2$

$V_1 d_1 = V_2 d_2$

$V_2 = \frac{V_1 d_1}{d_2}$

2. According to Kepler, the period of revolution (T) of a planet and its mean distance (r)from the sun are related by the equation. [2]

(1) $T^3 r^3$ = constant (2) $T^2 r^{-3}$ = constant (3) Tr^3 = constant (4) $T^2 r$ =constant

Ans : $T^2 \alpha\ r^3$

$T^2 = k r^3$

$\frac{T^2}{r^3} = K$

$T^2 r^{-3} = K$

3. A satellite revolves around the earth in an elliptical orbit . Its speed [2]

(1) Is the same at all points in the orbit.
(2) Is greatest when it is closest to the earth
(3) Is greatest when it is farthest from the earth
(4) Goes on increasing or decreasing continuously depending upon the mass of the satellite

Ans: mVr = constant

Vr = constant

$V \alpha \frac{1}{r}$

Speed is greatest when it is closest to the earth.

4. The force of gravitation is [3]

(1) Repulsive force (2) electrostatic force (3) conservative force (4)non-conservative force

Ans: Gravitational force is conservative force.

5. If the distance between two masses is doubled, the gravitational attraction between them [4]

(1) is doubled (2) becomes four times (3) is reduced to half (4) is reduced to a quarter

Ans: F = $\frac{Gm_1m_2}{r^2}$

$r_2 = 2r_1$

$F\alpha\frac{1}{r^2}$, $\frac{F_2}{F_1} = \frac{r_1^2}{r_2^2}$, $F_2 = \frac{r_1^2}{4r_1^2}$

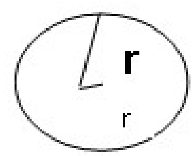
$F_2 = \frac{F_1}{4}$

6. **If the earth losses its gravity, then for a body** **[1]**
(1)weight becomes zero, but not the mass
(2) mass becomes zero, but not the weight
(3)both mass and weight become zero
(4)neither mass nor weight become zero
Ans: Given gravity is zero, Weight is zero, Mass remains same.

7. **The first experimental value of G is given by** **[1]**
(1)Cavendish (2) Copernicus (3) Brook Teylor (4) none of these
Ans: Experimental value of G is given by Cavendish.

8. **The acceleration due to gravity is 'g' at a point distant 'r' from the centre of earth of radius 'R'. If r <R, then** **[1]**
(1) $g\alpha r$ (2) $g\alpha r^2$ (3) $g\alpha r^{-1}$ (4) $g\alpha r^{-2}$

Ans: $R \rightarrow$ radius of earth
$r \rightarrow$ distance of point from centre
Inside point (r < R)
g = $\frac{GM}{R^2}r$
g α r



9. Four particales each of mass M, are located at the verities of a square with side L. The gravitational potential due to this at the center of the square is **[1]**

(1) $-\sqrt{32}\frac{GM}{L}$ (2) $-\sqrt{64}\frac{GM}{L^2}$ (3) zero (4) $\sqrt{32}\frac{GM}{L}$

Ans: At centre

$V_1 = -\frac{GM}{L/\sqrt{2}} = -\sqrt{2}\frac{GM}{L}$

$V_1 = V_2 = V_3 = V_4$

$V_{centre} = V_1 + V_2 + V_3 + V_4$

$= 4V_1$

$= -4\left(\frac{\sqrt{2}GM}{L}\right)$

$= \frac{-\sqrt{32}GM}{L}$

10.The escape velocity of a sphere of mass M from the surface of earth having mass M and radius R is given by **[1]**

(1) $\sqrt{\frac{2GM}{R}}$ (2) $2\sqrt{\frac{GM}{R}}$ (3) $\sqrt{\frac{2GMm}{R}}$ (4) $\sqrt{\frac{GM}{R}}$

Ans: $V_e = \sqrt{\frac{2GM}{R}}$

11.For a satellite moving in an orbit around the earth, the ratio of magnitudes of its kinetic energy to potential energy is **[2]**

(1) 2:1 (2) 1:2 (3) 1:$\sqrt{2}$ (4) $\sqrt{2}$:1

Ans:

$$KE = \frac{1}{2} = mV_0^2$$

$$V_o = \sqrt{\frac{GM}{r}}$$

$r \to$ radius of orbit

$$KE = \frac{1}{2}m\frac{GM}{r} = \frac{GMm}{2r}$$

$$PE = -\frac{GMm}{r}$$

$$\frac{KE}{|PE|} = \frac{1}{2}$$

12.When a satellite going round the earth in a circular orbit of radius r with speed v loses some of its energy, then r and v changes as **[3]**

(1)r and v both will increase (2) r and b both will decrease
(3) r will decreases and v will increase (4) r will decreases and v will decrease

Ans: Revolving satellite looses some energy radius of orbit decreases. So, velocity increases.

13.If r represents the radius of the orbit of a satellite of mass m moving around a planet of a mass M, the square of orbital velocity (V_0) of the satellite is given by **[4]**

(1) $V_0^2 = g\frac{M}{r}$ (2) $V_0^2 = \frac{GMm}{r}$ (3) $V_0^2 = \frac{Gr}{M}$ (4) $V_0^2 = \frac{GM}{r}$

Ans: Orbital Velocity

$$V_o = \sqrt{\frac{GM}{r}}$$

$$V_o^2 = \frac{GM}{r}$$

r = radius of orbit.

14.The period of a satellite in a circular orbit around a planet is independent of **[3]**

(1) the mass of the planet (2) the radius of the planet
(3) the mass of the satellite (4) the radius of the satellite

Ans: Time Period

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$r \rightarrow$ radius of Orbit

T is independent of mass of the satellite.

II. FILL IN THE BLANKS

1. **Kepler's second law regarding constancy of areal velocity of a planet is a consequence of the law of conservation of angular momentum.**
   Ans: Areal velocity = Constant
   External torque in zero
   So angular momentum is constant.

2. **If M is the mass of the earth and R is its radius, the ratio of the acceleration due to gravity on the surface of earth and the universal gravitational constant is** $\underline{\frac{M}{R^2}}$

Ans:
$$g = \frac{GM}{R^2}$$
$$\frac{g}{G} = \frac{M}{R^2}$$

3. **If R is the radius of the earth and g is the acceleration due to gravity on the earth's surface, then the mean density of the earth is** $\underline{\frac{3g}{4\pi RG}}$**.**

Ans: $g = \frac{GM}{R^2}$ M=Volume$\times$density, M=$\frac{4}{3}\pi R^3\rho$

$$g = \frac{G}{R^2}\times\frac{4}{3}\pi R^3\rho$$
$$g = \frac{4}{3}\pi RG\rho$$
$$g\alpha R$$
$$\rho = \frac{3g}{4\pi RG}$$

4. **The intensity of gravitational field at the centre of the spherical shell is zero.**
   Ans: Intensity of gravitational field at centre is Zero.

5. **The force of attraction due to a hollow spherical shell of uniform density, on a point mass situated inside it is <u>zero</u>.**

   Ans: Inside field is zero in hollow spherical shell. So force on mass kept inside is Zero.

6. **The acceleration due to gravity on the surface of earth is 'g'. Its value at a height equal to double the radius of the earth is $\underline{\frac{g}{9}}$**

   Ans: On the surface of earth g = $\frac{GM}{R^2}$

   At a height h from surface of earth

   $g_h = \frac{gR^2}{(R+h)^2}$ h=2R, $g_h = \frac{gR^2}{(2R+R)^2}, g_h = \frac{gR^2}{9R^2} = \frac{g}{9}$.

7. **As we go from the equator to the poles, the value of g <u>increases</u>.**

   Ans: The value of g is more at poles than equator as we go from equator to poles g increases.

8. **The escape velocity of a projectile from the earth's surface is (approximately) <u>11.2km/s.</u>**

   Ans: Escape velocity on the surface of earth is

   $$v_e = \sqrt{\frac{2GM}{R}}$$
   $$= \sqrt{2gR}$$
   $$= \sqrt{2 \times 9.8 \times 6400 \times 10^3}$$
   $$= 11.2 km/s$$

9. **A missile is launched with a velocity less than the escape velocity. The sign of sum of its kinetic and potential energy is <u>negative</u>.**

   Ans: If projected velocity is less than escape velocity

   Potential energy is negative

   Kinetic energy always positive

   Sum of potential energy and kinetic energy is Negative.

10. **The time period of satellite in an orbit very close to the surface of earth is (approximately) <u>85 minutes</u>.**

    Ans: $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

    Orbit close to earth surface

    R + h ≈ R

    $$T = 2\pi\sqrt{\frac{R^3}{GM}}$$
    $$= 2\pi\sqrt{\frac{R}{g}}$$
    $$= 2 \times 3.14\sqrt{\frac{6400 \times 10^3}{9.8}} \text{ sec}$$
    $$= 84.6 \min$$
    $$\approx 85 \min utes$$

11. If V_e is the escape velocity and V_o is the orbital velocity of a satellite for an orbit close to the earth's surface, then V_e is $\sqrt{2}$ times of V_o

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

Ans: $V_o = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

$$V_e = \sqrt{2}\sqrt{gR} = \sqrt{2}V_o$$

III. ONE WORD ANSWER QUESTIONS

1. **State Kepler's law of orbits.**

   Ans: All planets move in elliptical orbits with the sun situated at one of the foci of the ellipse.

2. **State Kepler's law of areas.**

   Ans: The line that joins any planet to the sun sweeps equal areas in equal intervals of time.

3. **What is the value of universal gravitational constant?**

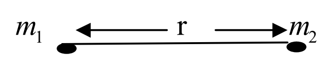
   Ans: $G = 6.67 \times 10^{-11}$ Nm^2 Kg^{-2}

4. **What is the weight of a body at the centre of the earth?**

   Ans: Weight at centre of earth is zero

5. **What is the gravitational potential energy of two point masses m_1 and m_2, which are kept at infinite distance from each other?**

   Ans:



$$U = \frac{-Gm_1m_2}{r}$$

$r = \infty$

$U = 0$

6. **Does the escape velocity of the projected body depend upon its mass?**

   Ans: $V_e = \sqrt{\frac{2GM}{R}}$

   V_e is independent of mass of projected body.

7. **What is the sign of kinetic energy of a satellite revolving around the earth?**

   Ans: Kinetic energy is always positive.

8. **Does the orbital velocity of a satellite depend upon its mass?**

   Ans: $V_0 = \sqrt{\frac{GM}{r}}$

   $r \rightarrow$ radius of orbit , V_0 is independent of mass of revolving body

9. **What is the time period of the moon revolving in an orbit around the earth approximately?**
   Ans: T = 27.3 days.

IV. VERY SHORT ANSWER QUESTIONS

1. **State the unit and dimension of the universal gravitational constant (G)**

Ans: From the universal law of gravitation

$$F = G\frac{Mm}{d^2}$$

$$\Rightarrow G = \frac{F.d^2}{Mm} = \frac{N.m^2}{kg^2}$$ is unit for gravitational constant

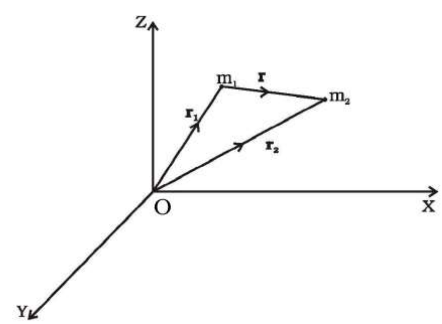
For dimension: $G = \frac{\left[MLT^{-2}\right]\left[L^2\right]}{\left[M^2\right]}$

$\therefore$ Dimension of $G = \left[M^{-1}L^3T^{-2}\right]$

2. **State the vector form of Newton's law of gravitation**

Ans: The force F on a point mass 'm_2' due to another point mass 'm_1' has the magnitude

$$|F| = G\frac{m_1m_2}{r^2}$$



It can be expressed in vector form as

$$F = G\frac{m_1m_2}{r^2}(-\hat{r})$$

$$F = -G\frac{m_1m_2}{|r^3|}\bar{r}$$

$$\bar{r} = \overline{r_2} - \overline{r_1}$$

3. **If the gravitational force of earth on the moon is F, what is the gravitational force of moon on earth? Does these forces to form an action-reaction pair?**

   Ans: If the gravitational force of earth on the moon is F, then the gravitational force of moon on earth is "$-F$"

   Yes, the gravitational forces form an action-reaction pair i.e., obeys Newton's third law.

4. What would be the change in acceleration due to gravity(g) at the surface, if the radius of earth decreases by 2% keeping the mass of the earth constant?

Ans: $g = \frac{GM_E}{R_E^2}$

Here 'G' constant, given M_E is also constant

$$\therefore g \propto \frac{1}{R_E^2} \text{ Let } GM_E = K = \text{Constant}$$

$$g = \frac{K}{R_E^2} \rightarrow (1)$$

$$\Delta g = -\frac{2K}{R_E^3}\Delta R_E \rightarrow (2)$$

$$\frac{\Delta g}{g}\times 100 = \frac{\frac{-2K}{R_E^3}\Delta R_E}{\left(K/R_E^2\right)}\times 100$$

$$\frac{\Delta g}{g}\times 100 = \frac{-2K}{R_E^3}\Delta R_E \times \frac{R_E^2}{K}\times 100$$

$$= -2\left[\frac{\Delta R_E}{R_E}\right]\times 100$$

$$\text{Given } \frac{\Delta R_E}{R_E}\times 100\,\% = -2\%$$

$$\therefore \frac{\Delta g}{g}\times 100 = -2\times(-2) = +4\%$$

i.e., As radius of earth decreased by 2% implies acceleration due to gravity increases by 4%

5. As we go from one planet to another, how will

(a) The mass and
(b) The weight of a body change?
Ans:

(a) The mass of a body is constant.

(b) Weight of a body first decreases gradually with height, becomes zero and again increases due to gravitational field of another planet.

6. **Keeping the length of a simple pendulum constant, will the time period be the same on all planets? Support your answer with reason**

Ans: No, time period changes from planet to planet. From oscillation lesson, we know that time period of the simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$

In the given hypothesis $'l'$ is constant

$\therefore T \propto \frac{1}{\sqrt{g}}$

T depends on g where

$g = \frac{GM}{R^2}$

Here M and R changes ,

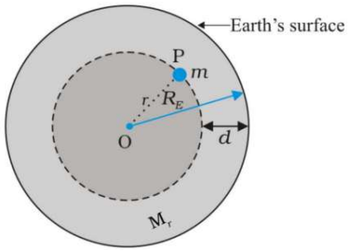
From planet to planet $'g'$ value changes, So time period also changes.

7. **Give the equation for the value of $'g'$ at a depth $"d"$ from the surface of earth. What is the value of $'g'$ at the centre of earth?**

Ans: Let mass and radius of the earth are M_E and R_E then acceleration due to gravity at a depth $"d"$ from the surface of earth is

$$g(d) = g\left[1 - \frac{d}{R_E}\right]$$

Diagram:



At the centre of earth $d = R_E$

$$\therefore g(centre) = g\left[1 - \frac{R_E}{R_E}\right] = 0$$

8. What are at factors that make "g" the least at the equator and maximum at the poles?

Ans: The acceleration due to gravity on earth surface $(g) = \frac{GM_E}{R_E}$ Here 'R_E' is earth radius

(i) In reality earth is not in perfect spherical shape, it is an oblate spheroid i.e., slightly fattened at the poles and bulging at the equator. This difference in radius affects the value of acceleration due to gravity.

(ii) The earth's rotation creates a centrifugal force, this force acts outwards, this effect is dominated at the equator because rotational speed is highest. The combined effect of the large radius and centrifugal force at the equator results lowest 'g' value at the equator.

The smaller radius at the pole and the absence of significant centrifugal force contribute to the highest 'g' value at the poles.

9. "Hydrogen is in abundance around the sun but not around earth" Explain?

Ans: As per kinetic theory gases $V_{rms} = \sqrt{\frac{3RT}{M}}$

Where R = universal gas constant

T = Absolute temperature

M = Molar mass of the gas

$\therefore V_{rms}$ of hydrogen gas at room temp = 2.4km/sec

(as per page no: 154) The escape velocity of a planet $(v) = \sqrt{\frac{2GM}{R}}$

R= Radius of the planet, M= Mass of the planet

$\therefore$ Escape velocity of sun's surface = 617.6km/sec

Escape velocity of earth surface = 11.2km/sec

Escape velocity of sun > escape velocity of earth > RMS Value of hydrogen gas

Difference between escapes velocity of the sun and rms velocity of hydrogen is very high compared to difference between escape velocity of earth and rms velocity of hydrogen, so more hydrogen is around the sun compared to earth.

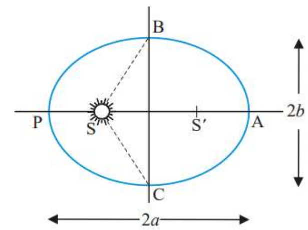
$\therefore$ On the surface of sun abundance hydrogen gas exist but not around the earth.

V. SHORT ANSWER QUESTIONS:

1) State Kepler's law of planetary motion.

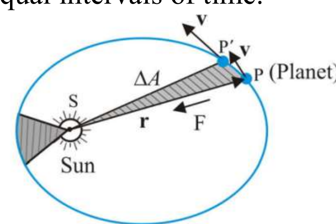
Ans: The three laws of Kepler can be stated as follows:

1) **Law of orbits**:- All planets move in elliptical orbits with the sun situated at one of the foci of the ellipse.



2) **Law of areas:-**
The line that joins any planet to the sun sweeps equal areas in equal intervals of time.

Areal velocity is constant



3) **Law of periods:-**
The square of the time period of revolution of a planet is proportional to the cube of the semi major axis of the ellipse placed out by the planet.
$T^2 \alpha a^3$ where 'a' is semi major axis.

2) Derive the relation between acceleration due to gravity (g) at the surface of a planet and the gravitational constant (G).

Ans: According to Newton's law of universal gravitational, the force (F) between two objects with masses M (planet) and m (object) separated by a distance R is given by

$F = G\frac{Mm}{R^2} \rightarrow (1)$ where $'G'$= Universal gravitational constant

When an object of mass $'m'$ is on the surface of a planet of mass $'M'$ and radius $'R'$, the gravitational force acting on it is the force that causes it to accelerate due to gravity (g)

$\therefore F = mg \rightarrow (2)$ (From Newton's Second law)

Equation (1) and (2) represent the same force expressions

$$\therefore G\frac{Mm}{R^2} = mg$$

$$\Rightarrow g = \frac{GM}{R^2}$$

3) How does the acceleration due to gravity(g) chances for the same values of height (h) and depth (d).

Ans: The acceleration due to gravity (g) above the earth surface with height (h) is

$$g(h) = g\left[1 - \frac{2h}{R_E}\right] \text{ (as per Eq 7.15 in page no.151)}$$

The acceleration due to gravity (g) at a depth (d) from the earth surface is

$$g(d) = g\left[1 - \frac{d}{R_E}\right]$$

For same value of height 'h' and 'd' i.e., h=d

$$\therefore g(h) = g\left[1 - \frac{2h}{R_E}\right]$$

$$g(d) = g\left[1 - \frac{h}{R_E}\right]$$

$$g_h = g - \frac{2gh}{R_E}$$

$$g_h - g = -\frac{2gh}{R_E}$$

But $$|g_h - g| = \frac{2gh}{R_E}$$

$$g_d - g = -\frac{gh}{R_E}$$

$$|g_d - g| = \frac{gh}{R_E}$$

$$g_h < g_d$$

for small and same values of d and h the acceleration due to gravity decreases more at height compared to depth from the surface of earth.

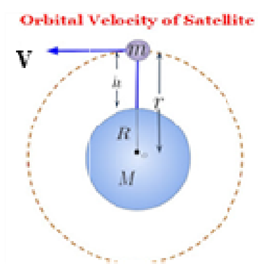
4) What is orbital velocity? Obtain an expression for it.

Ans: Orbital velocity is the speed at which a satellite must move to stay in orbit around a planet or other celestial body. Consider a satellite of mass 'm' moving around the planet (let planet mass=M and radius R) with an height 'h' in a circular orbit. Let the speed of the satellite = V.

The centripetal force required for the orbit is $F_{centripetal} = \frac{mv^2}{(R+h)} \rightarrow (1)$ is directed towards the centre.

r = R + h where ‘r’ is called radius of orbit.

This centripetal force is provided by the gravitational force, which is



$$F(Gravitation) = \frac{GMm}{(R+h)^2} \rightarrow (2)$$

Equating RHS of equations (1) and (2) $\frac{mv^2}{(R+h)} = \frac{GmM}{(R+h)^2}$

$$\Rightarrow V^2 = \frac{GM}{(R+h)}$$

$$\therefore \text{Orbital velocity (V)} = \sqrt{\frac{GM}{(R+h)}}$$

Orbit close to earth’s surface R + h $\approx$ R

$$V = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

5) What is escape velocity? Obtain an expression for it.

Ans: Escape velocity is the minimum speed an object needs to launched from a celestial body to overcome its gravitational pull and escape into space, never to return. Suppose energy of the object is the sum of potential and kinetic energy.

Let W_1 denotes that gravitational potential energy of the object at infinity then

Total energy of the projectile at infinity

$$E(\infty) = W_1 + \frac{1}{2} m v_f^2 \text{ \_\_\_\_\_\_\_\_\_\_\_\_ } (1)$$

If the object was thrown initially with a speed $'V_i'$ from a point at a distance $"h + R_E"$ from the centre of the earth (R_E = radius of the earth), its energy initially

$$E(h+R_E) = \frac{1}{2}mv_i^2 - \frac{GM_E m}{(h+R_E)} + W_1 \rightarrow (2)$$

Principle of conservation of energy, $E(h+R_E) = E(\infty)$

$$W_1 + \frac{1}{2}m V_f^2 = \frac{1}{2}m V_i^2 - \frac{GM_E m}{(h+R_E)} + W_1$$

$$\therefore \frac{1}{2}mV_i^2 - G\frac{M_E m}{(R_E+h)} = \frac{1}{2}mV_f^2$$

RHS is +Ve quantity with minimum value zero

$$\therefore \frac{1}{2}mV_i^2 - G\frac{M_E m}{(R_E+h)} \geq 0$$

The minimum value of above $\Rightarrow \frac{1}{2}m(V_i)^2_{\min} - \frac{GM_E m}{(R_E+h)} = 0$

$$\therefore \frac{1}{2}m(V_i)^2_{\min} = \frac{GM_E m}{(R_E+h)}$$

$$(V_i)_{\min} = \sqrt{\frac{2GM_E}{(R_E+h)}}$$

If objects is thrown from the surface of earth $\Rightarrow h = 0$

$$\therefore (V_i)_{\min} = EscapeVelocity = \sqrt{\frac{2GM_E}{R_E}}$$

But $g = \frac{GM_E}{R_E^2} \Rightarrow GM_E = gR_E^2$

$$\therefore (V_i)_{\min} = \sqrt{2gR_E}$$

6) If two places are at the same height from the mean sea level, one is a mountain and other is in air. At which place will 'g' be greater? State the reason for your answer.

Ans: The value of 'g' will be greater on the mountain, even if both places are at the same height above the sea level.

The acceleration due to gravity $(g) = \frac{GM_E}{R_E^2}$. The value of 'g' at a point is determined by the gravitational pull of the earth. On a mountain, the additional mass of the mountain itself also contributes to the gravitational pull at that location. This is a local effect meaning the gravitational pull is slightly stronger near the mountain's surface compared to the air at the same elevation.

7) The weight of an object is more at the poles than at the equator. At which of these can we get more sugar for same weight? State the reason for your answer

Ans: Weight of a body is given by W =mg

It is given that, weight of a body at poles = weight of the body at equator

i.e $m_p\, g_p = m_e\, g_e$ …………….(1)

From $g = \frac{GM}{R^2}$ we have $g \alpha \frac{1}{R^2}$, But $R_{poles} < R_{Equator}$

$g_{poles} > g_{Equator}$ ……………..(2)

From (1) and (2) $m_e > m_p$

Hence more sugar is obtained at the equator.

8) If a nut becomes loose and gets detached from a satellite revolution around the earth, will it fall down to earth or will it revolve around earth? Give reason for your answer

Ans: When the satellite is revolving around the Earth with orbital velocity then every part of the satellite possess the same velocity, V = $\sqrt{\frac{GM}{r}}$ radius of orbit is ‘ r ’.

When a nut is detached from a satellite, it will have same orbital velocity as that of the satellite due to inertia. Hence it continues to revolve around the Earth in the same orbit.

9) An object projected with a velocity greater than or equal to 11.2km/sec will not return to earth explain the reason.

Ans: Escape velocity of a earth is given by $V_e = \sqrt{\frac{2GM}{R}}$, where G is universal gravitational constant. ‘M’ is mass of the earth and ‘R’ is the radius of the Earth.

The escape velocity of earth is 11.2km/sec. Therefore, when an object projected with a velocity greater than or equal to 11.2km/sec then it will escape from gravitational field of Earth. Their after the body will not return to the Earth.

VI. LONG ANSWER QUESTIONS:

1) Define gravitational potential energy and derive an expression for it associated with two particles of masses m_1 and m_2

Ans: The force of gravity is a conservative force and we can calculate the potential energy of a body arising out of this force called the gravitational potential energy. Consider points close to the surface of earth, at distances from the surface much smaller than the radius of the earth. Hence the force of gravity is constant equal to mg, directed towards the centre of the earth.

If we consider a point at a height h_1, from the surface of the earth and another point vertically above it at a height “h_2” from the surface, the work done in lifting the particle of mass "m" from the first to the second position is denoted by W_{12}.

W_{12} = Force × displacement

$W_{12} = \text{mg}\left(h_2 - h_1\right) \rightarrow (1)$

If we associate a potential energy W(h) at a point at a height ’h’ above the surface such that

W(h) = mgh + $W_0 \rightarrow (2)$ where ‘W_0‘ is constant

$W_{12} = W(h_2) - W(h_1) \rightarrow$ (3) In equation (2) if h = 0 then W_0 is the potential energy on the surface of the earth

We know that the point outside the earth, the force of gravitation on a particle directed towards the centre of the earth is

$$F = \frac{GM_E m}{r^2} \rightarrow (4)$$

Where M_E = mass of earth

m= mass of the particle

r is distance from centre of the earth to mass $'m'$

The work done in lifting a particle from $r = r_1$ to $r = r_2$ $(r_2 > r_1)$

Along a vertical path $W_{12} = \int_{r_1}^{r_2} F.dr$

$$W_{12} = \int_{r_1}^{r_2} \left[\frac{GM_E m}{r^2}\right] dr$$

$$W_{12} = -GM_E m\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

In place of Equation (2), we can this associate a potential energy $W(r)$ at a distance

'r' such that

$$W(r) = \frac{-GM_E m}{r} + W_1 \text{ Valid for } r > R$$

One conventionally set W_1 equal to zero, so that the potential energy at a point is just the amount of work done in displacing the particle from infinity to that point.

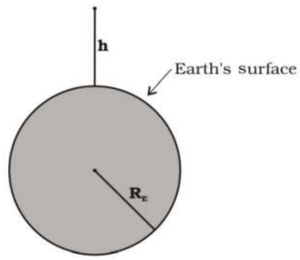
The gravitational potential energy associated with two particles of masses m_1 and m_2 separated by separated by distance $'r'$ is given by

V= $-\frac{Gm_1 m_2}{r}$ (If we choose V=0 as $r \rightarrow \infty$)

2) Derive an expression for the variation of acceleration due to gravity (a) above and (b) below the surface of the earth

Ans: Consider a point mass $"m"$ at a height $'h'$ above the surface of the earth as shown in following figure. The radius of the earth is denoted by $"R_E"$. Since this point is outside the earth, its distance from the centre of the earth is $(R_E + h)$. If $F(h)$ denoted the magnitude of the force on the point mass $"m"$

$$F(h) = G.\frac{M_E m}{(R_E + h)^2} \rightarrow (1)$$



The acceleration experienced by the point mass is $\frac{F(h)}{m} = g(h)$

$$\therefore\ g(h) = \frac{F(h)}{m} = \frac{1}{m} \times G\frac{M_E m}{(R_E + h)^2}$$

$$g(h) = \frac{GM_E}{(R_E + h)^2} \rightarrow (2)$$

We know that the value of $'g'$ on the surface of earth

$$\text{g}=\frac{GM_E}{R_E^2} \rightarrow (3)$$

Equation (2) is

$$g(h) = \frac{GM_E}{(R_E + h)^2} = \frac{GM_E}{\left[R_E\left(1 + \frac{h}{R_E}\right)\right]^2}$$

$$g(h) = \frac{GM_E}{R_E^2\left[1 + \frac{h}{R_E}\right]^2}$$

$$= \frac{g}{\left(1 + \frac{h}{R_E}\right)^2} = g\left(1 + \frac{h}{R_E}\right)^{-2}$$

For $\frac{h}{R_E} \ll 1 \Rightarrow$ From Binomial expression

$g\left(h\right)=g\left[1-\frac{2h}{R_E}\right]$ is equation for variation of $'g'$ above the surface of earth with height $'h'$ Consider a point mass $'m'$ at a depth $'d'$ below the surface of the earth as shown in figure , so its distance from the centre of the earth is $\left(R_E-d\right)$

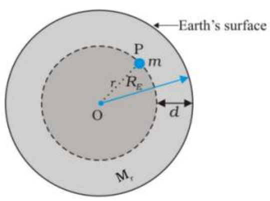
The earth can be thought of as being composed of a smaller sphere of radius $\left(R_E-d\right)$ and a spherical shell of thickness $"d"$. As far as the smaller sphere of radius $\left(R_E-d\right)$ is concerned, the point mass is outside it.

M_S $\rightarrow$ mass of the small sphere .

Mass of the sphere is proportional to the cube of its radius

$$\frac{M_s}{M_E}=\frac{\left(R_E-d\right)^3}{R_E^3}$$

$$M_S = \frac{(R_E-d)^3}{R_E^3}M_E$$



$\therefore$ Force on the point mass F (d) = $\frac{GM_S m}{(R_E-d)^2}$

$$F(d) = \frac{Gm\ (R_E-d)^3}{R_E^3}M_E \times \frac{1}{(R_E-d)^2}$$

$$F(d)=GM_E m\frac{\left(R_E-d\right)}{R_E^3}$$

Hence the acceleration due to gravity as a depth (d) is $g(d)=\frac{F\left(d\right)}{m}$

$$g(d)=\frac{1}{m}.GM_E m.\frac{\left(R_E-d\right)}{R_E^3}$$

$$g(d)=\frac{GM_E}{R_E^2}\left(\frac{R_E-d}{R_E}\right)$$

We know that $\frac{GM_E}{R_E^2}=g$

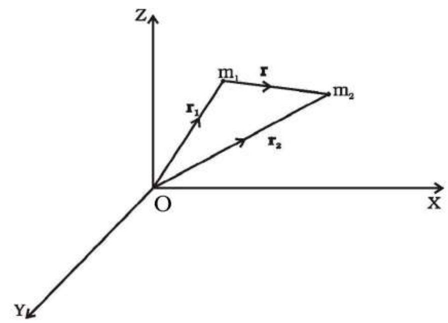
$$\therefore\ g\left(d\right)=g.\frac{\left(R_E-d\right)}{R_E}$$

$g(d)=g\left(1-\frac{d}{R_E}\right)$ This gives the variation of $'g'$ with depth $'d'$ from the earth surface.

3) State Newton's Universal law of gravitation. Explain how the value of the gravitational constant (G) can be determined by cavendish method

Ans: The force F on a point mass 'm_2' due to another point mass 'm_1' has the magnitude

$$|F| = G\frac{m_1 m_2}{r^2}$$



It can be expressed in vector form as

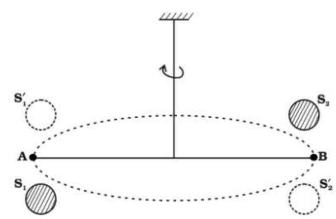
$$\boldsymbol{F} = G\frac{m_1 m_2}{r^2}\left(-\hat{r}\right)$$

$$\boldsymbol{F} = -G\frac{m_1 m_2}{\left|r^3\right|}\bar{r}$$

$$\bar{r} = \overline{r_2} - \overline{r_1}$$

Gravitational force is attractive i.e., the force $\overline{F}$ is along $-\bar{r}$

The value of the gravitational constant "G" entering the universal law of gravitation can be determined experimentally and this was first done by English scientist Henry cavendish in 1798. The apparatus used by him is shown in following Figure.



The bar AB has two small lead spheres attached as its ends, the bar is suspended from a rigid support by a fine wire. Two large lead spheres are brought close to the small ones but on opposite sides as shown.

The big spheres attract the near by small ones by equal and opposite forces there is no net force on the bar but only a torque which is clearly equal to F times the length of the bar, where 'F' is the force of attraction between a big sphere and its neighbouring small sphere. Due to this torque the suspended wire gets twisted till such time as the restoring torque of the wire equals the gravitational torque.

If 'θ' is the angle of twist of the suspended wire the restoring torque is proportional to 'θ' equal to "$\tau\theta$" where τ is the restoring couple per unit angle twist.

The gravitational force between the spherical balls is the same as if their masses are concentrated as their centres.

Thus if 'd' is the separation between the centres of big and small ball, M and m their masses, the gravitational force between big sphere and small sphere is small ball is $F = G\frac{Mm}{d^2}$

If L is the length of the bar AB, then the torque arising out of F is F multiplied by L.

At equilibrium, this is equal to the restoring torque and hence $G\frac{Mm}{d^2}L = \tau\theta$

Observation of 'θ' thus enables one to calculate 'G' from the equation.

Since cavendish's experiment, the measurement of 'G' has been refined and the currently accepted value is G = 6. 67 x 10^{-11} Nm^2/kg^2.

VII. PROBLEMS :

1.Two spherical balls each of mass 1kg are placed at 1 cm apart. Find the gravitational force of attraction between them.

Ans: Given $m_1 = m_2 = 1kg$ and $d = 1cm = 10^{-2}m$ gravitational constant

$G = 6.67x10^{-11}\ Nm^2 / kg^2$

Gravitational force of attraction, $F = \frac{Gm_1m_2}{d^2}$

$$= \frac{6.67 \times 10^{-11} \times 1 \times 1}{(10^{-2})^2} = 6.67 \text{ x } 10^{-7}\ N$$

2. The mass of a ball is four times the mass of another ball. When these balls are separated by a distance of 10 cm, the force of gravitation between them is 6.67 x 10^{-7}N. find the masses of the two balls.

Ans: let m_1 and m_2 be the masses of two balls and given $m_2 = 4\ m_1$ and

$d = 10cm = 10^{-1}\ m$

$$F = \frac{Gm_1m_2}{d^2}$$

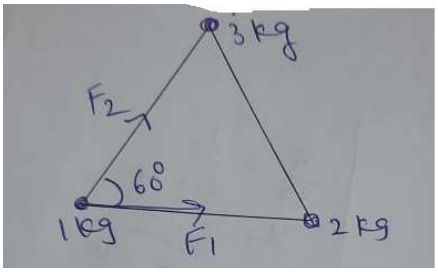
$$6.67X10^{-7} = \frac{6.67X10^{-11}Xm_1X4m_1}{(10^{-1})^2}$$

$$\Rightarrow 4m_1^2 = 100, \quad \Rightarrow m_1^2 = 25$$

$$\Rightarrow m_1 = 5kg \text{ and } m_2 = 4m_1 = 20kg$$

3. Three spherical balls of masses 1kg, 2kg, 3kg are placed at the corners of an equilateral triangle of side 1m. find the magnitude of gravitational force exerted by 2kg and 3 kg masses on the 1kg mass.

Ans: Let F_1 be the force exerted by 2kg on 1kg mass.



$$F_1 = \frac{Gm_1m_2}{d^2} = \frac{G(1)(2)}{1^2} = 2G$$

Let F_2 be the force exerted by 3kg on 1kg mass

$$F_2 = \frac{Gm_1m_3}{d^2} = \frac{G(1)(3)}{1^2} = 3G$$

Resultant force $F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$$= \sqrt{(2G)^2 + (3G)^2 + 2(2G)(3G)\cos 60^0}$$

$$\Rightarrow F_R = \sqrt{19}\,G \text{ N}$$

4.At a certain height above the earth's surface, the acceleration due to gravity is 4% of its value at the surface of earth. Determine the height.

Ans: Given g_h = 4% of g $\Rightarrow$ $g_h = \frac{4}{100}$g and radius of the earth R = 6400 km

We know $g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$$\frac{4}{100}g = \frac{g}{\left(1+\frac{h}{R}\right)^2} \Rightarrow \left(1+\frac{h}{R}\right)^2 = 25$$

$$1 + \frac{h}{R} = 5$$

$$\Rightarrow \frac{h}{R} = 4$$

$\Rightarrow$ =4R =4(6400) = 25600km.

5. A satellite is orbiting the earth at a height of 1000km. find its orbital speed.

Ans: Orbital velocity V = $\sqrt{\frac{GM}{R+h}}$

$$\sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6400 + 1000) \times 10^{3}}} = 7.354 \text{ km/sec}$$

6. A satellite is revolving around the earth in an orbit at a height equal to the radius of earth.

Find its (i) orbital speed and (ii) period of revolution.

Ans: Given h = R = 6400 km

(i) Orbital speed V = $\sqrt{\frac{GM}{R+h}}$

$$\sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6400 + 1000) \times 10^{3}}} = 5.592 \text{ km / sec}$$

(ii) Period of revolution T = $\frac{2\pi(R+h)}{v}$

$$= 2 \times 3.14 \left(\frac{6400+6400}{5592 \times 36 \times 10^{2}}\right) \times 10^{3} = 4\text{hr}$$

7. The gravitational force of attraction between two objects decreases by 36% when the distance between them is increased by 4m. find the original distance between them.

Ans: Given, $F_2 = \frac{64}{100} F_1$ and $d_2 = d_1 + 4m$

We know that $F = \frac{Gm_1m_2}{d^2}$ and $F \alpha \frac{1}{d^2}$

$$\frac{F_2}{F_1} = \left(\frac{d_1}{d_2}\right)^2 \Rightarrow \frac{64}{100} = \left(\frac{d_1}{d_1+4}\right)^2$$

$$\Rightarrow d_1 = 16m$$

8.Two satellites are revolving round the earth at different heights. The ratio of their orbital speeds

is 2:1. If one of them is at a height of 100km. what is the height of the other satellite.

Ans: Given $\frac{V_1}{V_2} = \frac{2}{1}$ and $h_1 = 100$ km and $R_E = 6400$ km

Orbital velocity $V_0 = \sqrt{\frac{GM}{R+h}} \Rightarrow V_0 \alpha \frac{1}{\sqrt{R+h}}$

$$\frac{V_1}{V_2} = \sqrt{\frac{R+h_2}{R+h_1}} \Rightarrow h_2 = 19,600km$$

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UNIT – 8

MECHANICAL PROPERTIES OF SOLIDS

I. MULTIPLE CHOICE QUESTIONS

1. Among the following, the materials with highest elasticity is (4)

1) copper 2) Aluminium 3) Brass 4) Steel

Ans : Steel metal Young's modulus is more than copper, Aluminium, Brass metals

2. The type of stress acts tangentially to a body's surface is (2)

1) Hydraulic stress 2) Shearing stress 3) Compressive stress 4) Tensile stress

Ans : The restoring force per unit area developed due to the applied tangential force is known as shearing stress

3. If the length of a wire is stretched to twice its original length, the strain in the wire is (2)

1) **2** 2) **1** 3) **1/2** 4) **1/4**

Ans : $Strain = \frac{\Delta L}{L} = \frac{L_2 - L_1}{L_1} = \frac{2L_1 - L_1}{L_1} = \frac{L_1}{L_1} = 1$

4. Two metal wires of the same material have lengths in the ratio 1 : 2 and radii in the ratio 1 : 2. The ratio of their elongations under the same applied force is (2)

1) **1 : 2** 2) **2 : 1** 3) **1 : 4** 4) **4 : 1**

Note : In text book the ratio of radii read as 1 :2 instead of 2 : 1.

Ans : $Y = \frac{Fl}{Ae}$ $Y = \frac{Fl}{\pi r^2 e}$

$l_1 : l_2 = 1 : 2$ $r_1 : r_2 = 1 : 2$

$e = \frac{Fl}{Y\pi r^2}$, $\therefore \frac{e_1}{e_2} = \frac{l_1}{l_2}\left[\frac{r_2^2}{r_1^2}\right]$

$= \frac{1}{2}\left(\frac{2}{1}\right)^2$

$= \frac{1}{2}\left(\frac{4}{1}\right)$

$= \frac{2}{1}$

= 2 : 1

5. **Determine the volume contraction of a solid copper cube with a volume of $10^{-3}m^3$ when subjected to a hydraulic pressure of $7\times10^6 Pa$ (Bulk modulus of copper $k=140\times10^9 Nm^{-2}$)** **(2)**

1) $5\times10^{-7}m^3$ 2) $5\times10^{-8}m^3$ 3) $0.5\times10^{-8}m^3$ 4) $5\times10^{-5}m^3$

Ans : $B=\dfrac{P}{\dfrac{\Delta V}{V}}=\dfrac{PV}{\Delta V}$

$$\Delta V=\frac{PV}{B}$$

$$\Delta V=\frac{7\times10^6\times10^{-3}}{140\times10^9}$$

$$\Delta V=\frac{7\times10^3}{14\times10^{10}}=\frac{1}{2}\times10^{-7}=0.5\times10^{-7}$$

$$\Delta V=5\times10^{-8}m^3$$

6. **Poisson's ratio depends only on** **(3)**

1) Stress 2) Strain
3) Nature of the material 4) force

7. **The maximum stress that a material can withstand without breaking is** **(2)**

1) yield strength 2) ultimate tensile strength
3) Elastic limit 4) Breaking stress

8. **Steel is more elastic than rubber, because** **(2)**

Ans: 1) Young's modulus of rubber is greater than that of steel

2) Young's modulus of steel is greater than that of rubber

3) For the same applied force, the elongation of steel is more

4) For the same applied force, the elongation of rubber is less

9. **A force of 30 KN is applied along its length on a steel rod of cross sectional area 3cm^2. The stress on the rod is** **(3)**

1) $10^2 Nm^{-2}$ **2)** $10^4 Nm^{-2}$ **3)** $10^8 Nm^{-2}$ **4)** $10^6 Nm^{-2}$

Ans : Stress = $\dfrac{F}{A}$ F = 30KN = $30\times10^3 N$

A = 3cm^2 = $3\times\left(10^{-2}m\right)^2=3\times10^{-4}m^2$

Stress = $\dfrac{30\times10^3}{3\times10^{-4}}=10^8 Nm^{-2}$

II. FILL IN THE BLANKS

1. The property of a body to regain its original shape after deformation is called ________

A. elasticity

2. The SI unit of Young's modulus is _______

A. Pascal or Nm^{-2}

3. Materials that can undergo large strains are called _____

A. Elastomers

4. An example of elastomer is _________

A. Rubber or tissue of aorta

5. The typical value of Poisson's ratio for steel is ___________

A. between 0.28 and 0.30

6. When a wire is stretched, it shows an increase in length and a decrease in ____

A. diameter

7. The strain produced by hydraulic pressure is called ______

A. Volume strain

8. The strain perpendicular to the applied force is called _____

A. Lateral strain

9. The reciprocal of the bulk modulus is called ________

A. Compressibility

10. The ratio of shearing stress to shearing strain is called ______

A. Shear modulus

III. ONE WORD ANSWER QUESTIONS

1. What is the proportionality constant in Hooke's law?

A. Modulus of elasticity

2. What is the term for the energy stored in a stretched wire ?

A. Elastic potential energy

3. In the stress-strain curve, what is the point beyond which permanent deformation occurs?

A. yield point

4. What is the dimensional formula of stress?

A. $[ML^{-1}T^{-2}]$

5. Which modulus measures resistance to compression?

A. Bulk modulus

6. What is the equation for elastic potential energy stored in a stretched wire?

A. $U = \frac{1}{2} \times stress \times strain \times volume$

7. What is poission's ratio?

A. $\sigma = \frac{(\Delta d / d)}{(\Delta L / L)}$ or $\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}}$

or

The ratio of the lateral strain to the longitudinal strain in a stretched wire is called possion's ratio

8. What is the SI unit of strain?

A. Dimensionless, strain has no units.

9. What is the maximum possible height of a mountain on earth?

A. 10 Km

10. What is the property of a material that resists a change in shape?

A. Rigidity

IV. VERY SHORT ANSWER QUESTIONS

1. What is elasticity?

A. The property of a body by virtue of which it tends to regain its original size and shape when the applied force is removed is called elasticity.

2. What is elastic deformation ?

A. When the applied force is removed which the body regain its original size and shape, this property of a body is called elasticity and the deformation caused is known as elastic deformation

3. What are elastomers ? Give an example

A. The materials, which can be stretched to cause large strains are called elastomers.

Ex. Tissue of aorta, or rubber

4. What is plasticity ?

A. If you apply force to a lump of putty or mud, they have no gross tendency to regain their previous shape, and they get permanently deformed. Such substances are called plastic and this property is called plasticity.

5. State Hooke's law of elasticity

A. For small deformations the stress and strain are proportional to each other

Stress α Strain

Stress = K (strain)

K = modulus of elasticity

6. Define modulus of elasticity

A. For small deformations the stress and strain are proportional to each other. The ratio between stress and strain is called modulus of elasticity

7. State the units and dimensions of stress

A. CGS units : dyne cm^{-2}

SI units : Nm^{-2} (or) pascal or Pa

Dimensional Formula $ML^{-1}T^{-2}$

8. State the units and dimensions of modulus of elasticity.

A. CGS units : dyne cm^{-2}

SI units : Nm^{-2} (or) pascal or pa

Dimensional formula $ML^{-1}T^{-2}$

9. What is lateral strain?

A. The strain perpendicular to the applied force is called lateral strain

10. What is poisson's ratio? What is the value of poisson's ratio for steel?

A. The ratio of the lateral strain to the longitudinal strain in a stretched wire is called poisson's ratio. The value of poisson's ratio for steel is between 0.28 and 0.30

V. SHORT ANSWER QUESTIONS

1. Define Hooke's law of elasticity, proportionality limit, permanent set, and breaking stress.

A. **Hooke's Law of elasticity : -** For small deformations the stress and strain are proportional to each other. This is known as Hooke's law.

Stress α Strain

Stress = K strain

K = modulus of elasticity

Proportionality limit : - The maximum stress developed in a body till it obey's Hooke's law. The body regains its original dimensions when the applied force is removed.

Permanent set : - When the load is removed beyond yield point, the body does not regain its original dimension. Then the material is said to have a permanent set.

Breaking stress : - The maximum stress the material can bear before it breaks.

2. Define Young's modulus, Bulk modulus and shear modulus.

A. **Young's modulus :** The ratio of tensile stress (σ) to the longitudinal strain (ε) is defined as Young's modulus (Y)

$$Y = \frac{\sigma}{\varepsilon} = \frac{tensile\ stress}{longitudinal\ strain}$$

Bulk modulus :- The ratio of hydraulic stress to the corresponding hydraulic strain is called bulk modulus (B)

$$B = -\frac{p}{\left(\frac{\Delta V}{V}\right)} = \frac{hydraulic\ stress}{hydraulic\ strain}$$

Shear modulus : - The ratio of shearing stress to the corresponding shearing strain is called the shear modulus (G)

$$G = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta x}{L}\right)} = \frac{shearing\ stress}{shearing\ strain}$$

3. Define stress and explain the types of stress

A. **Stress : -** The restoring force per unit area is called stress

Longitudinal stress (Tensile stress) : - The normal restoring force acting per unit area to change the length of a body is called longitudinal stress.

Shearing stress (Tangential stress):- The tangential restoring force acting per unit area to change the shape of the body is called shearing stress

Hydraulic stress :- (Bulk stress):- The normal force restoring force acting per unit area to change the volume of a body is called hydraulic stress.

4. Define strain and explain the types of strain

Ans **Strain:-** It is a ratio of change in dimension to the original dimension of the body

$$\text{Strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

Types of strain : -

1. **Longitudinal strain : -** The ratio of change in the length (ΔL) to the original length (L) of the body is known as "longitudinal strain"

   Longitudinal strain = $\frac{\Delta L}{L}$

2. **Tangential (or) Shearing strain :-** The ratio of relative displacement between the opposite faces (Δx) and vertical distance between the faces (L) is known as shearing strain

   Shearing strain = $\frac{\Delta x}{L} = \tan\theta$

3. **Volume strain : -** The ratio of change in volume (ΔV) to the original volume (V) is known as volume strain

   Volume strain = $\frac{\Delta V}{V}$

5. Explain why steel is preferred to copper, brass and aluminium is heavy – duty machines and structural design?

Ans: A large force is required for steel to produce small change in length. To increase the length of a thin steel wire of 0.1 cm^2 cross sectional area by 0.1%, a force of 2000N is required. The force required to produce the same strain in aluminium, brass and copper wires having the same cross – sectional area are 690N, 900N and 1100 N.

It means that steel is more elastic than copper, brass and aluminium. It is for this reason that steel is preferred in heavy – duty machines and in structural designs

6. While constructing buildings and bridges, a pillar with distributed ends is preferred to a pillar with rounded ends. Why?

Ans: A pillar with rounded ends supports less load than that with a distributed shape at the ends. The precise design of a bridge or a building has to take into account the conditions under which it will function, the cost and long period, reliability of usable materials.

7. Explain why the maximum height of a mountain on earth is approximately 10 km.

Ans: A mountain base is not under uniform compression and this provides some shearing stress to the rocks under which they can flow. The stress due to all the material on the top should be less than the critical shearing stress at which the rocks flow.

At the bottom of a mountain of height h, the force per unit area due to the weight of the mountain is $h\rho g$ where ρ is the density of the material of the mountain and g is the acceleration due to gravity. The material at the bottom experiences this force in the vertical direction and the sides of the mountain are free

There is a shear component approximately hρg it self.

The elastic limit for a typical rock is $30\times10^{7}\,Nm^{-2}$

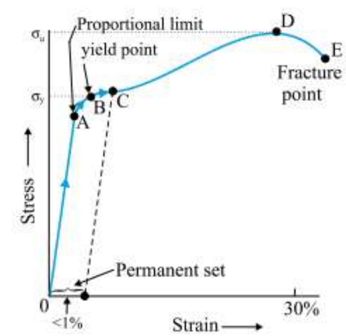
hρg = $30\times10^{7}\,Nm^{-2}$

h = $30\times10^{7}\,Nm^{-2}$ / $30\times10^{3}\,Kgm^{-3}\times10ms^{-2}$

h = 10Km

VI. LONG ANSWER QUESTIONS

1. Define stress and strain. Describe the behaviour of a wire under a gradually increasing load.



Ans: **Stress :-** The restoring force per unit area is known as stress

Strain : - It is the ratio of change in dimension to the original dimension

Behaviour of a wire under gradually increasing load:-

1. **Proportionality limit (O-A) : -** In this region, the curve is linear and Hooke's law is obeyed. The body regains its original dimensions when the applied force is removed. The solid behaves as an elastic body.
2. **Elastic limit (A-B) :-** In this region, stress and strain are not proportional even though the body still returns to its original dimension when the load is removed. The point B is known as yield point
3. **Permanent set (point C):-** When the load is removed, the body does not regain its original dimension. In this case, the material is said to have a permanent set.
4. **Ultimate tensile strength (D – E) : -** The point D on the graph is the ultimate tensile strength of the material.
5. **Fracture point :-** Additional strain is produced, even by a reduced applied force fracture occurs at point E.

2. Explain the concept of elastic potential energy in a stretched wire and derive its expression.

Ans: **Elastic potential energy in a stretched wire : -** "When a wire is put under a tensile stress, work is done against the inter atomic forces. This work is stored in the wire in the form of elastic potential energy"

Expression for elastic potential energy : Consider a wire of original length L and area of cross section A is subjected to a deforming force F along the length of the wire.Let the length of the wire be elongated by l

Young's modulus $Y = \frac{FL}{Al}$

$$F = \frac{YAl}{L}$$

Work done due to further elongation of small length dl,

Work done dw = $F \times dl$

$$= \frac{YAl}{L} \times dl$$

The total amount of work done in increasing the length of the wire from L to $L + l$, that is from $l = 0$ to $l = l$ is

$$W = \int_0^l \frac{YAl}{L} \times dl = \frac{YA}{2}\frac{l^2}{L}$$

$$W = \frac{1}{2} \times Y \times \left(\frac{l}{L}\right)^2 AL$$

$W = \frac{1}{2} \times$ Young's modulus $\times$ (Strain)^2 $\times$ volume of the wire

$W = \frac{1}{2} \times$ stress $\times$ strain $\times$ volume of the wire. This work is stored in the wire in the form of elastic potential energy (U).

$\therefore$ The elastic potential energy per unit volume of the wire U is

$$U = \frac{1}{2} \text{ x stress x strain}$$

VII. PROBLEMS

1. A copper wire of 1mm diameter is stretched by applying a force of 10N. Find the stress in the wire.

Ans. Diameter D = 1mm = $10^{-3} m$

Radius $r = \frac{D}{2} = 0.5 \times 10^{-3} m$

Force F = 10N

$$Stress = \frac{F}{A} = \frac{F}{\pi r^2}$$

$$= \frac{10}{\frac{22}{7}\left(0.5\times10^{-3}\right)}$$

$$= \frac{10}{3.14\times0.5\times10^{-3}}$$

$= 1.273 \times 10^{7}\ Nm^{-2}$

2. A tungsten wire of length 20cm is stretched by 0.1 cm, Find the strain on the wire.

Ans. Length L = 20cm= 20×10^{-2} m

Change in length $\Delta l = 0.1cm = 0.1\times10^{-2} m$

Strain $= \frac{\Delta L}{L} = \frac{0.1\times10^{-2}}{20\times10^{-2}} = 0.005$

3. If an iron wire is stretched by 1% what is the strain on the wire?

Ans. $\frac{\Delta l}{l} = 1\%$

Strain = $\frac{\Delta l}{l} = 1\% = \frac{1}{100} = 0.01$

4. A brass wire of diameter 1mm and length 2m is stretched by applying a force of 20N. The increase in length is 0.51mm. Find i) the stress ii) the strain iii) the Young's modulus of the wire.

Ans. D = 1mm

$r = \frac{D}{2} = 0.5\times10^{-3} m$

F = 20 N , L = 2m, $\Delta L = 0.51mm$

$= 0.51\times10^{-3} m$

i) Stress = $\frac{F}{A} = \frac{F}{\pi r^2} = \frac{20}{3.14\times\left(0.50\times10^{-3}\right)^2}$

$= 2.546\times10^{7} N/m^2$

ii) Strain = $\frac{\Delta L}{L} = \left(\frac{0.51\times10^{-3}}{2}\right) = 0.255\times10^{-3}$

iii) Young's modulus Y = $\dfrac{FL}{A\Delta L} = \dfrac{F}{\pi r^2 \Delta L}$

$$Y = \frac{20\times 2}{3.14\times(0.50\times10^{-3})^2\times0.51\times10^{-3}}$$

$$Y = 9.984 \times 10^{10}\ \text{N/m}^2$$

5. There are two wires of same material. Their radii and lengths are both in the ratio 1:2. If the extensions produced are equal. What is the ratio of loads?

Ans. $l_1 : l_2 = 1:2$

$r_1 : r_2 = 1:2$

$= F\alpha \dfrac{r^2}{l}$

$= \dfrac{F_1}{F_2} = \left(\dfrac{r_1}{r_2}\right)^2\left(\dfrac{l_2}{l_1}\right)$

$= \left(\dfrac{1}{2}\right)^2\left(\dfrac{2}{1}\right) = \dfrac{1}{4}\times\dfrac{2}{1} = \dfrac{1}{2}$

$= F_1 : F_2 = 1:2$

6. Two wires of different materials have the same lengths and areas of cross-section. What is the ratio of their increase in length when forces applied are the same ?
Given $Y_1 = 0.9 \times 10^{11}$ Nm^{-2}, $Y_2 = 3.6 \times 10^{11}$ Nm^{-2}

Ans: $Y_1 = 0.9\times10^{11}\,Nm^{-2}$

$Y_2 = 3.6\times10^{11}\,Nm^{-2}$

$\text{Y} = \dfrac{\text{FL}}{\text{A}\,\Delta\text{l}}$

$\Delta L \propto \dfrac{1}{Y}$

$\dfrac{\Delta L_1}{\Delta L_2} = \dfrac{Y_2}{Y_1} = \dfrac{3.6\times10^{11}}{0.9\times10^{11}} = \dfrac{4}{1}$

$\Delta L_1 : \Delta L_2 = 4:1$

7. **A metal wire of length 2.5m and area of cross –section 1.5 x 10 $^{-6}$ m^2 is stretched through 2mm. If its Young Modulus is 1.25 x 10 ^{11} N/m^2 , find the tension in the wire.**

Ans: L = 2.5m, A = 1.5 × 10^{-6} m^2

$\Delta L = 2\times10^{-3}m$, Y = 1.25 × 10^{11} N/m^2

Tension F = $\frac{YA\Delta L}{L} = \frac{1.25\times10^{11}\times1.5\times10^{-6}\times2\times10^{-3}}{2.5}$

F = 150N

8. **If an aluminium wire and a steel wire of the same length and cross section are joined end to end. The composite wire is hung from a rigid support and a load is suspended from the free end. If the increase in length of the composite wire is 1.35mm, find the ratio of the (i) stress in the two wires (ii) strain in the two wires.**
Given Y $_{Al\ =\ 0.7\ x\ 10}$ ^{11} Nm^{-2} , Y $_{steel}$ = 2 x 10 ^{11} Nm^{-2}

Ans : $\Delta L_1 + \Delta L_2 = 1.35mm$ ---(1),

$\Delta L_1 = \frac{FL_1}{AY_1}$, $\Delta L_2 = \frac{FL_2}{AY_2}$

$\frac{\Delta L_1}{\Delta L_2} = \frac{FL_1}{AY_1}\times\frac{AY_2}{FL_2}$

$\frac{\Delta L_1}{\Delta L_2} = \frac{Y_2}{Y_1}$

$\frac{\Delta L_1}{\Delta L_2} = \frac{2\times10^{11}}{7\times10^{10}} = \frac{20}{7}$ ---(2)

From (1), (2) equations

$\Delta L_1 = 1mm$

$\Delta L_2 = 0.35mm$

i) $\frac{Stress\ 1}{Stress\ 2} = \frac{Y_1}{Y_2}\times\frac{\Delta L_1}{\Delta L_2} = \frac{7\times10^{10}}{2\times10^{11}}\left(\frac{1}{0.35}\right) = 1$

ii) $\frac{Strain1}{Strain2} = \frac{\Delta L_1}{\Delta L_2} = \frac{1}{0.35} = \frac{100}{35} = \frac{20}{7}$

9. **A 2cm cube of some substance has the upper face displaced by 0.15cm due to a tangential force of 0.3N while keeping the lower face fixed. Calculate the rigidity modulus of the substance.**

Ans : $L = 2\times10^{-2}m$ $A = L^2 = 40\times10^{-4}m^2$

F = 0.3 N

$\Delta L = 0.15 \times 10^{-2} m$

$$G = \frac{F}{A\theta} = \frac{F}{A\left(\frac{\Delta L}{L}\right)} = \frac{F}{L(\Delta L)} = \frac{0.3}{2\times10^{-2}\times0.15\times10^{-2}}$$

G = 10^4 N/m^2

10. A spherical ball of volume 1000 cm^3 is subjected to a pressure of 10 atmosphere. The change in volume is 10 $^{-2}$ cm^3. If the ball is made of iron, find its bulk modulus.

Ans: V = 1000cm^3 = 1000 × 10^{-6} m^3 = 10^{-3} m^3

$P = 1\,atm = 1\times10^5\,N/m^2$

$-\Delta V = 10^{-2}\,cm^3 = 10^{-8}\,m^3$

Bulk modulus $B = \frac{-PV}{\Delta V}$

$$B = \frac{10^5 \times 10^{-3}}{10^{-8}} = 10^{10}\,N/m^2$$

11. A copper cube of side length 1cm is subjected to a pressure of 100 atmosphere. Find the change in its volume if the bulk modulus of copper is 1.4 x 10 ^{11} Nm^{-2}. Given 1 atm = 1 x 10 ^{5} Nm^{-2}

Ans: L = 1cm = 10^{-2}m

Volume **V =** L^3 = 10^{-6} m^3

$P = 100atm = 100\times10^5 = 10^7\,N/m^2$

$B = 1.4\times10^{11}\,N/m^2$

$$B = -\frac{pV}{\Delta V}$$

$$-\Delta V = \frac{pV}{B} = \frac{10^7\times10^{-6}}{1.4\times10^{11}}$$

$-\Delta V = 0.7143\times10^{-10}\,m^3$

12. Determine the pressure required to reduce the given volume of water by 2 %. Given bulk modulus of water. = 2.2 x 10 ^{9} Nm^{-2}.

Ans: $B = 2.2 \times 10^{9} N/m^{2}$

$-\frac{\Delta V}{V} = 2\%$

$B = -\frac{PV}{\Delta V}$, $P = -B\frac{\Delta V}{V}$

$P = 2.2 \times 10^{9} \times \frac{2}{100}$

$P = 4.4 \times 10^{7} N/m^{2}$

13. A steel wire of length 20 cm is stretched to increase its length by 0.2cm. Find the lateral strain in the wire if the Poisson's ratio for steel is 0.19

Ans: $L = 20cm = 20 \times 10^{-2} m$

$\Delta L = 0.2 \times 10^{-2} m$

$\sigma = 0.19$

$$\sigma = \frac{Lateral\ strain}{Longitudinal\ strain\left(\frac{\Delta L}{L}\right)}$$

Lateral strain = $\frac{\sigma \times \Delta L}{L}$

$= \frac{0.19 \times 0.2 \times 10^{-2}}{20 \times 10^{-2}}$ = 0.0019

****

UNIT – 9

MECHANICAL PROPERTIES OF FLUIDS

I. MULTIPLE CHOICE QUESTIONS

1. The rise of a liquid in a capillary tube does not depends on (3)

1) angle of contact **2)** density

3) outer radius of the capillary tube **4)** surface tension

2. The type of flow is characterized by parallel layers of fluid moving without mixing is (1)

1) Streamline flow **2)** Turbulent flow

3) Circular flow **4)** Rotational flow

3. The angle of contact between pure water and clean glass is (2)

1) 90^0 **2)** 0^0 **3)** 135^0 **4)** 180^0

4. Bernoulli's principle is based on the conservation of (3)

1) Momentum **2)** Angular momentum **3)** Energy **4)** Volume

5. Among the following, the wrong statement about pressure is (1)

1) It is a vector quantity

2) It is a scalar quantity

3) Pressure at sea level is 1.013 x 10^5 pa

4) Dimensions of pressure are $ML^{-1} T^{-2}$

Ans : It is a vector quantity is the wrong statement. **Reason**:It has no directional property.

6. The gauge pressure at a depth of 10m in a lake is (2)

1) $10^6 Pa$ **2)** $10^5 Pa$ **3)** $10^4 Pa$ **4)** Zero

Ans : $P_g = P - Pa = \rho gh$

$$P_g = 10^3 \text{ kgm}^{-3} \times 10 \text{ m/s}^2 \times 10 \text{ m} = 10^5 \text{ pa}$$

7. Surface tension arises due to (4)

1) gravitational forces **2)** Pressure differences

3) viscosity **4)** Cohesive forces

8. **Bernoulli's principle states that an increase in the speed of a fluid occurs simultaneously with a decrease in** **(3)**

1) Temperature **2)** Viscosity **3)** Pressure **4)** Flow rate

9. **The terminal velocity of a sphere of radius R falling in a viscous fluid is proportional to (4)**

1) R **2)** R^3 **3)** $\sqrt{R}$ **4)** R^2

10. **1 torr is equal to** **(3)**

1) $1.013\times10^5 Pa$ **2)** 760Pa **3)** 133Pa **4)** 1013Pa

II. FILL IN THE BLANKS

1. **The excess pressure $P - P_a$ at a depth h is called ___________**

Ans. Gauge pressure

2. **1 bar is equal to __________P_a**

Ans. 10^5

3. **Hydraulic lift and hydraulic brakes are based on _____ law**

Ans. Pascal's

4. **The study of fluids in motion is known as __________**

Ans. fluid dynamics

5. **The Path taken by a fluid particle under a steady flow is called a _____**

Ans. Streamline

6. **The Bernoulli's equation in general can be written as ______**

Ans. $P + \frac{1}{2}\rho\vartheta^2 + \rho gh = \text{constant}$ (Here ϑ is Velocity)

7. **The maximum velocity of a sphere falling in a viscous fluid is called________**

Ans. Terminal velocity

8. **The SI unit of surface tension is _________**

Ans. N/m

9. **The spherical shape of raindrops is due to______**

Ans. Surface tension

10. **The aerodynamic lift experienced by a spinning ball moving in air is called__**

Ans. Magnus effect

III. ONE WORD ANSWER QUESTIONS

1. What is relative density?

Ans. The relative density of a substance is the ratio of its density to the density of water at $4^0 c$

2. Name the device used to measure atmospheric pressure

Ans. Barometer

3. Write the equation of continuity

Ans. $A_P \vartheta_P = A_R \vartheta_R = A_Q \vartheta_Q$ or $A \vartheta$ = constant

4. Mention the speed limit, beyond which steady flow of liquid becomes turbulent ?

Ans. Critical speed

5. What is the speed of efflux from a hole at the bottom of a tank filled with water to a height h, which is open at the top?

Ans. Speed $\vartheta = \sqrt{2gh}$

6. What is the property of a fluid to resist relative motion between its layers?

Ans. Viscosity

7. Which principle is involved in the dynamic lift of an airplane?

Ans . Bernoulli's principle.

8. What happens to the viscosity of liquids with increase in temperature ?

Ans. decreases

9. What is the SI unit of the coefficient of viscosity?

Ans. Poiseiulle (or) NSm^{-2} (or) Pa s (or) $Kgm^{-1}s^{-1}$

10. What is surface tension?

Ans. Force per unit length acting in the plane of the interface between the plane of the liquid

IV. VERY SHORT ANSWER QUESTIONS

1. Define average pressure. Is it a scalar or a vector quantity?

A. The normal force acting per unit area $P_{ave} = \frac{F}{A}$. It is a scalar quantity.

2. What is Magnus effect?

A. The difference in the velocities of air results in the pressure difference between the lower and upper faces and there is a net upward force on the ball. This dynamic lift due to spinning is called magnus effect.

3. Why are liquid drops and bubbles spherical?

A. The surface tension of a liquid tends to have the least surface area. For a given volume the surface area is minimum for a sphere, hence liquid drops and bubbles are spherical shape.

4. Give the expression for the excess pressure in a liquid drop.

A. Excess pressure in a liquid drop $P_i - P_o = \frac{2S}{r}$

S = Surface tension

r = radius of the liquid drop

5. Give the expression for the excess pressure in an air bubble inside the liquid

A. Excess pressure in an air bubble inside the liquid

$$P_i - P_o = \frac{2S}{r}$$

Where S = Surface tension , r = radius of air bubble

6. Give the expression for the excess pressure in a soap bubble in air

A. Excess pressure in a soap bubble is $P_i - P_o = \frac{4S}{r}$

Where S = Surface tension, r = radius of the soap bubble

7. What is the angle of contact ?

A. The angle between tangent to the liquid surface at the point of contact and solid surface inside the liquid is called angle of contact.

8. Mention two examples that obey Bernoulli's theorem and justify them

A. 1. In cricket we notice that a spinning ball deviates from its parabolic trajectory as it moves through air

2. When the aerofoil moves against the wind, there is an upward force resulting in a dynamic lift of the wings and this balances the weight of the plane.

9. When water flows through a pipe, which layers move fastest and slowest?

A. When water flows through a pipe the layers near the axis of the tube are fastest and at walls of the tube are slowest.

10. Explain why terminal velocity is higher for a body with a larger surface area.

A. Terminal velocity $\vartheta_t \alpha r^2$

Surface area $A = 4\pi r^2$ when surface area increases, r^2 also increases.Then the terminal velocity is also increases

11. What is the net force acting on a ball that is moving in air?

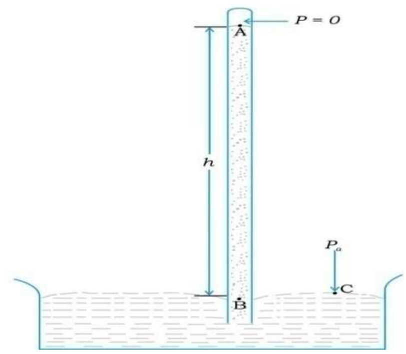
a) Without spin b) with spin

A. a) Net force acting on a ball that is moving in air without spin is zero

b) Net force acting on a ball that is moving in air with spin is upward.

V. SHORT ANSWER QUESTIONS

1. What is atmospheric pressure? How is it determined using a barometer?



Ans. Atmospheric pressure:- Atmospheric pressure at any point is equal to the weight of a column of air of unit cross sectional area extending from that point to the top of the atmosphere.

1 atm = 1.013×10^{5} *pascals*

Barometer : - A long glass tube closed at one end and filled with mercury is inverted into a trough of mercury. This device is known as mercury barometer.

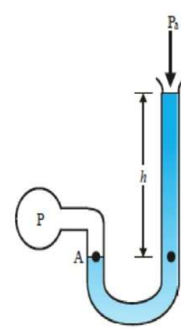
The space above the mercury column in the tube contains only mercury vapour whose pressure P is so small that it may be neglected. The pressure inside the column at point A must equal the pressure at point B, which is at same level

Pressure at B = atmospheric pressure = P_a

$P_a = h\rho g$ = Pressure at A

In this experiment height of the mercury column in the barometer is 76 cm at sea level (1atm).

2. What is guage pressure ? How is a manometer used for measuring pressure differences?



(b) The open tube manometer

Ans. **Guage pressure : -** Guage pressure is the difference of the actual pressure and the atmospheric pressure.

Guage pressure = $P - P_a$

Manometer : - It consists of a U tube containing a suitable liquid. One end of the tube is open to the atmosphere and other end is connected to the system whose pressure we want to measure.

The pressure at A = the pressure at B

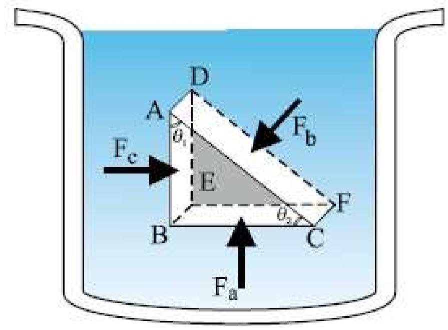
$P = P_a + h\rho g$

$P - P_a = h\rho g$

$\therefore$ Guage pressure = $P - P_a = h\rho g$

Guage pressure is proportional to difference in heights (h) of mercury levels in manometer.

3. State Pascal's law and verify it with the help of an experiment



Ans.. **Pascal's : -** The pressure in a fluid at rest is the same at all points if they are at the same height

Verification:- Consider ABC – DEF is an element in the form of a right angled prism in the interior of a fluid at rest. This element is very small so that the effect of the gravity is the same at all points.

The areas of faces BEFC, ADFC and ADEB are A_a, A_b, A_c.

The fluid exerts pressures P_a, P_b, P_c on this element corresponding to the normal forces F_a, F_b, F_c.

Then $F_b \sin\theta_2 = F_c$, $F_b \cos\theta_2 = F_a$ (by equilibrium)

$A_b \sin\theta_2 = A_c$, $A_b \cos\theta_2 = A_a$ (by geometry)

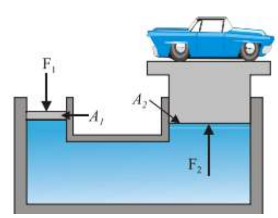
Thus $\frac{F_b}{A_b} = \frac{F_c}{A_c} = \frac{F_a}{A_a}$; $\therefore P_b = P_c = P_a$

Hence pressure exerted is same in all directions in a fluid at rest.

4. Explain the working principles of hydraulic lift and hydraulic brakes.

Ans. Hydraulic lift and hydraulic brakes are based on the Pascal's law.

Hydraulic lift : - In a hydraulic lift two pistons are separated by the space filled with a liquid. A piston of small cross section A_1 is used to exert a force F_1 directly on the liquid. The pressure $P = \frac{F_1}{A_1}$ is transmitted through the liquid to a larger piston of area A_2.



The upward force exerted on large piston is $F_2 = PA_2 = \frac{F_1 A_2}{A_1}$

$$\frac{F_2}{F_1} = \frac{A_2}{A_1}$$

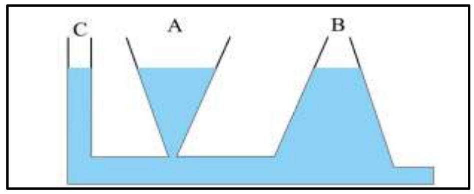
By changing the force at A_1, the platform can be moved up or down.

Hydraulic brakes:- When we apply a little force on the pedal with our foot the master piston moves inside the master cylinder. The pressure caused is transmitted through the brake oil to act on a piston of large area.

A large force acts on the piston and is pushed down expanding the brake shoes against brake lining. In this way a small force on the pedal produces a large retarding force on the wheel.

5. What is the hydrostatic paradox?

Ans. The liquid pressure is same at all points at the same depth and is independent of base area.

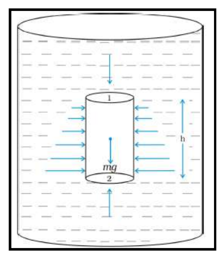


Consider three vessels A,B and C of different shapes.

They are connected at the bottom by a horizontal pipe. On filling with water the level in the 3 vessels is the same, though they hold different amounts of water. This is so because water at the bottom has the same pressure below each section of the vessel.

6. Explain how pressure varies with depth in a liquid.

Ans. Consider a fluid at rest in a container. In figure point 1 is at a height h above a point 2. The pressure at points 1 and 2 are P_1 and P_2. As the fluid is at rest the horizontal forces should be zero. The resultant vertical forces should balance the weight of the element. Pressure at top



(P_1 A) acting downward, pressure at bottom (P_2A) acting upward

$(P_2 - P_1) A = mg$ – (1)

mass of the fluid(m) = $\rho v = \rho hA$

$\therefore (P_2 - P_1)A = \rho hAg$

$P_2 - P_1 = \rho gh$ ---(2)

Pressure difference depends on the verticle distance h.

If point 1 is shifted to the top of the fluid, which is open to the atmosphere, P_1 may be replaced atmospheric Pressure (P_a) and P_2 by P.

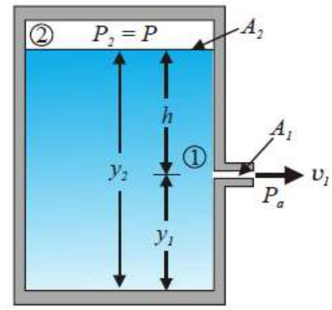
$P - P_a = \rho gh$

$P = P_a + \rho gh$

Thus the pressure P, at depth below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by an amount of ρgh.

7. What is Torricelli's law ? How the speed of efflux is determined experimentally?

Ans. ***Torricelli's law : -*** The efflux velocity of a liquid through small hole of a tank is equal to the velocity acquired by a freely falling body from a height which is equal to that of liquid level from small hole.



Consider tank containing a liquid of density ρ with small hole in its side at a height y_1 from the bottom (at(1)). The air above the liquid, whose surface is at height y_2 is at pressure P (at (2)). From the equation of continuity

$$\vartheta_1 A_1 = \vartheta_2 A_2$$

$$\vartheta_2 = \frac{A_1}{A_2} \vartheta_1$$

If the cross sectional area of the tank A_2 is much larger than that of the hole ($A_2 >> A_1$), then we may take the fluid at rest at the top i.e $\vartheta_2 = 0$ Now we applying the Bernoulli's equation at point 1 and 2 and noting that at the hole $P_1 = P_a$, $P_2 = P$

$$P_a + \frac{1}{2}\rho\, \vartheta_1{}^2 + \rho\, gy_1 = P + \rho\, gy_2$$

Taking $y_2 - y_1 = h$ we have $\vartheta_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}}$

If the tank is open to the atmosphere, then $P = P_a$

$$\vartheta_1 = \sqrt{2gh}$$

ϑ_1 = speed of efflux.

8. Explain dynamic lift with examples.

Ans. ***Dynamic lift :*** - Dynamic lift is the force that acts on a body by virtue of its motion through a fluid.

Ex : - 1. ***Ball moving without spin : -*** Streamlines are equally distributed above and below the ball. The velocity above and below the ball is same resulting in Zero pressure difference. There is no upward or downward force on the ball.

Ball moving with spin : - The streamlines are more crowded above the ball than below. Pressure is less at top of the ball and more below the ball. There is a net upward force on the ball.

2. Dynamic lift also acts on an aircraft wing.

9. Explain surface tension and surface energy.

Ans. ***Surface tension*** : - Surface tension is a force per unit length (or surface energy per unit area) acting in the plane of the interface between the plane of the liquid and any other substance.

Surface tension S = $\frac{Force}{length} = \frac{F}{l}$

SI unit : Nm^{-1}

Dimensional formula : $[ML^0T^{-2}]$

Surface Energy : - Surface energy is the energy of the interface between two materials, and depends on both of them.

The additional potential energy due to molecular forces per unit surface area is called surface energy

Surface energy = $\frac{work\ done}{Area}$

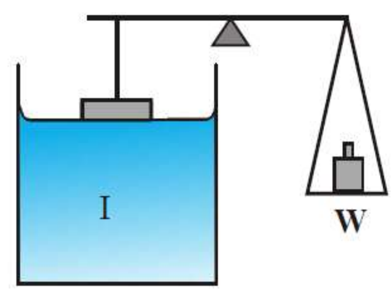
SI unit : Jm^{-2}

Dimensional formula $[ML^0T^{-2}]$

Surface tension is equal to the surface energy per unit area of the liquid interface and is also equal to the force per unit length exerted by the fluid on the movable bar.

10. Explain how surface tension is measured experimentally.

Ans. A flat vertical glass plate, below which a vessel of some liquid is kept, forms one arm of the balance. The plate is balanced by weights on the other side, with its horizontal edge just over water. The vessel is raised slightly till the liquid just touches the glass plate and pulls it down a little because of surface tension. Weights are added till the plate just clears water. Suppose the additional weight required is W, the surface tension of the liquid -air interface is

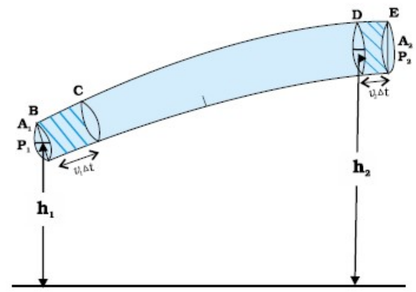


$S_{la} = (W/2l) = (mg/2l)$

m = extra mass, l = length of the plate edge.

VI. LONG ANSWER QUESTIONS

1. State Bernoulli's principle. From conservation of energy in a fluid flow through a tube, derive Bernoulli's equation. Given an application of Bernoulli's theorem



Ans. ***Bernoulli's Principle : -*** In a streamline flow, the sum of the pressure (P), the kinetic energy per unit volume $\left(\frac{\rho\vartheta^2}{2}\right)$, and the potential energy per unit volume (ρ gh) remains a constant.

Derivation:- Consider a non viscous, incompressible fluid is flowing through the pipe in a steady flow. Consider the flow at 2 regions, BC and DE. In short time interval Δt, this fluid would have moved. Suppose ϑ_1 is the speed at B and ϑ_2 is the speed at D, moved distance B to C is $\vartheta_1\Delta t$, in same time moved distance D to E is $\vartheta_2\ \Delta t$.

Pressures P_1 and P_2 act at areas of cross sections A_1 and A_2. The work done on the fluid at left end (BC) is $W_1 = P_1 A_1 (\vartheta_1 \Delta t) = P_1 \Delta V$, ($\Delta V$ is the Change In Volume)

The work done by the fluid at other end (DE) is $W_2 = P_2 A_2 (\vartheta_2 \Delta t) = P_2 \Delta V$

The work done by the fluid is $- P_2\Delta V$

(or) the work done on the fluid is $- P_2\Delta V$.

So the total work done on the fluid is $W_1 - W_2 = (P_1 - P_2)\Delta V$

If the density of the fluid is is ρ and $\Delta m=\rho A_1 \vartheta_1\Delta t =\rho\Delta V$ is the mass passing through pipe in time, then change in gravitational potential energy is $\Delta U=\rho g\, \Delta V\, (h_2-h_1)$

The change in its kinetic energy is $\Delta K= \frac{1}{2}\rho\Delta V\, (\vartheta_2^2 - \vartheta_1^2)$

We can employ the work energy theorem to this volume of the fluid and this yields

$$(P_1-P_2)\, \Delta V = \frac{1}{2}\rho\Delta V\, (\vartheta_2^2 - \vartheta_1^2) + \rho g\, \Delta V\, (h_2-h_1)$$

Now we devide each term by Δt,we can rearrange the above terms to obtain

$$P_1 + \frac{1}{2}\ \rho\, \vartheta_1^2 + \rho\, gh_1 = P_2 + \frac{1}{2}\ \rho\, \vartheta_2^2 + \rho g\, h_2$$

$P + \frac{1}{2}\rho\, \vartheta^2 + \rho gh$ = constant. This a Bernoulli's equation.

Bernoulli's theorem is proved.

Application:- It explains the dynamic lift on the wings of aeroplanes

2. Define the coefficient of viscosity. Explain Stoke's law and derive the expression for terminal velocity. Discuss the conditions under which rain drop attains terminal velocity.

Ans. ***Coefficient of viscosity : -*** The ratio of shearing stress to the strain rate.

$$\eta = \frac{\frac{F}{A}}{\frac{\vartheta}{l}} = \frac{Fl}{\vartheta A}$$

Stoke's law :- Viscous force (F) is proportional to (i) the velocity of the object (ii) the radius of the spherical body (iii) coefficient of viscosity fluid (η)

$$F \propto na\vartheta,\ F = 6\pi na\vartheta$$

This is known as Stoke's law

Expression for ϑ_t : - We consider a rain drop in air. (sphere). It accelerates initially due to gravity, Finally when viscous force plus buoyant force becomes equal to force due to gravity. The net force becomes zero and so does the acceleration. The sphere(rain drop) then descends with constant velocity. This is called terminal velocity (ϑ_t)

$$F_V + F_B = W$$

$$6\pi\eta r\vartheta_t = \frac{4}{3}\pi r^3(\rho - \sigma)g$$

$$\vartheta_t = \frac{\frac{4}{3}\pi r^3(\rho-\sigma)g}{6\pi\eta r}$$

$$\vartheta_t = \frac{2}{9}r^2\frac{(\rho-\sigma)g}{\eta}$$

Conditions : - i) A rain drop in air accelerates initially due to gravity. As its velocity increases, the retarding force also increases.

ii) When viscous force plus buoyant force becomes equal to force due to gravity. The net force becomes zero and hence the acceleration becomes zero. The rain drop then descends with a constant velocity. Then it attains terminal velocity.

VII. PROBLEMS

1. Calculate the work done in blowing a soap bubble of diameter 0.6cm against the surface tension force. (Surface tension of soap solution $T = 2.5\times10^{-2}Nm^{-1}$)

Ans : Given D = 0.6cm = $0.6\times10^{-2}m$ and surface tension T = 2.5×10^{-2} Nm^{-1}

$$r = \frac{D}{2} = 0.3\times10^{-2}m$$

Work done $W = 8\pi r^2 T$

$$= 8\times3.14\times\left(0.3\times10^{-2}\right)\times\left(0.3\times10^{-2}\right)\times2.5\times10^{-2}$$

$W = 5.652 \times 10^{-6}J$

Note: Surface tension may be denoted as S or T.

2. **How high does methyl alcohol rise in a glass tube of diameter 0.06 cm? (Surface tension T of methyl alcohol = $0.023Nm^{-1}$, density = $0.8g/cm^3$, angle of contact is zero)**

Ans : Given diameter D = 0.06 cm

Radius (r) = $\frac{D}{2} = \frac{0.06}{2} = 0.03 \text{ x } 10^{-2}$m

$r = 3 \text{ x } 10^{-4}$ m

angle of contact $\theta = 0$

$T = 0.023 \text{ Nm}^{-1}$

Density $\rho = 0.8\times10^{-3}kg/m^3$

T or $S = \dfrac{hr\rho g}{2\cos\theta}$

$$h = \frac{2S\cos\theta}{r\rho g}$$

$$h = \frac{2\times0.023}{3\times10^{-4}\times0.8\times10^3\times9.8}$$

$$h = \frac{0.046}{23.52\times10^{-1}} = \frac{0.46}{23.52}$$

h = 0.0196m

h = 0.02 m

h = 2cm

Note: Surface tension may be denoted as S or T.

3. **What should be the radius of a capillary tube if water rises to a height of 6cm in it? (Surface tension of water $T = 7.2\times10^{-2}Nm^{-1}$)**

Ans : $S = 7.2\times10^{-2}N/m$

$h = 6\times10^{-2}m$

$\rho = 10^3kg/m^3$

$S = \frac{hr\rho g}{2}$

$r = \frac{2S}{h\rho g}$

$r = \frac{2 \times 7.2 \times 10^{-2}}{6 \times 10^{-2} \times 10^{3} \times 9.8}$

$r = \frac{14.4}{58.8} \times 10^{-3}$

$r = 0.24 \times 10^{-3} m$

r = 0.24mm

4. Find the depression of the meniscus in a capillary tube of diameter 0.4 mm dipped in mercury. (Density of mercury $\rho = 13.6 \times 10^{3} kgm^{-3}$, surface tension of mercury = $0.49 Nm^{-1}$ and angle of contact = 135^{0}

Ans : $r = \frac{D}{2} = \frac{0.4}{2} mm = 0.2 \times 10^{-3} m$

$\rho = 13.6 \times 10^{3} kg/m^{3}$, S = 0.49 N/m , $\theta = 135^{0}$, $\cos = 135^{0} = -\sin 45 = -\frac{1}{\sqrt{2}}$

$S = \frac{hr\rho g}{2\cos\theta}$, $h = \frac{2s\cos\theta}{r\rho g}$, $h = 2 \times 0.49 \times \left(-\frac{1}{\sqrt{2}}\right) \times \frac{1}{0.2 \times 10^{-3} \times 13.6 \times 10^{3} \times 9.8}$

h = 0.026m

5. If the diameter of a soap bubble is 10mm and its surface tension is 0.04 Nm^{-1}, find the excess pressure inside the bubble.

Ans : $r = \frac{d}{2} = \frac{10}{2} = 5mm = 5 \times 10^{-3} m$

S = 0.04 N/m

Excess pressure in side the bubble $P_{excess} = \frac{4S}{r}$

$P = \frac{4 \times 0.04}{5 \times 10^{-3}}$

$P = 0.032 \times 10^{3}$

$P = 32$ N/m^2

6. If the work done to form a bubble of radius R is W, find the energy required to increase its radius to 2R.

Ans : $R_1 = R, \qquad R_2 = 2R,$

Work done to form a bubble W = $8\pi R^2 S$

Energy required to increase its radius to 2R is

$W^1 = 8\pi\,(R_2^2 - R_1^2)S$

$W^1 = 8\,\pi\,(\,4\,R^2 - R^2\,)\,S \quad \therefore (R_1 = R, \quad R_2 = 2R)$

$= 3 \times 8\,\pi\,R^2\,S$

$W^1 = 3W$

7. If two soap bubbles of radii R_1 and R_2 (in vacuum) coalesce under isothermal conditions, find the radius of the resulting bubble. Take T as the surface tension of soap solution.

Ans : R_1, R_2 are radii of 1st and 2nd soap bubble.R is the radius of new bubble.

Work done in forming soap bubble of radius R_1 is W_1 and work done in forming soap bubble of radius R_2 is W_2

We have $W_1 = 8\,\pi\,R_1^2\,T$, $W_2 = 8\,\pi\,R_2^2\,T$ and $W = 8\,\pi\,R^2 T$

Total work done $W = W_1 + W_2$

$$8\pi R^2 T = 8\pi R_1^2 T + 8\pi R_2^2 T$$

$$R^2 = R_1^2 + R_2^2$$

$$R = \sqrt{R_1^2 + R_2^2}$$

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UNIT – 10

THERMAL PROPERTIES OF MATTER

I. MULTIPLE CHOICE QUESTIONS

1. The triple point of water is **(2)**

(1) $273.16\,^0$ C (2) 273. 16 K (3) 273. $16\,^0$ F (4) 0.15K

Ans : The triple point of water is 273.16 K

2. The temperature of an object is 60^0 C . Its value in Fahrenheit scale is **(3)**

(1) 120^0F (2) 130^0F (3) 140 ^{0} F (4) 110 ^{0} F

$$\frac{C-0}{100-0}=\frac{F-32}{180}$$

Ans : $\frac{C}{5}=\frac{F-32}{9}$

$$F-32=\frac{60X9}{5}=F=140^0F$$

3. An iron bar of length 10m is heated from 0^0C to 100^0C. If the coefficient of linear thermal expansion of iron is 10 x 10 $^{-6}$/C, the increase in the length of bar is **(2)**

(1) 0.5m (2) 1.0cm (3) 1.5cm (4)2.0cm

Ans : $\Delta l = l\alpha_1 = (10)\times(10\times10^{-6})(100) = 10^{-2}m = 1cm$

4. A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly. **(2)**

(1) Its speed of rotation increases
(2) Its speed of rotation decreases
(3) Its speed of rotation remains the same.
(4) Its speed increases because its moment of inertia increases.

Ans : In this case angular momentum L = Iω = constant $\therefore \omega \alpha \frac{L}{I}$

5. At temperature 6000K the dominant wavelength of radiation of a body is 5000^0A, the dominant wavelength corresponding to radiation of same body at temperature 3000K is

(1) $10{,}000^0$A (2) 1000^0A (3) 4000^0A (4) $12{,}000^0$A **(1)**

$$\lambda_1T_1=\lambda_2T_2$$

Ans : $\lambda_2=\frac{\lambda_1T_1}{T_2}=\frac{(6000)(5000)}{3000}=10000^0A$

6. The rate of heat per unit area of a cylindrical aluminum rod in steady state is 400 W/m^2. The thermal conductivity of aluminum is 200 W/m – K. The temperature gradient along the rod is **(3)**

(1) 1 K/m (2) 4 K/m (3) 2 K/m (4) 6 K/m

Ans : H = 400 W/m^2

K = 200 w/m- k

A = 1m^2

$\frac{dT}{L} = ?$

$H = KA(\frac{dT}{L})$; $\frac{dT}{L} = \frac{H}{KA} = \frac{400}{200} = 2K/m$

7. The amount of heat energy required to change 200gm water at 0^0 C to 100^0 C of water at atmospheric pressure is (specific heat of water S = 1k Cal/kg-k). (3)

(1) 20 Cal (2) 20 M Cal (3) 20 K Cal (4) 200 K Cal

Ans : $Q = ms\Delta t$

m = 200gms

= 200 x 10^{-3} kg

Δt = 100^0 C

S = 1K. cal/kg-k

$Q = ms\Delta t$

Q = 200 x 10^{-3} x 1 x 10^{3} x 100

= 20000 cal

= 20 K Cal

8. Two liquids A and B are at temperatures 32^0C and 24^0C . When equal masses of these liquids are mixed, the temperature of mixture is found to be 28^0C. The ratio of specific heats of A and B is (3)

(1) 3 :2 (2) 2 : 3 (3) 1: 1 (4) 4 : 3

Ans : $mS_A(32-28) = mS_B(28-24)$ $S_A : S_B = 1:1$

9. A piece of iron is heated in a flame. It first becomes dull red then becomes reddish yellow and finally turns to white hot. The correct explanation for the above observation is possible by using

(3)

(1) Newton's Law of Cooling (2) Stefan's Law (3) Wien's Displacement law

(5) Kirchhoff's Law

Ans : We know that Wien's displacement law is, $\lambda_m T = b\ (constant)$

As temperature increases wavelength for which intensity is maximum decreases. Initially T is low, so λ_m is more which corresponds to red as T increases λ turns to yellow and then to bluish white.

10. Two metal rods 1 and 2 of same lengths have same temperature difference between their ends. Their thermal conductivities are K_1 and K_2 and cross-sectional areas A_1 and A_2 respectively. If the rate of heat conduction in 1 is four times that in 2, then (2)

(1) $K_1 A_1 = K_2 A_2$ (2) $K_1 A_1 = 4K_2 A_2$ (3) $K_1 A_1 = 2K_2 A_2$ (4) $4K_1 A_1 = K_2 A_2$

$H_1 = 4H_2$

Ans : $\frac{K_1 A_1 \Delta t}{l} = 4\frac{K_1 A_1 \Delta t}{l} \Rightarrow K_1 A_1 = 4K_2 A_2$

II. FILL IN THE BLANKS

1. Heat is the form of energy transferred between two (or more) systems or a system and its surroundings by virtue of _________

Ans : temperature difference.

2. A measure of temperature is obtained by using ___________

Ans : thermometer.

3. On the Fahrenheit scale, there are _____equal intervals between two ice point and steam point.

Ans : 180

4. 0 K is equal to _______

Ans : -273.15^0C

5. Water will ________ on heating between 0^0C and 4^0C.

Ans : Contracts

6. If α_l and α_V are coefficient of linear and volume expansion of material of given solid then $\frac{\alpha_V}{\alpha_l}$ = _______

Ans : 3

7. A device in which heat measurement can be done is called ________

Ans : Calorimeter

8. The change from solid state to vapour state without passing through the liquid state is called ________

Ans : sublimation.

9. A mode of heat transfer by actual motion of matter is called _________

Ans : convection.

10. ___________law is useful for estimating the surface temperatures of celestial bodies like the moon, sun and other stars.

Ans : Wien's Displacement Law

III. ONE WORD ANSWER QUESTIONS

1. What is the SI unit of heat ?

Ans : Joule (J)

2. Write the relation connecting Kelvin temperature (T) and the Celsius temperature t_c.

Ans : $T = t_c + 273.15$

3. What is the value of the universal gas constant (R) in SI system ?

Ans : $R = 8.31\ Jmol^{-1}\ K^{-1}$

4. What is thermal expansion ?

Ans The increase in the dimension of a body due to the increase in the temperature is called thermal expansion.

5. At what temperature does water have the maximum density ?

Ans : Water has the maximum density at 4 ^{0}C.

6. **Define specific heat capacity of the substance**.

Ans : It is defined as the amount of heat per unit mass absorbed or given off by the substance to change its temperature by one unit.

7. **What is the SI unit of specific heat capacity?**

Ans : The SI unit of specific heat capacity is J /Kg - K.

8. **Why is water used as a coolant in automobile radiators?**

Ans : As it has the highest specific heat capacity compared to other substances.

9. **What is the effect of an increase in pressure on the boiling point of a liquid ?**

Ans : Boiling point increases with increase in pressure.

10. **What are the different ways in which heat energy transfer takes place ?**

Ans : conduction, convection and radiation.

11. **What is the SI unit of thermal conductivity ?**

Ans : The SI unit is $J\ s^{-1} m^{-1}\ K^{-1}$ or $W\ m^{-1}\ K^{-1}$

12. **Give an example of forced convection.**

Ans : The common examples of forced convection systems are : forced –air heating systems in home , the human circulatory system, the cooling system of an automobile engine.

13. **What is the Stefan – Boltzmann constant value in SI units** ?

Ans : Its value in SI units is $5.67 \times 10^{-8}\ W\ m^{-2}\ K^4$.

IV. VERY SHORT ANSWER QUESTIONS :

1. **Distinguish between heat and temperature**

HEAT	TEMPERATURE
1.It is the form of energy transferred between two systems due to temperature difference.	1. It is a relative measure of hotness (or) coldness.
2.SI unit is Joule (J)	2.SI unit is Kelvin (k)

2. What are the lower and upper fixed points in Celsius and Fahrenheit scales ?

Ans : **Celsius Temperature Scale :**

Lower fixed point (Ice point) : $0\ ^0C$

Upper fixed point (Steam point) : $100\ ^0C$.

Fahrenheit Temperature scale :

Lower fixed point : 32^0F

Upper fixed point : $212^0\ F$.

3. Do the values of coefficients of expansion differ, where the temperatures are measured on centigrade scale or on Fahrenheit scale ?

Ans : Yes, the values of coefficients of expansion (α) differed with respect to scale of temperature.

Reason : $\alpha_l = \left(\frac{\Delta l}{l \Delta T}\right)$, i.e 1^0 C rise in temperature in Celsius scale is not equal to 1^0 F rise in temperature of Fahrenheit scale. $\alpha_F = \alpha_C \times \frac{5}{9}$

4. Can a substance contract on heating ? Give example.

Ans : Yes

Example : Water contracts on heating between 0^0 C and 4^0 C.

5. Why gaps are left between rails on a railway track?

Ans : To allow linear expansion between the rails in summer. .

6. What is latent heat of fusion ?

Ans : The amount of heat required to change the state of unit mass substance from solid to liquid at constant temperature and pressure is called latent heat of fusion.

7. What is latent heat of vaporization ?

Ans: The amount of heat required to change the state of unit mass substance from liquid to gas at constant temperature and pressure is called latent heat of vaporization.

8. Why utensils are coated black? Why the bottoms of the utensils are made of copper ?

Ans : As the black bodies are good absorbers, the bottom of the utensils for cooking food are blackened, so that they absorb maximum heat from fire.

Being a good conductor of heat, copper promotes the distribution of heat over the bottom of the utensils for uniform cooking.

9. State Wien's displacement law .

Ans : The minimum wave length corresponding to the maximum energy is inversely proportional to absolute temperature of body.

λ_m T = constant

10.Ventilators are provided in rooms just below the roof. Why ?

Ans . To keep the room cool, ventilators are provided just below the roof to allow less density hot air escape from the room through ventilators, so as to allow cool air to enter the room through windows due to natural convection.

11. Does the body radiate heat at 0^0 K ? Does it radiate at $0\ ^0$C.

Ans. At T = 0^0 K, the body does not radiate heat energy.

At t = 0^0C i.e. T = 273.15 K, the body radiate heat energy.

12. State the different modes of transmission of heat, which of these modes require medium ?

Ans . These are three distinct modes of heat transfer

1. Conduction
2. Convection
3. Radiation.

Conduction and convection modes of heat transfer require medium.

13.Define Co-efficient of thermal conductivity and temperature gradient.

Ans : Coefficient of Thermal conductivity : (Definition) .

In steady state, the rate of flow of heat (H) per unit area (A) and per unit temperature gradiant of a metallic bar is called co-efficient of thermal conductivity.

Temperature gradient :

The temperature difference between the ends (C, D)of metallic bar per unit length is called temperature gradient.

Temperature gradient = $\left(\frac{T_C - T_D}{L}\right)$ Where L is length of metallic bar, T_C , T_D are the temperatures at the ends of metallic bar.

14. Define emissivity

Ans : The ratio of rate of energy emitted by a body to rate of energy emitted by a black body at same temperatures is called emissivity.

15.Define absorptive power of a body. What is the absorptive power of a perfect black body ?

Ans : The ratio between energy absorbed by a body to energy incident on a same body per unit area and unit time is called absorptive power of a body.

Absorptive power of a black body is one.

16.State Newton's Law of cooling.

Ans : The rate of cooling $\left(\frac{-dQ}{dt}\right)$ of a body is directly proportional to the difference of temperature of the body and the surroundings.

$$\frac{-dQ}{dt} \ \alpha \ (T_2 - T_1)$$

17.State the conditions under which Newton's law of Cooling is applicable?

Ans : 1. The temperature difference between body and surroundings should be small.

2.Loss of heat is only by radiation.

3.The temperature of surroundings remains constant.

18. The roof of building is often painted white during summer. Why ?

Ans : White paint is a bad absorber of heat. Hence the roof of building is often painted white, to keep the room in cool during summer.

V. SHORT ANSWER QUESTIONS

1. **Explain Celsium and Fahrenheit scales of temperature.Obtain the relation between Celsius and Fahrenheit scales of temperature.**

   Ans :

   - For any temperature scale, two reference points are needed.
   - The point at which pure water freezes at standard pressure is called ice point (or) freezing point.
   - The point at which pure water boils at standard pressure is called steam point (or) boiling point.
   - The ice point and steam point are taken as two reference points in any temperature scale.

- ❖ On Celsius scale, the ice point is 0^0 C and the steam point is 100^0C. There are 100 equal intervals between two reference points.
- ❖ On Fahrenheit scale, the ice point is 32^0F and the steam point is 212^0 F. There are 180 equal intervals between two reference points.

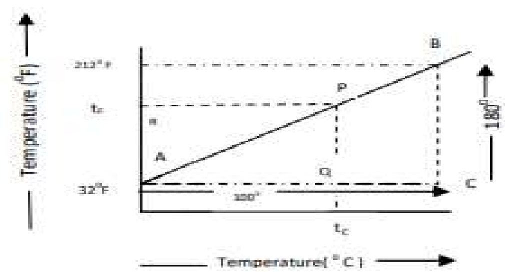
Relation between Celsius and Fahrenheit scales :

A plot of Fahrenheit temperature (t_F) versus celsius temperature is a straight line.

From above graph,

$$\frac{t_F - 32}{t_C - 0} = \frac{180}{100}$$

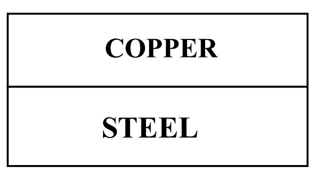
$$\frac{t_F - 32}{180} = \frac{t_C - 0}{100}$$



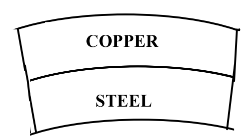
The above formula is used to convert a temperature between two scales.

2. **Two identical rectangular stripes one at copper and other of steel are revited together to form a compound bar. What will happen on heating.**

Ans : Coefficient of linear expansion of copper is 1.7 x 10 $^{-5}$ /K
Coefficient of linear expansion of steel is 1.2 x 10 $^{-5}$ /K
Compare the values of for copper and steel we find that copper expands more than steel.
A straight compound bar of two identical rectangular strips of copper and steel is heated, then copper strip expands more than steel strip. Then this compound bar bends and form a convex shape such that copper strip is outside.



Compound bar before heating



Compound bar after heating

3. **Pendulum clocks generally go fast in winter and slow in summer, Why ?**

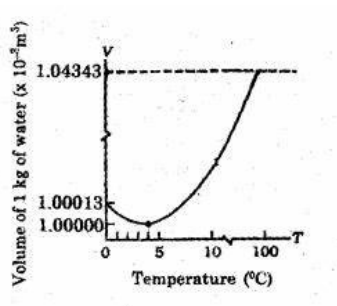
Ans : The time period of pendulum clock is $T = 2\pi\sqrt{\frac{l}{g}}$

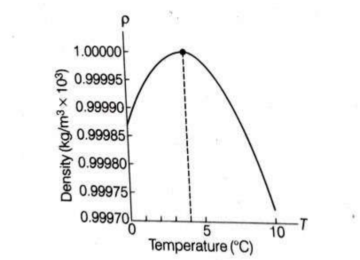
- The time period of pendulum clock (T) is directly proportional to the square root of length ($\sqrt{l}$) of pendulum.
- During winter, the length of pendulum clock contracts, so time period of pendulum clock decreases, so it makes fast oscillations. Hence pendulum clock goes fast.
- During summer, the length of pendulum of clock expands, so time period of pendulum clock increases, so it makes slow oscillations. Hence pendulum clock goes slow.
-

4. In what way is the anomalous behavior of water advantageous to acquatic animals ?

Ans : Water exhibits an anamolous behaviour :

It contracts on heating between 0^0C and $4\ ^0$ C. The volume of a given amount of water decreases (density increases) as it is cooled from room temperature, until its temperature reaches $4\ ^0$ C. Below $4\ ^0$C, the volume increases and the density decreases.





Variation of density of water w.r.t. temperature

This means that water has maximum density at $4\ ^0$C . This property has an important environmental effect . ie. Bodies of water, such as lakes and ponds, freeze at the top first . As a lake cools towards $4\ ^0$C. Water near the surface loses energy to the atmosphere, becomes denser and sinks the warmer, less dense water near the bottom rises. However, once the colder water on top reaches temperature below $4\ ^0$C. It becomes less dense and remains at the surface where it freezes. If water did not have this property, lakes and ponds would freeze from the bottom up, which would destroy acquatic animals.

5. Explain conduction, convection and radiation with examples.

Ans : Conduction :

Conduction is the mechanism of transfer of heat between two adjacent poarts of a body because of their temperature difference.

Example : One end of a metallic rod is put in a flame, here heat transfer takes place by conduction from hot end to cold end.

Convection :

(i) Convection is a mode of heat transfer by actual motion of matter. It is possible only in fluids.

(ii) Convection can be natural (or) forced convection.

(iii) In natural convection, when a fluid is heated from below, the hot part expands and becomes less dense because of buoyancy it rises, upper colder part replaces it.

(iv) In forced convection, material is forced to move by pump (or) by other physical means :

Examples of natural convection :

1) See breeze during the day
2) Land breeze during the night
3) Trade wind

Examples of forced convection :

1) Forced –air heating system in home.
2) The human circulatory system
3) The cooling system of an automobile engine.

Radiation :

- Radiation is a mode of heat transfer needs no medium. This heat energy so transferred by electromagnetic waves.
- The amount of heat that a body can absorb by radiation depends on the colour of the body.
- Black bodies absorb and emit radiant energy better than bodies of lighter colour. So we wear white (or) light coloured clothes in summer.

Example :

Heat is transferred to the earth from the Sun through empty space.

6. State and explain Newton's Law of cooling. State the conditions under which Newton's Law of Cooling is applicable.

Ans : The rate of loss of heat $\left(\frac{-dQ}{dt}\right)$ of a body is directly proportional to the difference of temperature ($T_2 – T_2$) of the body and surroundings.

$$\left(\frac{-dQ}{dt}\right)\alpha\left(T_2-T_1\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

If temperature falls by a small around dT_2 in time dt. Then the amount of heat lost is

$dQ = m\ s\ d\ T_2$

Rate of loss of heat is given by

$$\left(\frac{dQ}{dt}\right)=ms\frac{dT_2}{dt}\ldots\ldots\ldots\ldots\ldots(2)$$

From equation (1) & (2)

$$-ms\frac{dT_2}{dt}=K(T_2-T_1)$$

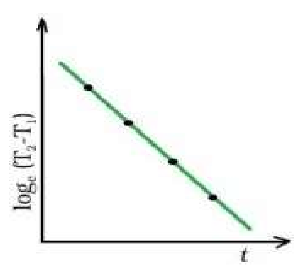
$$\frac{dT_2}{T_2-T_1}=\frac{-K}{ms}=-Kdt \qquad (\therefore K=\frac{K}{ms})$$

On integrating

$Log_c = (T_2 – T_1) = - Kt + C$

$T_2 = T_1 + e^{-Kt+C}$

A graph is plotted between $\log_e (T_2 – T_1)$ and time (t)



The nature of the graph is observed to be a straight line having a negative slope.

Conditions under which Newton's Law of Cooling is applicable :

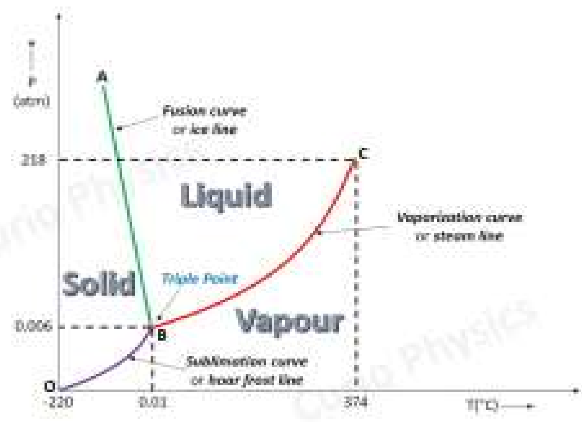
1. The temperature difference between body and surroundings should be small.
2. Loss of heat is only by radiation.
3. The temperature of surroundings remains constant.

7. Explain Triple point of water, with the help of pressure – temperature graph.

Ans : The phase diagram (or) P – T diagram of water is as shown in the figure.
This phase diagram divided P – T plane into a solid region, the vapour region and the liquid region. The regions are separated by the curves such as sublimation curve (BO) fusion curve (AO) and vaporization curve (CO).

The points on sublimation curve represent states in which solid and vapour phases co-exist. Points on the fusion curve AO represents states in which solid and liquid phase co-exist. Points on the vapourization curve CO represents states in which the liquid and vapour phases co-exist.

The temperature and pressure at which the fusion curve, the vaporization curve and sublimation curve meet and all the three phases of a substance co-exist is called the triple point of substance. The triple point of water is at temperature 273.16K and pressure 6.11 x 10^{-3} pa .



VI. LONG ANSWER QUESTIONS

1. Explain thermal conductivity and coefficient of thermal conductivity.

A copper bar of thermal conductivity 401 W/ (mk) has one end at 104 ^{0}C and other end at 24^0 C. The length of the bar is 0.10 m and the cross- sectional area is 1.0 x 10^{-6} m^2. What is rate of heat conduction along the bar ?

Note : Read cross sectional area 1.0 x 10^{-6} m^2 instead of 1.0 x 10 $^{-6}$ m^{-2} in text book.

Ans : Thermal conductivity : The ability of the material to conduct heat is called thermal conductivity.

Consider a metallic bar of length L and uniform cross – section A with its two ends maintained at different temperatures T_C and T_D respectively. ($T_C > T_D$).

Assume the ideal condition that the sides of the bar are fully insulated so that no heat is exchanged between the sides and surroundings.

After some time, a steady state is reached, at which heat supplied at a constant rate at 'C' is equal heat received at a same rate at 'D'.

In this steady state, the rate of flow of heat (or) heat current H is directly proportional to the temperature difference (T_C - T_D) and the area of cross section A and is inversely proportional to length 'L'.

$$H \alpha (T_C - T_D)$$

$$H \alpha A$$

$$H \alpha \frac{1}{L}$$

$$H \alpha \frac{A(T_C - T_D)}{L}$$

$$H = KA\frac{(T_C - T_D)}{L}$$

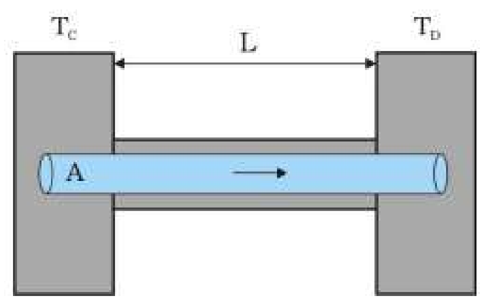


Fig. 10.14 Steady state heat flow by conduction in a bar with its two ends maintained at temperatures T_C and T_D; ($T_C > T_D$).

The constant of proportionality 'K' is called coefficient of thermal conductivity (or) thermal conductivity

The greater the value of K for a material, the more rapidly will it conduct heat. By observing thermal conductivities of some materials, we can conclude that thermal conductivity of solids are greater than liquids and thermal conductivity of liquids are greater than gases. So solids are better conductors than liquids and liquids are better conductors than gases.

Problem :

Thermal conductivity K = 401 W/(mk)
Temperature difference (T_c - T_D) = 104 -24 = 80^0 C.
Length of copper bar L = 0.1m
Area of cross – section A = 1 x 10^{-6} m^2

$$H = KA\frac{(T_C - T_D)}{L}$$

$$H = \frac{401 \times 10^{-6} \times (80)}{0.1}$$

$$H = \frac{401 \times (80) 10^{-6}}{1 \times 10^{-1}}$$

$H = 32080 \times 10^{-5}$

$H = 0.3208$ W

2. State and explain Newton Law of Cooling. State the conditions under which Newton's Law of Cooling is applicable. A body cools down from 60^0C and 50^0C in 5 minutes and to 40^0C in another 8 minutes. Find the temperature of the surroundings.

Ans : The rate of loss of heat $\left(\frac{-dQ}{dt}\right)$ of a body is directly proportional to the difference of temperature ($T_2 - T_2$) of the body and surroundings.

$$\left(\frac{-dQ}{dt}\right) \alpha \left(T_2 - T_1\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

If temperature falls by a small around dT_2 in time dt. Then the amount of heat lost is $dQ = m\, s\, d\, T_2$

Rate of loss of heat is given by

$$\left(\frac{dQ}{dt}\right) = ms\frac{dT_2}{dt} \ldots\ldots\ldots\ldots\ldots(2)$$

From equation (1) & (2)

$$-ms\frac{dT_2}{dt} = K(T_2 - T_1)$$

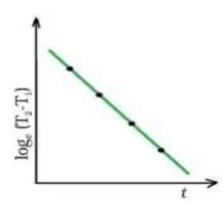
$$\frac{dT_2}{T_2 - T_1} = \frac{-K}{ms} = -Kdt \quad (\therefore K = \frac{K}{ms})$$

On integrating

$Log_c = (T_2 - T_1) = -Kt + C$

$T_2 = T_1 + e^{-Kt+C}$

A graph is plotted between $\log_e (T_2 - T_1)$ and time (t)



The nature of the graph is observed to be a straight line having a negative slope.

Conditions under which Newton's Law of Cooling is applicable :

1. The temperature difference between body and surroundings should be small.
2. Loss of heat is only by radiation.
3. The temperature of surroundings remains constant.

Problem : If the temperature of body falls from T_1 to T_2 in time – interval t, temperature of surroundings is T_0. According to Newton's Law of Cooling.

$$\frac{dT}{dt} = K(T - T_0)$$

$$\frac{T_1 - T_2}{t} = K(\frac{T_1 - T_2}{2} - T_0)$$

A body cools down from 60^0 C to 50^0 C in 5 minutes

$$\frac{60-50}{5} = K(\frac{60+50}{2} - T_0)$$

$$\frac{10}{5} = K(55 - T_0)$$

$2 = K(55 - T_0)$ ....(1)

The same body cools down from 50^0 C to 40^0 C in 8 minutes

$$\frac{50-40}{8} = K(\frac{50+40}{2} - T_0)$$

$$\frac{10}{8} = K(45 - T_0) \text{ ....(2)}$$

Divide equation(1) with equation.... (2)

$$\frac{2}{\left(\frac{10}{8}\right)} = \frac{K(55 - T_0)}{K(45 - T_0)}$$

$$\frac{16}{10} = \frac{55 - T_0}{45 - T_0}$$

$16(45 - T_0) = 10(55 - T_0)$

$720 - 16T_0 = 550 - 10T_0$

$16T_0 - 10T_0 = 120 - 550$

$6T_0 = 170$

$T_0 = \frac{170}{6} = 28.3^0$ C.

VI. PROBLEMS :

1. What is the temperature for which the readings on Kelvin and Fahrenheit scales are same ?

Ans : Given K = F ,

$$\frac{K - 273.15}{100} = \frac{F - 32}{180}$$

$$F - 273.15 = \frac{5}{9}(F - 32)$$

$$F - 273.15 = \frac{5F}{9} - \frac{5x32}{9}$$

$$\frac{4F}{9} = 273.15 - 17.11$$

$$F = \frac{9}{4}(255.38) = 574.6$$

$$K = F = 574.6^0 F (or) 574.6K$$

2. Find the increase in temperature of aluminum rod if its length is to be increased by 1%.

Ans :

$$\alpha_{Al} = 25 \times 10 - 6 /^0 C$$

$$Percentage\ increase\ in\ length\ = \frac{\Delta l}{l} \times 100 = 1\%$$

$$\Delta l = l\alpha(t_2 - t_1),$$

$$\frac{\Delta l}{l} = \alpha(t_2 - t_1)$$

$$\frac{\Delta l}{l} \times 100 = \alpha(t_2 - t_1) \times 100$$

$$t_2 - t_1 = \frac{1}{25x10^{-4}} = \frac{10^4}{25} = 400^0 C$$

3. How much steam at 100^0 C is to be passed into water of mass 100g at 20^0 C to raise its temperature by 5 ^{0}C?(Latent heat of steam is 540 cal/g and specific heat of water is

1 cal/g ^{0} C ?

Ans : In the method of mixtures,

Heat lost by steam = heat gained by water

$m_s\ L_s + m_s\ s\ (100\text{-}t) = m_w\ s\ (t - 20)$

Where m_s is the mass of steam, L_s is the latent heat of steam, s is the specific heat of water and m_w is the mass of water.

Here L_s = 540 cal/g ; s = 1cal /g^0 C;

m_w is the mass of water.

Here L_s = 540 cal/g ; s = 1 cal/ g^0 C ;

m_w = 100g , t = 20 + 5 = 25 ^{0} C

m_s x 540 + m_s x 1 (100-25) = 100 x 1 (25 – 20)

$615m_s = 500$

$$m_s = \frac{500}{615} = 0.813g$$

4. A body cools from 60 ^{0} C to 40 ^{0} C in 7 minutes. What will be its temperature after next 7 minutes if the temperature of its surroundings is 10 ^{0}C ?

Ans : $60^0\ C \xrightarrow[7\ min]{} 40^0\ C \xrightarrow[7\ min]{} x^0$ C, O_s = 10 ^{0}C

According to Newton's Law of Cooling

$$\frac{d\theta}{dt} = K\left[\left(\frac{\theta_1 + \theta_2}{2}\right) - \theta_s\right]$$

$$i.e\left(\frac{\theta_2 - \theta_1}{t}\right) = K\left[\frac{\theta_2 + \theta_1}{2} - \theta_s\right]$$

For decrease of temperature 60 ^{0}C to 40 ^{0}C

$$\frac{(60-40)}{7} = K\left[\left(\frac{60+40}{2}\right) - 10\right]$$

$$\frac{20}{7} = K[50-10]$$

$$K = \frac{20}{7\times 40} = \frac{1}{14} \text{.............(1)}$$

For decrease of temperature 40 ^{0} C to x^0 C

$$\frac{\theta_2 - \theta_1}{t} = \text{K}\left[\frac{\theta_2 + \theta_1}{2} - \theta_s\right]$$

$$\frac{40-x}{7} = \text{K}\left[\frac{40+x}{2} - 10\right] \cdots\cdots(2)$$

From (1) & (2)

$$\frac{40-x}{7} = \frac{1}{14}\left[\frac{40+x}{2} - 10\right]$$

$$\Rightarrow \frac{40-x}{7} = \frac{1}{14}\left[\frac{20+x}{2}\right]$$

$$\Rightarrow 4\,(40 - x) = 20 + x$$

By simplify it $x = \dfrac{140}{5} = 28^0 C$

5.If the maximum intensity of radiation for a black body is found at 2.65 µm what is the temperature of the radiating body ? (Wien's constant = 2.9 x 10 $^{-3}$ mK).

Ans :

$$\lambda_m T = K;$$

$$2.65\times 10^{-6} \times T = 2.9\times 10^{-3}$$

$$T = \frac{2.9\times 10^{3}}{2.65} = 1094K$$

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UNIT – 11

THERMODYNAMICS

I. MULTIPLE CHOICE QUESTIONS

1. Identify the Joule's Equation of heat among the following **(1)**

(1) W = JH (2) J = WH (3) J = H/W (4) W = JH ^{2}

Ans : Joules Equation W = JH

2. Which of the following is not a thermo dynamical state variable ? **(2)**

(1)Internal energy (2) Work (3) Pressure (4) Volume

Ans : Work is not a state function . It is a path function.

3. Which of the following is an intensive thermodynamic variable ? **(1)**

(1)Pressure (2) Mass (3) Energy (4) Heat capacity

Ans : Intensive variables are pressure, temperature, density.

4. Which of the following is not true in an isothermal process of an ideal gas ? **(2)**

(1)Internal energy doesn't change (2) No heat enters or leaves the system

(3) Temperature remains constant (4) PV = constant

Ans : For isothermal process, U is constant, T is constant and PV = constant.

5. Which of the following is true in an adiabatic process of an ideal gas ? **(2)**

(1) Internal energy is constant (2) No heat enters or leaves the system

(3)Temperature is constant (4) PV = constant

Ans : For adiabatic process, Q is constant. i.e. No heat enters or leaves the system.

6. $dU + dW = 0.$ is valid for **(2)**

(1)isothermal process (2) Adiabatic process

(3) Isobaric process (4) Isochoric process

Ans : $dU + dW = 0$ *is* for adiabatic process. Since $dQ = 0$ for adiabatic.

7. Work done in isothermal process is given by .... **(1)**

(1) $W = \mu RT \ln\left(\frac{V_2}{V_1}\right)$ (2) $W = \frac{\mu R}{\gamma - 1}(T_1 - T_2)$

(3) $W = RT \ln\left(\frac{V_1}{V_2}\right)$ (4) $W = \frac{\mu R}{\gamma - 1}(T_1 - T_2)$

Ans : Work done in isothermal process. $W = \mu RT \ln\left(\frac{V_2}{V_1}\right)$

8. Work done in adiabatic process is given by **(1)**

(1) $W = \frac{\mu R}{\gamma - 1}(T_1 - T_2)$ (2) $W = \mu RT \ln\left(\frac{V_2}{V_1}\right)$ (3) $W = \frac{\mu R}{\gamma - 1}(T_1 + T_2)$ (4) $W = \frac{\mu R}{\gamma - 1}\ln(T_1 + T_2)$

Ans : Work done in adiabatic process. $W = \frac{\mu R}{\gamma - 1}(T_1 - T_2)$

9. The mathematical equation of the first law of thermodynamics is given by **(3)**

(1) dU = PdV – dQ (2) dU = PdV + dQ (3) dU= dQ – PdV (4) dU = dQ + VdP

Ans : First law of thermodynamics $dQ = dU + dW \Rightarrow dQ = dU + PdV$

10. Which of the following equations holds good in an isothermal process of an ideal gas **(2)**

(1) PV^{γ} = constant (2) PV = Constant (3) TV^{γ} = constant (4) $PT^{1-\gamma}$ = constant

Ans : Isothermal process follows PV = constant.

11. If a liquid in a thermos flask is vigorously shaken, then **(1)**

(1) Internal energy increases
(2) Internal energy doesn't change
(3)Heat transfer from flask to the surroundings
(4) Heat transfer from surroundings to flask.

Ans : Flask is shaken, then the internal energy of liquid increases.

12. The molar specific heat of a solid at constant pressure and ordinary temperatures is approximately ………….. **(1)**

(1) 3R (2) 2R (3) R (d) 4R

Ans : Molar specific heat of solid at constant pressure and ordinary temperatures is around 3R.

13. In a cyclic process, **(2)**

(1) dQ = 0 (2) dU = 0 (3) dW = 0 (4) dU ≠ 0

Ans : *dU = 0* for cyclic process. Since dQ = dW for a cyclic process.

14. Which of the following statements is correct regarding the second law of thermodynamics? **(4)**

(1)Heat can be transferred from colder object to hotter object.
(2) Heat can't be transferred from hotter object to colder object.
(3) Heat absorbed from the reservoir can be completely converted into work
(4) Heat absorbed from the reservoir can't be completely converted into work.

II. FILL IN THE BLANKS

1. **If C_P and C_v are specific heats at constant pressure and volume respectively for an ideal gas then ________**
   Ans : $C_P - C_v = R$.
2. **If there are no inter-molecular forces, then the internal energy is a function of_______**
   Ans : temperature for ideal gas.
3. **1 cal = _________**
   Ans : 4.186 Joules.
4. **The change in volume during isochoric process is______________**
   Ans : zero.
   Hint : V is constant for isochoric process. Hence $dV = 0$
5. **Relation between temperature (T) and volume (V) in adiabatic process is ________**
   Ans : $TV^{\gamma-1}$ = constant
6. **The amount of heat required to raise the temperature of 1 gram of water from 14.5 ^{0}C to 15.5^0 C is ________**
   Ans : Calorie.
7. **According _______________heat cannot flow itself from cold body to hot body.**
   Ans : second law of thermodynamics
8. **The change in internal energy during isothermal process is**
   Ans : zero.
   Hint : U is constant for isothermal process. Hence $dU = 0$
9. **Sign convention for work done is taken as _________when work done by the system.**
   Ans : Positive
   Hint : Work done w $= P (V_2 - V_1)$
   Work done by the system ($V_2 > V_1$), w = positive
   Work done on the system ($V_2 < V_1$), w = negative.
10. **The area under P-V curve represent __________**
    Ans : work done.
11. **First law of thermodynamics is a consequence of conservation of ___________**
    Ans : Energy

III. ONE WORD ANSWER TYPE QUESTIONS

1. **Which physical quantity remains constant in a thermal equilibrium ?**

   Ans : Temperature

2. **What is the measure of degree of hotness or coldness of a body ?**

   Ans : Temperature

3. **What is the SI unit of specific heat capacity ?**

   Ans : J/Kg.K (or) joule/ kilogram . Kelvin

4. **Give the relation between C_P, C_v, and γ**

   Ans : $\frac{C_P}{C_V} = \gamma$

5. **Which parameter is constant in an isothermal process ?**

   Ans : Temperature or internal energy

6. **Which parameter is constant in an adiabatic process ?**

   Ans : Total heat is constant

7. **Which parameter is constant in an isobaric process ?**

   Ans : Total pressure is constant

8. **Give an expression for the efficiency of a carnot engine .**

   Ans : $\eta = 1 - \frac{T_2}{T_1}$ (or) $\eta = 1 - \frac{Q_2}{Q_1}$

9. **Write the ideal gas equation for an isothermal process ?**

   Ans : $PV = nRT$ (or) PV = constant

10. **Write the ideal gas equation for an adiabatic process ?**

    Ans : PV^{γ} = constant

IV. VERY SHORT ANSWER TYPE QUESTIONS

1. **Define thermal equilibrium. How does it lead to Zeroth law of Thermodynamics.**
   Ans : If the temperature of the two systems are equal. They are said to be in thermal equilibrium.
   Zeroth law of Thermodynamics : Zeroth law essentially defines the concept of temperature as a measurable property that determines whether systems are in thermal equilibrium.

2. **Define Calorie. What is the relation between calorie and mechanical equivalent of heat ?**
   Ans : Calorie : Amount of heat required to raise the temperature of 1g of water from 14.5^0 C to 15.5^0 C.
   1 Cal = 4. 186 J/gram K. = 4186 J/Kg K
3. **What thermodynamic variables can be defined by (a) Zeroth law (b) First law**
   Ans : (a) temperature (b) Internal energy

4. **Define specific heat capacity of the substance. On what factors does it depend ?**
   Ans : The amount of heat required to raise the 1 gram of substance through 1^0C or 1 K is called specific heat capacity.

   (or)

   The heat capacity per unit mass is called specific heat.

   $S = \frac{1}{m}\left(\frac{\Delta Q}{\Delta T}\right)$

   It depends on nature of substance and temperature.

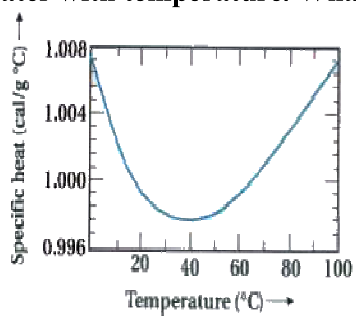
5. Define molar specific heat capacity.

Ans : The amount of heat required to raise the 1gram mole of substance through $1\ ^0C$ or 1 K is called molar specific heat capacity.

6. For a solid, what is the total energy of an oscillator ?

Ans : For one mole of solid, the total energy of an oscillator U = 3 R T = 3 KTN

7. Indicate the graph showing the variation of specific heat of water with temperature. What does it signify ?



Ans : The specific heat of water varies slightly with temperature in the range $0\ ^0C$ to $100\ ^0C$.

8. Define State variables and equation of a state.

Ans : The thermodynamic state of a system is described by pressure P, Temperature T, density ρ (intensive variables) and internal energy U, Volume V, total mass M (extensive variables) . These are called thermodynamic state variables. That connection between the thermodynamic state variables is called the equation of state.

9. In summer, when the valve of a bicycle tube is opened, the escaping air appears cold. Why ?

Ans : This happens due to adiabatic expansion of the air of the tube of the bicycle.

10. Why does the brake drum of an automobile get heated up while moving down at constant speed ?

Ans : The work done by the brake drum on wheel is converted into heat due to friction.

11. Can a room be cooled by leaving the door of an electric refrigerator open ?

Ans : No. A room cannot be cooled by leaving the door of refrigerator open, rather it will get slightly heated.

12. A sound wave is sent into a gas pipe. Does its internal energy change ?

Ans : Yes, A sound wave is sent into a gas pipe, internal energy increases.

13. **How much will be the internal energy change in (a) isothermal process (b) adiabatic process** ?

Ans : (i) the change in internal energy during isothermal process is zero $dU = 0$

(ii) During adiabatic compression internal energy increases

(iii) During adiabatic expansion internal energy decreases.

14. The coolant in a chemical or a nuclear reactor plant should have high specific heat. Why ?

Ans : In chemical and nuclear plants more heat is liberated.

To absorb this heat, the coolant should have a property of high specific heat for small rise in temperature since $S\alpha \frac{1}{dt}$

15. Explain the following process (a) Isochoric process (b) Isobaric process ?

Ans : Isochoric process : It takes place at constant volume. During this process , no work is done on or by the gas, its internal energy and its temperature changes.

Isobaric process : It takes place at constant pressure. During this process, temperature and internal energy changes.

V. SHORT ANSWER TYPE QUESTIONS

1. State Zeroth Law, First law and Second law of thermodynamics.

Ans : Zeroth law : If two systems (A, B) are in thermal equilibrium with third system (C) separately, then two systems (A, B) are thermal equilibrium with each other.

First law : The amount of heat supplied to system is equal to the algebraic sum of the change in internal energy of the system and the amount of external work done.

Second law : No process is possible whose sole result is the transfer of heat from a colder object to a hotter object.

2. Define two principal specific heats of a gas. Which is greater and why?

Ans : Molar specific heat capacity at constant pressure (C_P) : The amount of a heat required to raise the temperature of 1gm – mole of a gas through 1^0 C at constant pressure is called molar specific heat at constant pressure.

$$C_P = \frac{1}{\mu}\frac{\Delta Q}{\Delta T}$$

Molar specific heat capacity at constant volume (C_v) : The amount of heat required to raise the temperature of 1gm – mole of a gas through 1^0 C at constant volume is called molar specific heat at constant volume.

$$C_V = \frac{1}{\mu}\frac{\Delta Q}{\Delta T}$$

μ= Number of moles

$C_P > C_V$ Explanation : When a gas is heated at constant pressure, it expands . The heat supplied to it is used partly in raising its temperature and partly in doing work against the external pressure, if on the other hand, the gas is heated at constant volume, no work is done. Therefore, the heat supplied is to be used only in raising the temperature. Hence $C_P > C_V$.

3. Explain the following processes

(a)Cyclic process with examples

(b) Non cyclic process with examples

Ans : Cyclic Process : A process in which the system after passing through various stages returns to its initial state is called as a cyclic process.

For cyclic process $P-V$ graph is a closed curve, the area under $P-V$ graph gives work done by the substance. In a cyclic process there will be no change in the internal energy $\Delta U = 0$.

For cyclic process $\Delta Q = \Delta W$

Ex : heat engine.

Non cyclic process : A process which is not cyclic is called non – cyclic process. In non – cyclic process the system doesn't attaints its initial state. Irreversible process is a non –cyclic process. Work done in non- cyclic process is given by the area between the curve and volume axis.

Ex : 1. Diffusion of liquids

2. Free expansion of a perfect gas.

4. Distinguish between isothermal and adiabatic processes.

Ans :

ISOTHERMAL PROCESS	ADIABATIC PROCESS
1. Changes in volume and pressure of a gas taking place at constant temperature	1. Changes in volume and pressure of a gas taking place in a thermally isolated system.
2. Temperature of the gas remains constant	2.Temperature of the gas changes
3. The gas remains in good thermal contact with the surroundings and heat is exchanged	3.The gas is isolated from the surroundings and heat is not exchanged. $\Delta Q = 0$
4. Internal energy remains constant change in internal energy $\Delta U = 0$	4.Internal energy changes
5. Boyle's law PV = constant hold good	5. PV^{γ}= constant
6. Work done $w = \mu RT \ln\left(\frac{V_2}{V_1}\right)$	6. work done $w = \frac{\mu R}{\gamma - 1}[T_1 - T_2]$
7 .Melting and boiling process are the examples	6. Propagation of sound in gases, bicycle pump, gas turbine expansion are the examples.

VI. LONG ANSWER TYPE QUESTIONS

1. Define two specific heat capacities of a gas and derive the relation between them on the basis of first law of thermodynamics.

Ans : C_v : The amount of heat required to raise the temperature of 1gram mole of gas through 1^0 C at constant volume is called molar specific heat at constant volume.

$$C_V = \frac{1}{\mu}\left(\frac{\Delta Q}{\Delta T}\right)$$

C_p : The amount of heat required to raise the temperature of 1 gram mole of gas through 1^0 C at constant pressure is called molar specific heat at constant pressure.

$$C_P = \frac{1}{\mu}\left(\frac{\Delta Q}{\Delta T}\right)$$

From first law of thermodynamics Proof of $C_P - C_v = R$ $\Delta Q = \Delta U + PdV$

For 1 mole of gas $C_V = \left[\frac{\Delta Q}{\Delta T}\right]_V = \left[\frac{\Delta U}{\Delta T}\right]_V = \frac{\Delta U}{\Delta T}$ ……….(1)

Subscript V is dropped from above equation, since U depends only on temperature for ideal gas.

$$C_P = \left[\frac{\Delta Q}{\Delta T}\right]_P = \left[\frac{\Delta U}{\Delta T}\right]_P + P\left[\frac{\Delta V}{\Delta T}\right]_P \quad \ldots\ldots\ldots\ldots\ldots(2)$$

We know $PV = RT$

$$P\left[\frac{\Delta V}{\Delta T}\right]_P = R \ldots\ldots\ldots(3)$$

Sub (3) in (2)

$$C_P = \left[\frac{\Delta Q}{\Delta T}\right]_P + R$$

$$C_P = C_V + R \quad (\therefore \text{ from (1)})$$

$$C_P - C_V = R$$

2. Describe the working of carnot engine. Obtain an expression for its efficiency.

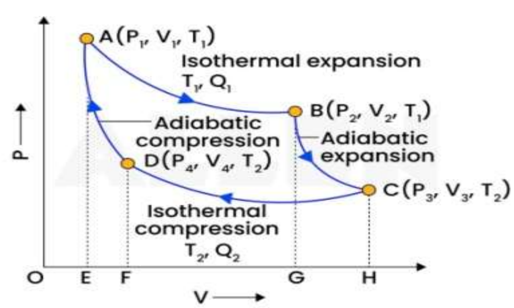
Ans : Carnot Engine : A reversible heat engine operating between two temperatures is called a carnot engine. A cycle operating it is known as Carnot's cycle. In their cycle the working substance (say an ideal gas) is taken through a cycle by means of two isothermals and two adiabatics. The four operations are shown in $P - V$ diagram

AB – Isothermal Expansion

BC – Adiabatic expansion

CD – Isothermal Compression

DA – Adiabatic compression



The work done in isothermal expansion $W_1 = Q_1 = \mu RT_1 ln\left(\frac{V_2}{V_1}\right)$………*(1)*

The work done in adiabatic expansion $W_2 = \frac{\mu R}{\gamma - 1}[T_1 - T_2]$…..………..(2)

The work done in isothermal compression $W_3 = -\mu RT_2 \ln\frac{V_3}{V_4}$ …………(3)

The work done in adiabatic compression $W_4 = -\frac{\mu R}{\gamma - 1}[T_1 - T_2]$…………(4)

Total work done by the gas in one complete cycle is W = $W_1 + W_2 + W_3 + W_4$

$$W = \mu RT_1 \ln\left(\frac{V_2}{V_1}\right) + \frac{\mu R}{\gamma - 1}\ln[T_1 - T_2] - \mu RT_2 \ln\left(\frac{V_3}{V_4}\right) - \frac{\mu R}{\gamma - 1}\ln[T_1 - T_2]$$

$$W = \mu RT_1 \ln\left(\frac{V_2}{V_1}\right) - \mu RT_2 \ln\left(\frac{V_3}{V_4}\right)$$

The efficiency $\eta = \frac{W}{Q_1}$

$$\eta = \frac{\mu RT_1 \ln\left(\frac{V_2}{V_1}\right) - \mu RT_2 \ln\left(\frac{V_3}{V_4}\right)}{\mu RT_1 \ln\left(\frac{V_2}{V_1}\right)} \quad \text{…………..(5)}$$

BC and AD curves are adiabatic curves

Hence $\frac{V_2}{V_1} = \frac{V_3}{V_4}$ ………………………………….(6)

Sub (6) in (5) $\eta = \frac{\mu RT_1 \ln\left(\frac{V_2}{V_1}\right) - \mu RT_2 \ln\left(\frac{V_2}{V_1}\right)}{\mu RT_1 \ln\left(\frac{V_2}{V_1}\right)}$

$$\Rightarrow \eta = \frac{T_1 - T_2}{T_1}$$

(or) $\eta = 1 - \frac{Q_2}{Q_1}$

3. Obtain expressions for the work done by an ideal gas in isothermal and adiabatic processes.

Ans : Let us consider certain mass of gas expands from volume V_1 to V_2 isothermally at constant temperature T. Let the pressure changes from P_1 to P_2.

The work done during the small change in volume dV at constant pressure P is

dw = P.dv

Total work done $W = \int dw = \int_{V_1}^{V_2} P.dv$ ……………(1)

For isothermal change $PV = \mu RT$ …………………..(2)

Sub (2) in (1) $W = \int_{V_1}^{V_2} \frac{\mu RT}{V} dV$

$$= \mu RT[\ln v]_{V_1}^{V_2}$$

$$= \mu RT[\ln V_2 - \ln V_1]$$

$$W = \mu RT \ln \frac{V_2}{V_1} \quad = 2.303\ \mu RT \log_{10}\left(\frac{V_2}{V_1}\right)$$

During the adiabatic change the state of an ideal gas from (P_1, V_1, T_1) and (P_1, V_2, T_2)

The work done during a small change in volume dV at a pressure P is dW= Pdv.

The total work done $W = \int_{V_1}^{V_2} PdV$

For adiabatic change PV^{γ}= constant = $K \Rightarrow P = \frac{K}{V^3}$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} = K \text{............}(2)$$

$$W=\int_{V_1}^{V_2}\frac{K}{V^{\gamma}}.dV$$

$$W=K\int_{V_1}^{V_2}V^{-\gamma}.dV$$

$$W=K\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_1}^{V_2}$$

$$W=\frac{1}{1-\gamma}\left[KV_2^{1-\gamma}-KV_1^{1-\gamma}\right]$$

$$=\frac{1}{1-\gamma}\left[P_2V_2^{\gamma}V_2^{1-\gamma}-P_1V_1^{\gamma}V_1^{1-\gamma}\right]$$

$$W=\frac{1}{1-\gamma}\left[P_2V_2^{\gamma+1-\gamma}-P_1V_1^{\gamma+1-\gamma}\right]$$

$$W=\frac{1}{1-\gamma}\left[P_2V_2-P_1V_1\right]$$

$$=\frac{1}{1-\gamma}\left[RT_2-RT_1\right]$$

$$W=\frac{R}{1-\gamma}\left[T_2-T_1\right]$$

$$(or)W=\frac{R}{\gamma-1}\left[T_1-T_2\right]$$

$$W=\frac{1}{1-\gamma}\left[P_2V_2^{\gamma+1-\gamma}-P_1V_1^{\gamma+1-\gamma}\right]$$

$$W=\frac{1}{1-\gamma}\left[P_2V_2-P_1V_1\right]$$

$$=\frac{1}{1-\gamma}\left[RT_2-RT_1\right]$$

$$W=\frac{R}{1-\gamma}\left[T_2-T_1\right]$$

$$(or)W=\frac{R}{\gamma-1}\left[T_1-T_2\right]$$

VII. PROBLEMS

1. **If a monatomic ideal gas of volume 1 litre at N. T. P is compressed (i) adiabatically to half of its volume, find the work done on the gas. Also find (ii) the work done if the compression is isothermal.**

Ans : Adibatic process $TV^{\gamma -1}$ = constant

$T_1 = 273$ K $\qquad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$

$T_2 = ?$

Given $V_2 = \frac{V_1}{2}$

$$T_2 = 273(2)^{\frac{5}{3}-1} = 273(2)^{\frac{2}{3}} = 431.6K \quad (\therefore \gamma = \frac{5}{3})$$

Number of moles (μ) $= \frac{1\ litre}{22.4\ litre} = \frac{1}{22.4}$

Work done $= \frac{\mu R}{\gamma - 1}\left[T_1 - T_2\right]$

$$= \frac{8.314}{22.4(\frac{5}{3}-1)}(273-431.6) = -89J$$

(ii) Isothermal process $W = 2.303\mu RT\log_{10}\left(\frac{V_2}{V_1}\right)$

$$= 2.303\left(\frac{1}{22.4}\right)8.314\times 273\log_{10}(0.5)$$

= - 70J

2. **Five moles of hydrogen when heated through 20 K expand by an amount of 8.3×10^{-3} m^3 under a constant pressure of 10^5N/m2. If C_V = 20 J mole K, find C_P**

Ans : Mayor's relation $C_P - C_V = R$

$\mu C_P \Delta T - \mu C_V \Delta T = \mu R \Delta T$

$\mu(C_P - C_V)\Delta T = P\Delta V$

$5(C_P - 20)(20) = 1 \times 10^5 \times 8.3 \times 10^{-3}$

$C_P - 20 = 8.3$

$C_P = 28.3$ J mol/K.

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UNIT – 12

KINETIC THEORY

I. MULTIPLE CHOICE QUESTIONS

1. To decrease the volume of a gas by 5% at constant temperature, the pressure should be **[2]**

(1) Decreased by 5.26% (2) Increased by 5.26%
(3) Decreased by 11% (4) Increased by 11%

Ans: $V_2 = \frac{95}{100}V_1$

At constant temperature

$P_1V_1 = P_2V_2$

$P_1V_1 = \frac{95}{100}V_1P_2$

$P_2 = \frac{100}{95}P_1$

$$\frac{P_2 - P_1}{P_1}X100 = \left(\frac{\frac{100}{95}P_1 - P_1}{P_1}\right)100$$

$= +\frac{5}{95}X100 = +5.26\%$

2. At NTP, sample of equal volume of chlorine and oxygen is taken. Now ratio of number of molecules is **[1]**

(1) 1:1 (2) 32:27 (3) 2:1 (4) 16:14

Ans: PV= N K T

P $\rightarrow$ Same

V $\rightarrow$ Same

T $\rightarrow$ Same

N $\rightarrow$ Same

$N_1 = N_2$

3. If pressure of a gas contained in a closed vessel is increased by 0.4% when heated by 1^0C, the initial temperature must be **[1]**

(1) 250K (2) 250^0C (3) 2500K (4) 25^0C

Ans: V $\rightarrow$ Constant

Given P $\propto$ T , $T_2 - T_1$ =1K

$$\frac{P_2 - P_1}{P_1}X100 = \frac{T_2 - T_1}{T_1}X100$$

$$0.4 = \frac{1}{T_1}X100$$

$$T_1 = \frac{100}{0.4} = \frac{1000}{4} = 250K$$

4. A perfect gas at 27^0C is heated at constant pressure to 327^0C. If original volume of gas at 27^0C is V then its volume at 327^0C. **[3]**

(1) V (2) 3V (3) 2V (4) V/2

Ans: P = Constant

$V \propto T$

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$T_1 = 273 + t_1 = 273 + 27 = 300K$

$T_2 = 273 + t_2 = 273 + 327 = 600K$

$$\frac{V_2}{V_1} = \frac{600}{300}$$

$V_2 = 2V_1 = 2V$

5. Under which of the following conditions is the law PV = RT obeyed most closely by a real gas **[3]**

(1)High pressure and high temperature (2) Low pressure and low temperature

(3) Low pressure and high temperature (4) High Pressure and low temperature

Ans: Real gas behaves as an ideal gas at high temperature and low pressure.

6. The equation of state for 5g of oxygen at a pressure P and temperature T, when occupying a volume V, will be **[1]**

(1) PV =(5/32)RT (2) PV = (5/8)RT (3) PV = (5/2)RT (4) PV = (5/16)RT

Ans: $PV = \frac{m}{M} RT$

m=5g, M=32g

Pv= $\frac{5}{32} RT$

7. A sample of gas is at 0^0c. The temperature to which it must be raised in order to double the r.m.s speed of the molecule is **[2]**

(1) 270^0C (2) 819^0C (3)1090^0C (4) 100^0C

Ans: $V_{rms} = \sqrt{\frac{3RT}{M}}$

$V_{rms} \, \alpha \, \sqrt{T}$

$V_{rms2} = 2\, V_{rms1}$

$$\sqrt{\frac{3RT_2}{M}} = 2\sqrt{\frac{3RT_1}{M}}$$

$T_2 = 2^2 T_1 \quad T_1 = 273K$

$T_2 = 4T_1$

$= 4X273$

$= 1092K$

$T_2 = 1092 - 273 = 819^0 C$

8. The r.m.s speed of gas molecules is given by **[2]**

(1) $1.414\sqrt{\frac{RT}{M}}$ (2) $1.732\sqrt{\frac{RT}{M}}$ (3) $1.414\sqrt{\frac{M}{RT}}$ (4) $1.732\sqrt{\frac{M}{RT}}$

Ans: $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\sqrt{3}\sqrt{\frac{RT}{M}}$

$= 1.732\sqrt{\frac{RT}{M}}$

9. The relation between the gas pressure P and average kinetic energy per unit volume E is **[4]**

(1) $P = \frac{1}{2}E$ (2) $P = E$ (3) $P = \frac{3}{2}E$ (4) $P = \frac{2}{3}E$

Ans: E $\rightarrow$ average Kinetic energy per unit volume

$E = \frac{KE}{V}$

$KE = \frac{3}{2}PV \qquad \Rightarrow \frac{KE}{V} = \frac{3}{2}P$

$E = \frac{3}{2}P \qquad \Rightarrow P = \frac{2E}{3}$

10. In kinetic theory of gases, a molecule of mass m of an ideal gas collides with a wall of vessel with velocity V. the change in the linear momentum of the molecule is **[1]**

(1) 2mV (2) mV (3) –mV (4) zero

Ans: Initial momentum P_i = mv . After an elastic collision, the molecule rebounds with the same speed but in the opposite direction.

$\therefore$ Final momentum P_f = - mv

Change in linear momentum $\Delta P = P_f - P_i$

$\Delta P = -mv - mv = -2mv$

$\therefore |\Delta P| = 2\ mv$

11. Pressure of an ideal gas is increased by keeping the temperature constant. The effect of kinetic energy of molecules is **[3]**

(1) Increase (2) Decrease (3) No change

(4) can't be determined.

Ans: $KE = \frac{3}{2}KT$

$T \rightarrow$ absolute temperature $\rightarrow$ constant, K is also constant

$KE = same$

12. Energy of all molecules of a monatomic gas having a volume V and pressure P is $\frac{3}{2}$ PV. The total translational kinetic energy of all molecules of a diatomic gas as the same volume and pressure.

(1) $\frac{1}{2}$ PV (2) $\frac{3}{2}$ PV (3) $\frac{5}{2}$ PV (4) 3PV **[2]**

Ans: For monoatomic gas

KE = $\frac{3}{2}$ PV

For diatomic gas average translational kinetic energy of gas is $\frac{3}{2}$ PV

13. The degree of freedom of a gas are f. then the ratio of two specific heats $\frac{C_P}{C_V}$ is given by **[1]**

(1) $\frac{2}{f}+1$ (2) $1-\frac{2}{f}$ (3) $1+\frac{1}{f}$ (4) $1-\frac{1}{f}$

Ans: $\frac{C_P}{C_V} = \gamma = 1+\frac{2}{f}$

f is degree of freedom

14. The relation between the molar specific heat at constant pressure and at constant volume of an ideal gas is [**2**]

(1) $C_p + C_v = R$ (2) $C_p - C_v = R$ (3) C_p/C_v = R (4) C_v/C_p = R

Ans: $C_p - C_v = R$

II. <u>FILL IN THE BLANKS</u>

1. If the molecular weights of two gases are M_1 and M_2 at a given temperature, then the ratio of their root mean square velocities will be ____________

Ans: $\sqrt{\frac{M_2}{M_1}}$

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{rms} \propto \frac{1}{\sqrt{M}}$$

$$\frac{V_{rms1}}{V_{rms2}} = \sqrt{\frac{M_2}{M_1}}$$

2. A gas is allowed to expand isothermally. Then the root mean square velocity of the molecules will ___________

Ans: remains unchanged

Isothermal process
Temperature is constant

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

V_{rms} is same

3. The pressure is exerted by the gas on the walls of the container because ________
Ans : On collision with walls there is a change in momentum.

4. The ratio of mean kinetic energy of hydrogen and oxygen gas molecules at a given temperature is ____

Ans: 1:1
Temperature is constant

$$V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3KT}{m_0}}$$

$KE_{molecule} = \frac{3}{2}$ KT
Kinetic energy of hydrogen and oxygen gas molecule is same.
∴ Ratio is 1 : 1

5. The mean kinetic energy per degree of freedom of gas molecule is __________

Ans: $\frac{1}{2}KT$

Mean kinetic energy per degree of freedom = $\frac{1}{2}KT$

6. The kinetic energy of one gm – mole of a gas at normal temperature and pressure nearly is ________

Ans: $3.4X10^3 J$

Mean KE of one gm = mole of gas

$$KE = \frac{3}{2}RT$$

$$= \frac{3}{2}X8.314X273$$

$$= 3.4X10^3 J$$

7. A monoatomic gas molecules has _________ degree of freedom.
Ans : Three

8. The number of translational degrees of freedom for a diatomic gas is _____
Ans: Three.

9. The value of C_v for one mole of Neon gas is _______

Ans: $\frac{3}{2}R$

$C_v = \frac{R}{\gamma - 1}$ $\quad \gamma = \frac{5}{3}$

$C_v = \frac{R}{\frac{5}{3}-1} = \frac{3}{2}R$

10. If the mean free path of the molecules is doubled then the pressure of the gas will become __________

Ans: Half

$\lambda \propto \frac{1}{P}$

$\frac{\lambda_1}{\lambda_2} = \frac{P_2}{P_1}$

$\lambda_2 = 2\lambda_1$

$P_2 = \frac{1}{2}P_1$

III. ONE WORD ANSWER QUESTIONS

1. **What is an ideal gas?**
   Ans: A gas which obeys all the gas laws at all temperature and pressure is called ideal gas.
2. **Write the ideal gas equation.**
   Ans: PV = nRT
3. **What is the nature of curve obtained when pressure verses reciprocal of volume is plotted for an ideal gas at constant temperature?**
   Ans: Straight line with slope K
   $P \propto \frac{1}{V}$, $P = K\frac{1}{V}$
4. **What is the nature of curve obtained when volume of an ideal gas is plotted against its absolute temperature at constant pressure?**
   Ans: Straight line passing through origin
5. **What is the value of universal gas constant?**
   Ans: R = 8.314 J / mol. K
6. **What is the value of Boltzmann constant?**
   Ans: $K = 1.38 \times 10^{-23}\ J\ K^{-1}$
7. **What is the average speed of five molecules of a gas having speeds** V_1, V_2, V_3, V_4 and V_5 **?**
   Ans: $V_{avg} = \frac{V_1 + V_2 + V_3 + V_4 + V_5}{5}$
8. **What is the RMS speed of five molecules of a gas having speeds** V_1, V_2, V_3, V_4 and V_5 **?**
   Ans: $V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + V_4^2 + V_5^2}{5}}$

IV. VERY SHORT ANSWER QUESTIONS

1. Define mean free path?

Ans: The mean free path l is the average distance covered by a molecule between two successive collisions.

$$l = \frac{1}{\sqrt{2}\pi n d^2}$$

$n \rightarrow$ number density

$d \rightarrow$ diameter of molecule

2. How does kinetic theory justify Avogadro's hypothesis and show that number in different gases is same?

Ans: Avogadro's hypothesis states that equal volumes of all gases under similar conditions of temperature and pressure contain equal number of molecules

$$P_1 V_1 = \frac{1}{3} m_1 N_1 \overline{V_1^2}$$
$$P_2 V_2 = \frac{1}{3} m_2 N_2 \overline{V_2^2}$$

Given $P_1 = P_2$

$V_1 = V_2$

$T_1 = T_2$

If gases are at same temperature the average kinetic energies of their molecules are equal

$$\frac{1}{2} m_1 \overline{V}_1^2 = \frac{1}{2} m_2 \overline{V}_2^2$$

So $N_1 = N_2$

3. **When does a real gas behave like an ideal gas?**

Ans: At low pressure and high temperature , a real gas almost behaves like an ideal gas.

4. State Boyle's law and Charle's law?

Ans: Boyle's Law Statement : keeping temperature constant the pressure of a given mass of gas varies inversely with volume

$$P \propto \frac{1}{V}$$

PV=Constant

Charles Law : For a fixed pressure, the volume of a given mass of gas is proportional to its absolute temperature

$$V \propto T \quad \Rightarrow \quad \frac{V}{T} = \text{constant}$$

5. State Dalton's law of partial pressures?

Ans: It states that the total pressure exerted by a mixture of non-reacting ideal gases is equal to the sum of the partial pressures of the individual gases in the mixture.

$$P = P_1 + P_2 + \ldots\ldots$$

6. Pressure of an ideal gas in container is independent of shape of the container explain?

Ans: Yes, the pressure of an ideal gas with in a container is independent of The container's shape then

$$PV = \frac{1}{3} m N\overline{V^2}$$

$m \rightarrow$ mass of each molecule

Pressure exerted by a gas is due to continuous collision of gas molecules with the container walls, each collision transfers of momentum and this transfers momentum is independent of shape.

7. Explain the concept degree of freedom for a molecules of a gas?

Ans: The degree of freedom is the number of possible ways in which a system can posses internal energy. The molecule of a mono atomic gas has only three translational degree of freedom. A diatomic molecule (rigid) has three translational and two rotational. Polyatomic molecule has 3 translational 3 rotational degree of freedom and certain number of vibration modes.

8. What is the expression between pressure and kinetic energy of a gas molecule?

Ans: The relationship between pressure and the average translational kinetic energy per unit volume $\left(\frac{E}{V}\right)$ of a gas is given by

$$P = \frac{2}{3}\frac{E}{V}$$

$$E = \frac{3}{2}PV$$

9. The absolute temperature of a gas is increased 3 times. What will be the increase in rms velocity of the gas molecule?

Ans: $V_{rms} = \sqrt{\frac{3KT}{m}}$

$$T_2 = 3T_1$$

$V_{rms} \propto \sqrt{T}$ $\quad m \rightarrow$ mass of a molecule

$$\frac{V_{rms2}}{V_{rms1}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{3T_1}{T_1}} = \sqrt{3}$$

$$V_{rms2} = \sqrt{3}\,V_{rms1}$$

V. SHORT ANSWER QUESTIONS

1. Explain the kinetic interpretation of temperature?

Ans: $P = \frac{1}{3}\frac{Nm}{V}\overline{V^2}$

$$PV = \frac{1}{3} \times \frac{2}{2} Nm\overline{V^2}$$

$PV = \frac{2}{3} \times E$, E $\rightarrow$ average translational kinetic energy of molecules in a gas.

$$E = \frac{1}{2} Nm\overline{V^2}$$

$$PV = \frac{2}{3}E$$

$$PV = NK_B T$$

$$E = \frac{3}{2}PV = \frac{3}{2}NK_B T$$

$$\frac{E}{N} = \frac{3}{2}K_B T,$$

$$\frac{E}{N} \alpha T$$

$\frac{E}{N} \rightarrow$ average kinetic energy of a molecule.

$$\frac{E}{N} \alpha T$$

T $\rightarrow$ absolute temperature of gas

As T increases $\frac{E}{N}$ also increases

Average kinetic energy of a molecule is independent of pressure, volume and the nature of gas.

If T = 0, $\frac{E}{N}$ = 0.

2. How specific heat capacity if mono atomic, diatomic and poly atomic gases can be explained on the basis of law of equi partition of energy.

Ans: Monatomic gases:

It has three translational degree of freedom the total internal energy of a mole of gas is

$$U = \frac{3}{2}N_A K_B T = \frac{3}{2}RT$$

molar specific heat at constant volume

$$C_V = \frac{dU}{dT} = \frac{3}{2}R$$

for ideal gas

$$C_p - C_V = R$$

$$C_p = C_V + R = \frac{3}{2}R + R = \frac{5R}{2}$$

$$\gamma = \frac{C_p}{C_V} = \frac{5}{3}$$

<u>Diatomic gas (rigid)</u>

Diatomic molecule has 3 translational and 2 rotational degree of freedom.

Total internal energy of one mole of gas

$$U = \frac{5}{2}N_A K_B T = \frac{5}{2}RT$$

$C_v = \frac{dU}{dT} = \frac{5}{2}$ R

$C_p = C_v + R = \frac{5}{2}$ R + R = $\frac{7}{2}$ R

$\gamma = \frac{7}{5}$

Diatomic molecule is not rigid but has in additional a vibrational mole.

$U = \frac{7}{2}RT$ for one mole of gas

$C_v = \frac{dU}{dT} = \frac{7}{2}$ R

$C_p = C_v + R = \frac{7}{2}$ R + R = $\frac{9}{2}$ R

$\gamma = \frac{C_p}{C_v} = \frac{9}{7}$

Polyatomic

Polyatomic molecule has three translational, three rotational degree of freedom and f vibrational modes
Internal energy of one mole of gas

$$U = \frac{6}{2}N_A K_B T + fK_B T N_A$$

$$= (3+f)RT$$

$$C_v = \frac{dU}{dT} = (3 + f)\ R$$

$$C_p = C_v + R = (4 + f)\ R$$

$$\gamma = \frac{C_p}{C_v} = \frac{4+f}{3+f}$$

3. Explain the concept of absolute zero of temperature on the basis of kinetic theory.

Ans: Root mean square speed of gas molecule

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{rms} \alpha \sqrt{T}$$

According to kinetic theory of gas, the average kinetic energy of a gas molecule is

$$E = \frac{3}{2}KT$$

$E \alpha$ T,

E = 0, at absolute zero the molecular motion stops

4. Prove that the average kinetic energy of a molecule of an ideal gas is directly proportional to the absolute temperature of the gas.

Ans: : $P = \frac{1}{3}\frac{Nm}{V}\overline{V^2}$

$$PV = \frac{1}{3} \text{ x } \frac{2}{2} Nm\overline{V^2}$$

PV = $\frac{2}{3}$ x E , E $\rightarrow$ average translational kinetic energy of molecules in a gas.

$$E = \frac{1}{2} Nm\overline{V^2}$$

$$PV = \frac{2}{3} E$$

$$PV = NK_B T$$

$$E = \frac{3}{2} PV = \frac{3}{2} NK_B T$$

$$\frac{E}{N} = \frac{3}{2} K_B T,$$

$$\frac{E}{N} \alpha T$$

$\frac{E}{N}$ $\rightarrow$average kinetic energy of a molecule.

5. When the pressure increases by 2%. What is the percentage decrease in the volume
Of a gas assuming Boyle's law is obeyed?

Ans: PV = constant at constant temperature.

$$P_1V_1 = P_2V_2$$

$$P_2 = \frac{102}{100} P_1$$

$$P_1V_1 = \frac{102}{100} P_1V_2$$

$$V_2 = \frac{100}{102} V_1$$

$$\frac{V_2 - V_1}{V_1} \text{ x } 100 = \left(\frac{\frac{100}{102} v_2 - v_1}{v_1} \right) 100$$

$$= \left(\frac{100 - 102}{102} \right) 100 = \frac{-2}{102} \text{ x } 100 = -1.96\%$$

6. What is the ratio of rms speed of oxygen and hydrogen molecules at the same temperature ?

Ans: $V_{rms} = \sqrt{\frac{3RT}{M}}$

T $\rightarrow$absolute temperature
T $\rightarrow$same
M $\rightarrow$ molecular weight

$$V_{rms} \alpha \frac{1}{\sqrt{M}}$$

$$\frac{V_{o_2}}{V_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{o_2}}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$

Ratio of rms speeds of oxygen and hydrogen is1:4

7. Four molecules of a gas have speeds 1,2,3 and 4 km/s. find the rms speed of the molecules.

Ans: $V_{rms} = \sqrt{\dfrac{V_1^2 + V_2^2 + V_3^2 + V_4^2}{4}}$

$V_{rms} = \sqrt{\dfrac{1^2 + 2^2 + 3^2 + 4^2}{4}}$

$= \sqrt{\dfrac{1+4+9+16}{4}}$

$= \sqrt{\dfrac{30}{4}}$

$= \sqrt{7.5} = 2.738$ km/s

8. If a gas has f degree of freedom. Find the ratio of C_P and C_V.

Ans: Total internal energy of a molecule for f degree of freedom

$$U = \frac{f}{2} K_B T$$

For one mole of gas

$$U = \frac{f}{2} N_A K_B T = \frac{f}{2} RT$$

$$C_V = \frac{dU}{dT} = \frac{f}{2} R$$

$$C_P = C_V + R = \frac{f}{2} R + R$$

$$= \left(\frac{f+2}{2}\right) R$$

$$\gamma = \frac{C_P}{C_V} = \frac{\left(\frac{f+2}{2}\right) R}{\frac{f}{2} R} = 1 + \frac{2}{f}.$$

9. Calculate the average KE of 1gm of Helium (molecular weight 4) at 127^{0}C. (Given R = 8.31 J/mol k)

Ans: Average Kinetic Energy of n moles of gas

$$KE = \frac{3}{2} nRT$$

$$T = 273 + 127 = 400K$$

n = $\dfrac{mass\ of\ gas}{molecular\ weight} = \dfrac{1}{4}$

∴ KE $= \frac{3}{2}$ x $\frac{1}{4}$ x 8.31 x 400 = 1246.5J

VI. LONG ANSWER QUESTIONS

1. Derive an expression for the pressure of an ideal gas in a container from kinetic theory and hence given kinetic interpretation of temperature.

Ans: Consider a gas enclosed in a cube of side "l"

A molecule of mass m with velocity (V_x, V_y, V_z) hits the planar wall parallel to yz plane of area A (= l^2)

since the collision is elastic, Velocity after collision is ($-V_x$, V_y, V_z)

Change in momentum of the molecule $= -mv_x-(mv_x) = -2mv_x$

Momentum imparted to this wall $=2mv_x$

The number of molecules with velocity (V_x, V_y, V_z) hitting the wall in time

$$\Delta t = \frac{1}{2} A V_x \, \Delta t \text{ x n}$$

$n \rightarrow$ number of molecules per unit volume

Total momentum imported to the wall by these molecules in Δt is

$$Q = (2mV_x)\left(\frac{1}{2} AV_x \; \Delta t \; n\right)$$

Force on the wall $= \dfrac{Q}{\Delta t}$

$$\text{Pressure} = \frac{force}{Area}$$

$$P = \frac{Q}{A\Delta t} = nmV^2_x$$

Total pressure due to all groups

$$P = nm\overline{V_x^2}$$

$\overline{V_x^2}$ is the average of V^2_x

$$\overline{V_x^2} = \overline{V_y^2} = \overline{V_z^2} = \frac{1}{3}\left[\overline{V_x^2} + \overline{V_y^2} + \overline{V_z^2}\right]$$

$$= \frac{1}{3}\overline{V^2}$$

$$P = \frac{1}{3} nm\overline{V^2}$$

$V \rightarrow$ Speed

$\overline{V^2} \rightarrow$ mean of the squared speed

$$P = \frac{1}{3} X \frac{2}{3} nm\overline{V^2}$$

$$\text{Here } n = \frac{N}{\text{Volume}} = \frac{N}{V}$$

$$P = \frac{1}{3} \text{ x } \frac{2}{2}\frac{N}{V} m\overline{V^2}$$

$$P = \frac{2}{3}\frac{E}{V} \quad (\therefore E = \frac{1}{2} N m \overline{V^2})$$

$$\therefore E = \frac{3}{2} PV = \frac{3}{2} NK_B T$$

$$\frac{E}{N} = \frac{3}{2} K_B T$$

Average transactional kinetic energy per molecule

$$\frac{E}{N} \propto T$$

As absolute temperature increases, average kinetic energy molecule also increases. Average kinetic energy of molecule is independent pressure, volume and nature of gas.

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UNIT – 13

OSCILLATIONS

I. MULTIPLE CHOICE QUESTIONS

1. Which one of the following represents the motion that is periodic but not simple harmonic?

(1) Motion of the earth around the sun **[1]**
(2) Oscillations of a loaded spring
(3) Oscillations of a balance wheel of watch
(4) Oscillations of liquid level in U-tube.

Ans: Motion of earth around the sun does'nt have to and fro motion. The acceleration does'nt proportional to displacement.

2. Identify the periodic function in the following **[4]**

(1) $e^{\omega t}$ (2) $e^{-\omega t}$ (3) $\log(\omega t)$ (4) $\sin \omega t + \cos \omega t$

Ans : The remaining may have undefined values at certain time.

3. The unit of frequency is **[4]**

(1)Sec (2) rad/sec (3) m/s (4) $(\sec)^{-1}$

Ans: Frequency $= \frac{1}{T} = \frac{1}{\sec}$

4. If the displacement of the particle in SHM is Y(t) = A sin ωt + Bcos ωt, then the amplitude of its motion is **[3]**

(1) A^2+B^2 (2)A+ B (3) $\sqrt{A^2+B^2}$ (4)AB

Ans: The resultant amplitude = $\sqrt{A^2+B^2}$, Y = ASin ω t + BCos ω t have phase difference of 90^0.

If phase difference is 0^0; resultant = A+B

If phase difference is 180^0; resultant = A-B

If phase difference is 90^0; resultant = $\sqrt{A^2+B^2}$

5. The quantity which does't vary periodically in SHM is **[4]**

(1) Displacement (2) velocity (3) acceleration (4) total energy

Ans: Displacement velocity, acceleration varies with time.

6. The expression for the maximum velocity in SHM is **[2]**

(1) $\omega^2 A$ (2) ωA (3) ωA^2 (4) $\omega^2 A^2$

Ans: Velocity v= $\omega\sqrt{A^2-x^2}$

For maximum velocity, x = 0, $V_{max} = \omega A$.
For minimum, x = A, $V_{min} = 0$.

7. The total energy of a simple harmonic oscillator is **[1]**

(1) $\frac{1}{2}KA^2$ (2) $\frac{1}{2}K^2A$ (3) $\frac{1}{2}K^2A^2$ (4) $\frac{1}{2}KA$

Ans: Total energy TE = $\frac{1}{2}m\omega^2A^2$, we know K= $m\omega^2$

$$TE = \frac{1}{2}KA^2$$

8. **The total energy of a simple harmonic oscillator is proportional to** **[1]**

(1) Square of the Frequency (2) velocity (3) amplitude (4) displacement

Ans: Total energy TE = $\frac{1}{2}m\omega^2A^2$, $\omega = 2\pi f$ (where f is frequency)

$= \frac{1}{2}m4\pi^2f^2A^2$

$= (2m\pi^2A^2)f^2$

9. The magnitude of maximum acceleration of a simple harmonic oscillator is given by **[2]**

(1) ωA (2) $\omega^2 A$ (3) ωA^2 (4) $\omega^2 A^2$

Ans: Acceleration $a = \omega^2 x$ For maximum x =A, $\Rightarrow a_{max} = \omega^2 A$

10. The time period of a simple pendulum is proportional to **[4]**

(1) Length of the pendulum (2)mass of the pendulum

(3) velocity of the pendulum (4) square root of the length of the pendulum.

Ans: Time period T = $2\pi\sqrt{\frac{l}{g}}$

II. FILL IN THE BLANKS

1. **For a simple harmonic oscillator, the restoring force is directly proportional to _____**

Ans: displacement.

For SHM $a \propto -x \Longrightarrow F \propto x$

2. **The general equation for simple harmonic motion is …………….**

Ans : $Y = ASin(\omega t + \varphi)$ or $Y = A\,Cos(\omega t + \varphi)$

General equation for SHM either sine function (or) cosine function

$$Y = ASin(\omega t + \varphi) \text{ or } Y = A\,Cos(\omega t + \varphi)$$

3. **In SHM, the value of acceleration at the mean position is _______.**

Ans: Zero

$a = -\omega^2 x$, at mean position $x = 0 \Rightarrow a = 0.$

4. **The potential energy of a Simple harmonic oscillator is maximum at ______position.**

   Ans: Extreme

   PE = $\frac{1}{2}m\omega^2 x^2$, at mean position x = 0 $\Rightarrow$PE = 0

   At extreme position x = A $\Longrightarrow$ PE = $\frac{1}{2}$ mω^2 A^2 *is maximum*

5. **The kinetic energy of Simple harmonic oscillator is maximum at ________ position.**

   Ans: Mean

   KE = $\frac{1}{2}m\omega^2(A^2 - x^2)$, at mean position x= 0 $\Rightarrow$ KE = $\frac{1}{2}$ mω^2 A^2 *is maximum*

   At extreme position x = A $\Rightarrow$ KE = 0

6. **The equation for the time period of a simple pendulum is T =_________**

   Ans: $2\pi\sqrt{\frac{l}{g}}$

   Time period T = $2\pi\sqrt{\frac{l}{g}}$.

7. **If the length of the pendulum is 100cm, then that pendulum is called___________**

   Ans : seconds pendulum.

   For seconds pendulum L= 1m =100cm, and Time period T = 2Sec.

8. **The time period of a seconds pendulum is _________**

   Ans : two seconds.

   Time period T = 2 Sec. then only it is called seconds pendulum.

9. **The frequency of a seconds pendulum is ___________**

   Ans: 0.5Hz.

   Frequency of seconds pendulum f = $\frac{1}{T} = \frac{1}{2} = 0.5Hz$

III. ONE WORD ANSWER QUESTIONS

1. **What is a period motion?**

   Ans: Repeated motion with fixed interval of time.

2. **Give any one example of a body whose motion is periodic but not oscillatory.**

   Ans: Rotation of Earth/ planet.

3. **What is the reciprocal of the time period?**

   Ans: Frequency

4. **Define amplitude**.

   Ans: Maximum displacement of the particle from mean position.

5. **Express the relation between angular frequency (ω) and time period (T).**

   Ans: $\omega = \frac{2\pi}{T}$

6. **What is the phase difference between displacement and acceleration of a particle executing simple harmonic motion?**

   Ans: π (or) 180^0

7. **What is the phase difference between displacement and velocity of a particle executing simple harmonic motion?**

   Ans: $\frac{\pi}{2}$ (or) 90^0

8. **What is the phase difference between velocity and acceleration of a particle executing simple harmonic motion?**

   Ans: $\frac{\pi}{2}$ (or) 90^0

9. **What is the maximum kinetic energy of a simple harmonic oscillator?**

   Ans: $KE_{\max} = \frac{1}{2} m\omega^2 A^2$

10. **What is the value of time period of a seconds pendulum?**

    Ans: Time period of seconds pendulum = 2 Sec

11. **What is the time period of a simple pendulum in an artificial satellite?**

    Ans: Infinity.

IV. VERY SHORT ANSWER QUESTIONS

1. **Give two examples of periodic motion which are not oscillatory.**

   Ans: The motion of a planet around the sun

   The motion of an electron around the nucleus

2. **The displacement in SHM is given by y = a sin(20t +4). What is the displacement when it is** increased by $\frac{2\pi}{\omega}$?

   Ans: Y = aSin (20t +4)

   Time period T = $\frac{2\pi}{\omega}$ increased, the displacement remains same.

3. **A girl is swinging seated in a swing. What is the effect on the frequency of oscillation, if she stands?**

   Ans: Frequency (n) = $\frac{1}{2\pi}\sqrt{\frac{l}{g}}$

   If girl stands, the length 'l' becomes decreases. Hence frequency of oscillation increases.

4. **The bob of a simple pendulum is a hallow sphere filled with water. How will the period of oscillation change, if the water begins to drain out of the hallow sphere?**

   Ans: The period of pendulum is same when bob is hollow (or) completely filled with water.

   As water flows out from the bob, the centre of gravity lowered. The length of pendulum increases. Hence time period increases.

When bob becomes empty. Again centre of gravity shifts upwards. The pendulum length decreases. Hence time period also decreases.

5. **The bob of a simple pendulum is made of wood. What will be the effect on the time period, if the wooden bob is replaced by an identical bob of aluminium?**

   Ans: Time period T = $2\pi\sqrt{\frac{l}{g}}$

   T is independent of mass of the bob. Hence time period remains constant.

6. **Will a pendulum clock gain or lose time when taken to the top of a mountain?**

   Ans: Time period T = $2\pi\sqrt{\frac{l}{g}}$

   At the top of mountain, the value of 'g' decreases. Hence time period increases. The pendulum will take longer time to complete one vibration. Hence pendulum clock will loose time on the mountain top.

7. **A pendulum clock gives correct time at the equator. Will it gain or lose time if it is taken to the poles? If so, why?**

   Ans: Time period T = $2\pi\sqrt{\frac{l}{g}}$

   g- value at pole is greater than the equator. Hence time period decreases. So pendulum clock gains time.

8. **What fraction of the total energy is KE when the displacement is half of the amplitude of a particle executing S.H.M?**

   Ans: Total energy E = $\frac{1}{2}m\omega^2A^2$, KE = $\frac{1}{2}m\omega^2(A^2 - y^2)$ (Here A → amplitude, y →displacement

   $$y = \frac{A}{2} \Rightarrow \frac{KE}{E} = \frac{3}{4}$$

9. **What happens to the energy of a simple harmonic oscillator if its amplitude is doubled?**

   Ans: Total energy E = $\frac{1}{2}m\omega^2A^2$

   A is doubled, then $E^1 = 4E$

10. **Can a simple pendulum be used in an artificial satellite? Why?**

    Ans: No. In artificial satellite g = 0. Hence we can't use simple pendulum.

V. SHORT ANSWER QUESTIONS

1. **Define simple harmonic motion? Give two examples.**

   Ans: Simple harmonic motion: A body is said to be simple harmonic, if it moves to and fro along a straight line, about its mean position. Such that its acceleration is proportional to its displacement but opposite in direction and is directed always towards the mean position.

   $$a\alpha - x$$

   Ex: Motion of a simple pendulum, Motion of mass attained to a spring

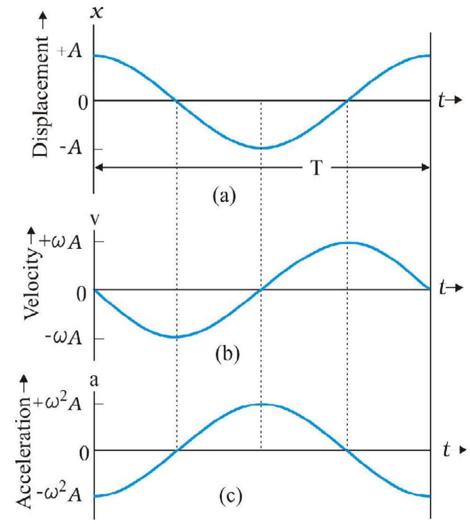
2. **Present graphically the variations of displacement, velocity and acceleration with time for a particle in SHM.**

<u>Ans:</u>

Displacement x(t) = A Cos ω t

v(t) = -Aω Sinωt

a(t)= -ω^2 Acos ωt



x(t) varies between A and –A

v(t) varies between ω A and –ω A

a(t) varies between ω^2A and - ω^2 A

velocity plot has a phase difference of $\frac{\pi}{2}$ with respect to displacement

Acceleration plot has a phase difference of π with respect to displacement.

3. **What is phase? Discuss the phase relations between displacement, velocity and acceleration in simple harmonic motion.**

Ans: Phase: Phase is defined as its state (or) position as regards to its position and direction of motion at that instant.

Displacement x(t) = $A\cos(\omega t-\varphi)$, $(\omega t-\varphi)$ is phase and φ is epoch

v(t)= $-A\omega\sin(\omega t-\varphi)$

a(t)= $-\omega^2 A\cos(\omega t-\varphi)$

phase difference between displacement and velocity = $\frac{\pi}{2}$

phase difference between velocity and acceleration = $\frac{\pi}{2}$

phase difference between displacement and acceleration = π

4. How does the energy of a simple pendulum vary as it moves from one extreme position to the other during its oscillations?

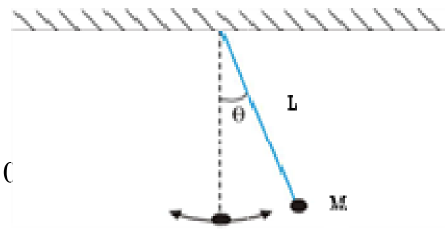
Ans: Total energy of SHO is TE = KE +PE

$$TE = \frac{1}{2}m(\omega^2)(A^2 - y^2) + \frac{1}{2}m\omega^2 y^2$$

$$TE = \frac{1}{2}m\omega^2 A^2$$

At mean position y = 0, $\Rightarrow$ PE = 0 and KE = $\frac{1}{2}m\omega^2 A^2$

$$TE = \frac{1}{2}m\omega^2 A^2$$



At extreme position y = A $\Rightarrow$ PE = $\frac{1}{2}m\omega^2 A^2$, and KE = (

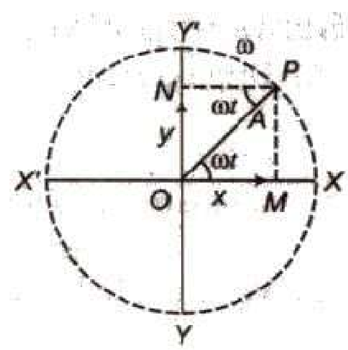
$$TE = \frac{1}{2}m\omega^2 A^2$$

$\therefore$ from mean position to extreme position KE is to be converted to PE.

5. Derive the expression for displacement, velocity and acceleration of a particle executes S.H.M.

Ans: Consider a particle p moves on the circumference of a circle of radius A with angular velocity ω. Let PN be the perpendicular drawn to the diameter YY^1 from P.

From fig sin $\theta = \frac{ON}{OP}$



$$\sin \omega t = \frac{y}{A}$$

$$Y = A \sin \omega t \text{ --------------(1)}$$

Velocity $V = \frac{dy}{dt} = A\omega \cos \omega t$

$$= A\omega\sqrt{1 - \sin^2 \omega t}$$

$$= A\omega\sqrt{1 - \frac{y^2}{A^2}}$$

$$V = \omega\sqrt{A^2 - y^2} \text{ ------------------(2)}$$

Acceleration $a = \frac{dv}{dt} = \frac{d}{dt}(A\omega \cos \omega t) = -A\omega^2 \sin \omega t$

$$a = -\omega^2 y \text{ ------------------------(3)}$$

VI. LONG ANSWER QUESTIONS

1. **Derive an expression for the kinetic energy and potential energy of a simple harmonic oscillator.**

Ans: Let the displacement of a particle executing SHM is y(t) = A sin ωt

Velocity V = $\frac{dy}{dt} = A\omega \cos \omega t$

$$v = A\omega\sqrt{1-\frac{y^2}{A^2}} = \omega\sqrt{A^2 - y^2}$$

Kinetic energy KE = $\frac{1}{2}mv^2 \quad = \quad \frac{1}{2}m\left(\omega\sqrt{1-y^2/A^2}\right)^2$

$$\text{KE} = \frac{1}{2}m\omega^2(A^2 - y^2)$$

At mean position y = 0, KE = $\frac{1}{2}m\omega^2 A^2$ is maximum.

At extreme position y = A, KE = o is minimum.

Potential energy: when the displacement of a particle executing oscillation, the restoring force also increases the restoring force in the opposite direction to the displacement. Therefore work is done in moving through the displacement, against restoring force. If F is restoring force at the displacement 'y'.

The average force against which work is done = $\frac{O+F}{2} = \frac{F}{2}$

Work done on the particle for the displacement (y) = Average force X displacement

$$\omega = \frac{F}{2} \text{X y}$$

$$\omega = \frac{may}{2} \text{---------------}1$$

The acceleration of a particle in SHM is a = - $\omega^2 y$ ---------------------2

Sub (2) in (1) the work done $\omega = \frac{1}{2}m\omega^2 y^2$

This work is stored in the form of potential energy .

$\therefore$ potential energy PE = $\frac{1}{2}m\omega^2 y^2$

At mean position y = 0 $\Rightarrow$PE = 0 minimum

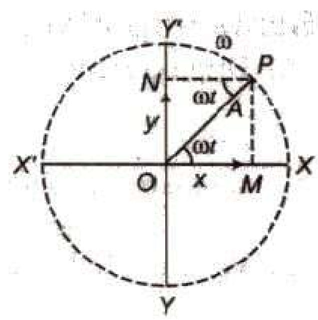
At extreme position y = A $\Rightarrow$ PE = $\frac{1}{2}m\omega^2 A^2$ maximum

2. **Define simple harmonic motion. Show that the motion of (point) projection of a particle performing uniform circular motion, on any diameter, is simple harmonic.**

Ans: Simple harmonic motion: A body is said to be simple harmonic if it moves to and fro along a straight line, about its mean position such that, at any point its acceleration is proportional to its displacement but opposite in direction and directed always towards the mean position.

Consider a particle p moving on the circumference of a circle of radius A with uniform angular velocity ω. Let 'o' be the centre of circle. Let PN be drawn perpendicular to the diameter YY^1 from P. A P moves on the circumference of the circle N moves on the diameter YY^1 to and fro about the centre 'o'.

The angular displacement at any instant $\theta = \omega t$.



$$\sin\theta = \sin\omega t = \frac{ON}{OP}$$

$$ON = OP\sin\omega t$$

$$y = A\sin\omega t$$

The velocity v = $\frac{dy}{dt} = \frac{d}{dt}(A\sin\omega t)$

$$= A\omega\cos\omega t$$

$$= A\omega\sqrt{1-\sin^2\omega t}$$

$$= A\omega\sqrt{1-\frac{y^2}{A^2}}$$

$$v = \omega\sqrt{A^2 - y^2}$$

The acceleration a = $\frac{dv}{dt} = \frac{d}{dt}(A\omega\cos\omega t)$

$$= -\omega^2 A\sin\omega t$$

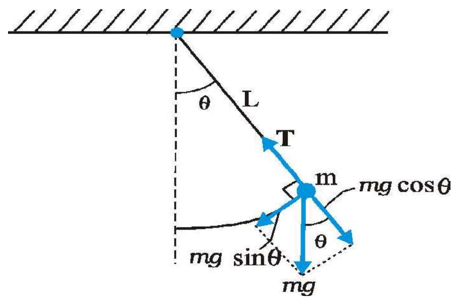
a= $-\omega^2 y$, $a\alpha - y$

hence the motion of 'N' is simple harmonic.

τ**Ans:** Consider simple pendulum, a small bob of mass 'm' tied to an inextensible mass less string of length L and other end of the string is fixed form a rigid support.

Once the bob is slightly displaced and released, it begins to oscillate about mean position. Let 'θ' be the angular displacement and T be the tension in the string.

The forces acting on the bob are
(a) tension T along the string.
(b) Weight mg acts vertically downwards.
The force mg can be resolved into two components as $mg\cos\theta$ and $mg\sin\theta$



At point P, $T = mg\cos\theta$, and $mg\sin\theta$ will provide restoring torque.

$\therefore$ the restoring torque τ = restoring force x perpendicular distance

$$= -mg\sin\theta \times L$$

$$\tau = -mgL\sin\theta \quad \text{-------------} \quad 1$$

Negative sign shows that the torque acts to reduce θ. Then

$\sin\theta$ is replaced by $\theta \Rightarrow \sin\theta \simeq \theta$

$$\therefore \tau = -mgL\theta \quad \text{----------} \quad 2$$

$$\tau = -k\theta$$

Where k = mgL is spring constant / force constant factor from eq (2) says that the bob will execute angular simple harmonic motion.

By Newton's Law of rotational note

$\tau = I\alpha$

Comparing with equation (2) $\therefore I\alpha = -mgL \times \theta$

$$\alpha = \frac{-mgL}{I} \times \theta \; \ldots\ldots\ldots \; 3$$

Comparing equation (3) with the equation $a = -\omega^2 x$

Here α resembles a and x resembles θ.

$$\therefore \omega^2 = \frac{mgL}{I} \qquad \omega = \sqrt{\frac{mgL}{I}}$$

Time period (t) : The time required for one oscillation is called time period.

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{mgL}} = 2\pi\sqrt{\frac{I}{mgL}}$$

$$\therefore T = 2\pi\sqrt{\frac{mL^2}{mgL}} = 2\pi\sqrt{\frac{L}{g}}$$ ($\therefore I = ML^2$ is the moment of inertia)

Seconds pendulum:- A pendulum whose time period is 2 seconds is called seconds pendulum.

$\therefore$ Time period of seconds pendulum T = 2 sec.

VII. PROBLEMS:

1. **A particle executes SHM such that, the maximum velocity during the oscillation is numerically equal to half of the maximum acceleration. What is the time period?**

Ans: $V_{max} = \frac{1}{2}a_{max}$

$$A\omega = \frac{1}{2}A\omega^2$$
$$\omega = 2$$
$$\frac{2\pi}{T} = 2 \Rightarrow T = \pi \sec$$

2. **A particle executing SHM has amplitude of 4cm and its acceleration at a distance of 1 cm form the mean position is 3 cm s^{-2}. What will be the velocity, when it is at a distance of 2cm from its mean position?**

Ans: Given A = 4cm, x_1=1cm,a = 3cm/s^2, x_2 = 2cm

$$a = \omega^2 x_1$$
$$3 = \omega^2(1)$$
$$\omega = \sqrt{3}$$
$$\therefore velocity = \omega\sqrt{A^2 - x_2^2}$$
$$= \sqrt{3}\sqrt{4^2 - 2^2} = \sqrt{36} = 6cm/\sec$$

3. **A simple harmonic oscillator has a time period of 2s. what will be the change in the phase 0.25s after leaving the mean position?**

Ans: T = 2sec

t = 0.25 sec

$$\sin\ \omega t = \sin\frac{2\pi}{T}t$$
$$\varphi = \omega t\ = \frac{2\pi}{T}\times t$$
$$= \frac{2\pi}{2}\times 0.25$$
$$\varphi = \frac{\pi}{4} radian$$

4. Calculate the change in the length of a simple pendulum of length 1m, when its period of oscillation changes from 2 s to 1.5 s.

Ans: $T \alpha \sqrt{L}$

$$\frac{T_1^2}{T_2^2} = \frac{l_1}{l_2} \Rightarrow l_2 = l_1 \times \frac{T_2^2}{T_1^2}$$

$$l_2 = 1m, T_2 = 1.5\sec, T_1 = 2\sec$$

$$\therefore l_2 = 1 \times \frac{(1.5)^2}{2^2} = 0.5625$$

$$\therefore \Delta l = l_1 - l_2 = 1 - 05625 = 0.4375m = \frac{7}{16} meter$$

∴ Length decreased by $\frac{7}{16}$ m.

5. A simple pendulum in a stationary lift has time period T. what would be the effect on the time period when the lift

(i) Moves up with uniform velocity

Ans: when lift moves up with uniform velocity $T = 2\pi\sqrt{\frac{l}{g}}$.

No change in time period.

(ii) Moves down with uniform velocity

Ans: when lift moves down with uniform velocity. No change in time period.

(iii) Moves up with uniform acceleration a

Ans: when lift moves up with uniform acceleration 'a'

$T = 2\pi\sqrt{\frac{l}{g+a}}$ therefore, time period decreases.

(iv) Moves down with uniform acceleration 'a'

Ans: when lift moves down with acceleration 'a'

$T = 2\pi\sqrt{\frac{l}{g-a}}$ therefore, time period increases.

(v) Begins to fall freely under gravity?

Ans: Lift falls freely a = g, $T = 2\pi\sqrt{\frac{l}{g-a}}$

Time period becomes infinite.

6. A body describes simple harmonic motion with amplitude of 5cm and a period of 0.2s. find the acceleration and velocity of the body when the displacement is.

Ans: (a)5cm (b) 3cm (c) 0 cm

A = 5cm = $5x10^{-2}$m

T = 0.2sec

(i) y = 5cm = 5 x 10^{-2}m

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 102\pi$$

Acceleration a = $-\omega^2 y = -(10\pi)^2 x5x10^{-2} = -5\pi^2 m/s^2$

Velocity v = $\omega\sqrt{A^2 - y^2} = 10\pi\sqrt{(5x10^{-2})^2 - (5x10^{-2})^2}$

(ii) Y = 3cm = $3x10^{-2}$m

Acceleration a = $-\omega^2 y = -(10\pi)^2 x3x10^{-2}$

$= -3\pi^2 m/s^2$

Velocity v= $\omega\sqrt{A^2 - y^2}$

$= 10\pi\sqrt{(5x10^{-2})^2 - (3x10^{-2})^2}$

= 0.4 π m/sec

(iii) $y = 0$

$a = -\omega^2 y = 0$

Velocity v = $\omega\sqrt{A^2 - y^2}$

$= 10\pi\sqrt{(5x10^{-2})^2 - 0^2}$

= 0.5 π m/sec

7. A freely falling body takes 2 seconds to reach the ground on a planet, when it is dropped from a height of 8 m. if the period of a simple pendulum is π seconds on the planet, calculate the length of the pendulum.

Ans: $u = 0, t = 2\sec,\ s = h = 8m$

$$s = ut + \frac{1}{2}at^2 \quad \Rightarrow g = 4m/s^2$$

$$T = 2\pi\sqrt{\frac{l}{g}},\ T = 2\pi\sqrt{\frac{l}{4}}$$

$$\Rightarrow l = 1m = 100cm$$

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UNIT – 14

WAVES

I. MULTIPLE CHOICE QUESTIONS

1. Which one of the following statement is true? (2)

1) Both light and sound waves in air are longitudinal
2) The sound waves in air are longitudinal while the light waves are transverse
3) Both light and sound waves in air are transverse
4)Both light and sound waves can travel in vacuum.

Ans :

Sound waves through solids, liquids and gasses are longitudinal in nature, whereas light waves are transverse.

2. The wave described by $y = 3\ \sin(10\pi x - 2\pi t)$, where x and y are in meters and t is in seconds, is a wave travelling along (4)

1)+ve x direction with frequency 3 Hz and wavelength λ= 0.2 m
2) -ve x direction with amplitude 3 m and wavelength λ= 0.2 m
3) -ve x direction with frequency 1 Hz and wavelength λ= 0.2 m
4) +ve x direction with frequency 1 Hz and wavelength λ= 0.2 m

Ans :

$y = a\sin(kx - \omega t)$ represents a progressive wave travelling in +ve X- direction.

Here $k = 10\pi$

$$\frac{2\pi}{\lambda} = 10\pi$$

$\lambda = 0.2m$ and $\omega = 2\pi$,

$2\pi\vartheta = 2\pi$

$\vartheta = 1$ Hz

3. Two waves are represented by the equations y_1= a $\sin(\omega t + kx + 0.57)$ m and
y_2= a $\cos(\omega t + kx)$ m , where x is in meter and t in sec. The phase difference between them is **(1)**

1) 1.0 rad
2) 1.25 rad
3) 1.57 rad
4) 0.57 rad

Ans :

The two waves can be written as

$y_1 = a\sin(\omega t + kx + 0.57)$ and $y_2 = a\sin\left(\frac{\pi}{2} + \omega t + kx\right)$

Phase difference $= \frac{\pi}{2} - 0.57 = 1.57 - 0.57 = 1.0$ rad

4. A transverse harmonic wave on a string is described by (2)

$y = 3\ \sin(36t + 0.018x + \frac{\pi}{4})$ where x and y are in cm and t in sec. which of the following is wrong.

1) Amplitude is 0.03 m
2) frequency is 36 Hz

3) speed of the wave is 20 m/s
4) It is travelling in -ve X –direction.

Ans : By comparing the equation $y = 3\sin(36t + 0.018x + \frac{\pi}{4})$ cm with the wave general equation $y = a\sin(\omega t + kx + \varphi)$

amplitude a = 3 cm =0.03

angular frequency ω = 36 rad/sec

$2\pi n = 36$

$n = 36/2\pi$ Hz

velocity of the wave $\vartheta = \frac{\omega}{K} = \frac{36}{0.018}$ = 2000 cm/s =20 m/s

The wave travelling in the –ve X direction is given by $y = a\sin(\omega t + kx + \varphi)$

all the options are correct except (b)

5. **If the tension in the string is doubled and the linear density is halved, then the ratio of initial and final speeds of a transverse wave along the string is** **(4)**

1) $1:\sqrt{2}$ 2) $\sqrt{2}:1$ 3) 2:1 4) 1:2

Ans :

The velocity of transverse wave is given by $\vartheta = \sqrt{\frac{T}{\mu}}$

$$\frac{\vartheta_1}{\vartheta_2} = \sqrt{\frac{T_1}{T_2}}\sqrt{\frac{\mu_2}{\mu_1}} = \sqrt{\frac{1}{2}}\sqrt{\frac{1}{2}} = \frac{1}{2}$$

6. **The transverse displacement of a string is given by $y(x,t) = 0.06\sin(\frac{2\pi}{3}x)\cos(120\pi t)$ where x and y are in m and t in sec. Interpret the wave as superposition of two waves travelling in opposite direction. What is the speed of each wave?**

1) 160 m/sec 2) 180 m/s 3) 200 m/s 4) 360 m/s **(2)**

Ans :

A stationary wave can be considered as superposition two transverse waves,

By comparing the equation with the standard stationary wave

$y = 2a\, sinkx\, cos\omega t$

$\omega = 120\pi$ and $k = \frac{2\pi}{3}$

velocity of transverse wave $\vartheta = \frac{\omega}{k} = \frac{120\pi}{\frac{2\pi}{3}}$ = 180 m/s

7. **Match the following an elastic wave to its mathematical equation** **(1)**

e) Stationary Wave	i) $y = 2\sqrt{x - vt}$
f) travelling harmonic wave	ii) $y = 2\cos 3x\ \sin 10t$
g) Neither travelling nor stationary	iii) $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$
h) Superposing of two stationary waves	iv) $y = \cos x\ \sin t + \cos 2x\ \sin 2t$

1) e-ii, f-iii, g-i ,h-iv 2)e-i ,f-iii , g-ii ,h-iv
3)e-i , f-ii ,g-iii ,h-iv 4)e-iv ,f-iii ,g-ii ,h-i

Ans : The general equation of stationary wave is $y = 2a\ sinkx\ cos\omega t$
harmonic wave equation is of the form $y = A \sin(kx - \omega t + \varphi)$
superposition of two stationary waves $y = cosx\ sint + cos2x\ sin2t$ is the superposition of two stationary waves.
$y = 2\sqrt{x - vt}$ doesn't represent a wave

8. A stone is dropped from the top of a tower of height h high splashes into the pond of water near the base of the tower. When the splash heard at the top, given that the speed of sound in air is V **(3)**

1) $\sqrt{2g\boldsymbol{h}} + \frac{\boldsymbol{h}}{V}$ 2) $\sqrt{2g\boldsymbol{h}} + \boldsymbol{h}V$ 3) $\sqrt{\frac{2\boldsymbol{h}}{g}} + \frac{\boldsymbol{h}}{V}$ 4) $\sqrt{\frac{2\boldsymbol{g}}{h}} + \frac{\boldsymbol{h}}{V}$

Ans : Time taken by the stone to fall in the pond at the bottom of the tower of height

h is $t_1 = \sqrt{\frac{2h}{g}}$

Time for the sound to travel a distance of h is $t_2 = \frac{h}{V}$ (since $t = \frac{d}{V}$)

Total time $t = t_1 + t_2$

$$t = \sqrt{\frac{2h}{g}} + \frac{h}{V}$$

9. The velocity of sound in air is given by $\boldsymbol{\vartheta} = \sqrt{\frac{\boldsymbol{\gamma P}}{\rho}}$, then which of the following is correct about the velocity of sound in air? **(3)**

i) dependent on pressure,
ii) Increases with temperature
iii) Increases with humidity

1) i& ii 2) i& iii 3) ii & iii 4) i,ii & iii

Ans : Velocity of sound in air doesn't depend on pressure, as the pressure increases density also increases and cancels out the effect.

Velocity of sound increases with temperature as $\vartheta = \sqrt{\frac{\gamma RT}{M}}$

Velocity of sound increases with humidity as the air has lower density

10. Why gases cannot support the propagation of transverse wave? **(2)**

1) gases are incompressible 2) gases have no shearing stress
3) density of gas is less 4) gases can flow

Ans : Transverse waves require a medium with the ability to maintain its shape and resist the deformation. Gases have no shearing stress.

11. **A pipe 20cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by 1275 Hz source?(Speed of sound in air is 340 m s^{-1})**

(1)

1)3rd 2) 5th 3) 7th 4) 9^{th}

Ans :

Fundamental frequency of closed pipe is $\nu_1 = \frac{\vartheta}{4L} = \frac{340}{4\times 20\times 10^{-2}} = 425\ Hz$

third harmonic $\nu_3 = 3\nu_1 = 3\times 425 = 1275\ Hz$

Hence 1275 Hz is third harmonic.

12. **Two vibrating tuning forks produce waves given by y_1= 5 sin $500\pi t$ and y_2= 3 sin $506\pi t$. Number of beats produced per sec.**

(2)

1) 2 2) 3 3) 4 4) 6

Ans :

By comparing the equations with $y = a\ \sin\omega t$
$\omega_1 = 500\ \pi$ and $\omega_2 = 506\ \pi$
$2\pi\nu_1 = 500\pi$ and $2\pi\nu_2 = 506\pi$
$\nu_1 = 250\ Hz$ and $\nu_2 = 253\ Hz$
number of beats/sec = $\nu_1 \sim \nu_2 = 3$

II. FILL UP THE BLANKS

1. **Mechanical waves, like sound waves require_______________ for propagation.**

   Ans : Medium

2. **If the constituents of the medium oscillate perpendicular to the direction of wave propagation, then we call that wave a__________________________.**

   Ans : Transverse wave

3. **What are the waves produced by a motor boat sailing in water______________.**

   Ans : Both Transverse and Longitudinal

4. **The distance between two successive points in a wave having same phase is called ____________________.**

   Ans : Wavelength

5. **The SI unit of angular frequency (ω) is___________________.**

   Ans : rad/sec

6. **In a______________________ wave, the displacement of an element of the medium is parallel to the direction of propagation of the wave.**

   Ans : Longitudinal

7. **The pressure variations in the propagation of sound waves are ____________ processes.**

   Ans : Adiabatic

8. **A travelling wave suffers a phase change of _________ radians on reflection at a rigid boundary.**
   Ans : π
9. **The equation y = 2a Sinkx Cosωt represents a stationary wave. The amplitude of the stationary wave is ______________________.**
   Ans : 2a Sinkx
10. **In a stationary wave, the points at which the amplitude is the largest are called____________.**
    Ans : Antinodes
11. **The lowest possible natural frequency of an oscillating system is called its_________________________**
    Ans : Fundamental frequency or First Harmonic
12. **___________________ Phenomenon is used in tuning musical instruments.**
    Ans : Beat

III. ONE WORD ANSWER QUESTIONS

1. **What does a wave represent?**
   Ans: A disturbance in the medium.
2. **Why sound wave require medium for their propagation?**
   Ans: Since they are mechanical waves.
3. **What is a longitudinal wave?**
   Ans: The vibrations of the particles of a medium are parallel to the propagation of wave.
4. **What is a transverse wave?**
   Ans: The vibrations of the particles of a medium are perpendicular to the propagation of wave.
5. **What is the phase difference between the incident and reflected waves, when the wave is reflected by a rigid boundary?**
   Ans: π or 180°
6. **What is the velocity of progressive wave, in terms of angular frequency (ω) and angular wave number (k).**
   Ans: $\vartheta = \frac{\omega}{k}$
7. **What is the SI unit of angular wave number or propagation constant (k)?**
   Ans: m^{-1}
8. **What is the fundamental mode or the first harmonic of an oscillatory system?**
   Ans: Lowest possible frequency of a vibrating system.
9. **A string is stretched between two rigid supports. What is the ratio of frequencies of vibration in such a string?**
   Ans: 1:2:3

10. **What is the ratio of harmonics produced by an air column in a long tube, closed at one end?**

Ans: 1:3:5

11. **What is the ratio of harmonics produced by an air column in a long tube, open at both ends?**

Ans: 1:2:3

12. **What is the distance between node and immediate antinode in a stationary wave?**

Ans: $\frac{\lambda}{4}$

IV. VERY SHORT ANSWER QUESTIONS

1. **Distinguish between longitudinal and transverse wave.**

Ans:

Longitudinal wave	Transverse wave
The vibrations of particles of the medium are parallel to the propagation of wave	The vibrations of particles of the medium are perpendicular to the propagation of wave
Compressions and rarefactions are formed.	Crests and troughs are formed.

2. **What is a stationary or standing wave?**

Ans: When two waves of same amplitude and frequency, travelling in opposite directions superimpose to produce stationary or standing wave.

3. **Using dimensional analysis obtain an expression for the speed of transverse wave in stretched string.**

Ans: Assume $\vartheta \propto T^a \mu^b$ (here T is tension in the string and μ is linear density)

$$LT^{-!} \propto (MLT^{-2})^a (ML^{-1})^b$$

Equating the powers of M,L and T on both sides

a = 1/2 and b = -1/2

so $\vartheta \propto T^{\frac{1}{2}} \mu^{\frac{-1}{2}}$

$$\vartheta = \sqrt{\frac{T}{\mu}}$$

4. **Using dimensional analysis obtain an expression for the speed of sound waves in a medium.**

Ans: Assume $\vartheta \propto E^a \rho^b$ (here E is modulus of Elasticity and ρ is density)

$$LT^{-!} \propto (ML^{-1}T^{-2})^a (ML^{-3})^b$$

Equating the powers of M,L and T on both sides

a = 1/2 and b = -1/2

so $\vartheta \propto E^{\frac{1}{2}} \rho^{\frac{-1}{2}}$

$$\vartheta = \sqrt{\frac{E}{\rho}}$$

5. **What is the principle of superposition of waves?**
   <u>**Ans**</u>:- If two or more waves are moving in the medium the resultant waveform is the sum of wave functions of individual waves.
   $y = y_1 + y_2 + y_3 + \ldots\ldots.$

6. **What do you understand by the term 'node' and 'antinode'?**
   <u>**Ans**</u>:- Nodes: The points at which the amplitude is zero are nodes.
   Antinodes: The points at which the amplitude is maximum are antinodes.

7. **What do you understand by natural frequency or normal mode of vibration?**
   <u>**Ans**</u>:-The frequency at which the system vibrates naturally when disturbed.
   Ex. When the tuning fork is struck, it vibrates with natural frequency.

8. **What are harmonics?**
   <u>**Ans**</u>:- The integral multiples of fundamental frequency are called harmonics.

9. **What are beats?**
   <u>**Ans**</u>:- If two harmonic sound waves of nearly equal frequency superimpose to produce waxing and waning of sound are called beats.

10. **Write down an expression for beat frequency and explain the terms therein.**
    <u>**Ans**</u>:- $\nu_{beat} = \nu_1 - \nu_2$
    Where ν_1 is the frequency of the first source
    ν_2 is the frequency of the second source.

<u>V. SHORT ANSWER QUESTIONS</u>

1. Discuss the speed of longitudinal sound waves in different media. Why solids have higher speeds of sound than in gas.?

Ans: Sound wave travels in solids, liquids and gases in the form of longitudinal waves.

In solids, the speed of longitudinal Waves is

$$\vartheta = \sqrt{\frac{Y}{\rho}}\,,$$ Where Y = Young's modulus

ρ = density of solid

In liquids, the speed of longitudinal wave is

$$\vartheta = \sqrt{\frac{B}{\rho}}\,,$$ where B =Bulk modulus

ρ = density of liquid

In gases, the speed of longitudinal wave is

$$\vartheta = \sqrt{\frac{\gamma P}{\rho}}\,,$$ Where P= Pressure

$$\gamma = \frac{C_P}{C_V}$$

ρ = density of gas

Solids have higher modulus of elasticity than gases, hence speed of sound is more in solid than gases.

2. What are Standing waves? Explain the modes of stretched string with examples.

Ans: **Standing waves:** When two waves of same amplitude and wavelength, travelling in opposite directions produce standing waves.

When a string is attached between two ends, plucked it may vibrate with one or two or more loops. Nodes and Antinodes are formed.

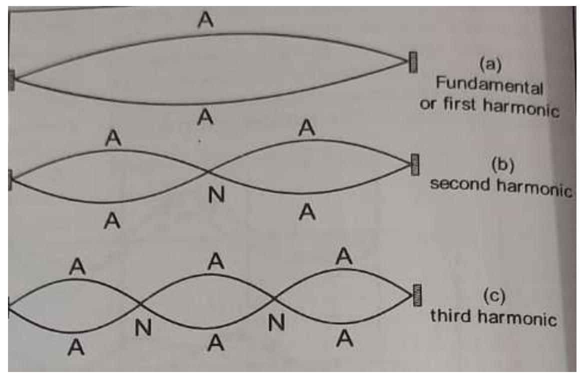
Possible wavelengths of stationary

Wave

$$\lambda_n = \frac{2L}{n}, \; n = 1, 2, 3\text{- - - -}$$

corresponding frequencies

$$\vartheta = n\frac{\vartheta}{2L}$$



for n = 1 , The lowest possible frequency is called fundamental mode

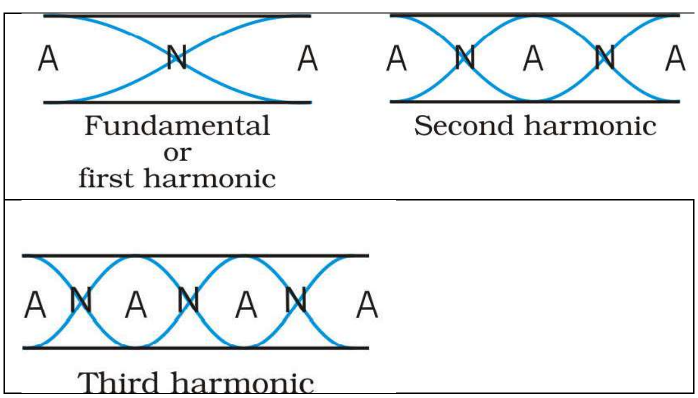
$$\nu_1 = \frac{\vartheta}{2L}$$

The n = 2 frequency is called second harmonic

The n = 3 frequency is called third harmonic etc

3. Explain the modes of vibrations in an air column in an open pipe.

Ans:



When the air in open pipe is vibrated, Antinode are formed at the open end, and nodes are form in the middle-pressure changes are largest at the nodes and while the displacement is minimum. At the open end, the anti node, it is just the other way

The possible wave length in an open pipe

$\lambda_n = \frac{2L}{n}$, n=1, 2, 3- - - -

The normal modes the natural frequencies of the system are

$\nu_n = n\left(\frac{\vartheta}{2L}\right)$

The lowest possible frequencies is called fundamental frequency

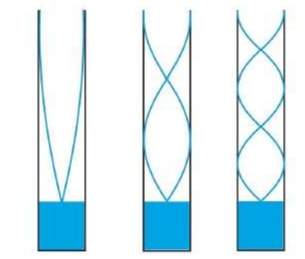
$\upsilon_1 = \left(\frac{\vartheta}{2L}\right)$ for n=1

Higher frequencies integral multiple of fundamental frequency

$\upsilon_2 = 2\left(\frac{\vartheta}{2L}\right), \quad \upsilon_3 = 3\left(\frac{\vartheta}{2L}\right)$ etc

4. Explain the modes of vibrations of air column in a closed pipe.

Ans:



When the air in a closed pipe is vibrated, node is formed at the closed end and antinode is formed at the open end. At the node the pressure changes are the largest, while the displacement is minimum. At the open end - the antinode, it is just the other way.

The possible wave length in a closed pipe

$\lambda = \frac{2L}{\left(n+\frac{1}{2}\right)} \quad \frac{\vartheta}{2L}$ for n= 0, 1, 2, 3 ----

The normal modes the natural frequencies of the system are

$\upsilon_n = \left(n+\frac{1}{2L}\right)\frac{\vartheta}{2L}$, for n= 0, 1, 2, 3 ----

The fundamental frequency corresponds to n=0 is given by

$\upsilon_1 = \left(\frac{\vartheta}{4L}\right)$

Higher frequency are odd harmonics, odd multiples fundamental frequency

$\upsilon_3 = 3\left(\frac{\vartheta}{4L}\right)$

$\upsilon_5 = 5\left(\frac{\vartheta}{4L}\right)$ etc.

5. What are beats? When do they occur? Derive an expression for beat frequency.

<u>Ans</u>: Beats: when two sound waves of slightly different in frequency interfere to produce waxing and waning of sound, called beats.

Beats/Sec = $\upsilon_1 - \upsilon_2$

Beats occur when two sound waves of nearly the same Frequency interfere with each other.
Let us assume two longitudinal waves
$S_1 = a\cos\omega_1 t$ and $S_2 = a\cos\omega_2 t$ interfere

The resultant displacement is $S = S_1 + S_2$

$$S = a[\cos\omega_1 t + \cos\omega_2 t]$$

$$S = 2a\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t \quad \cos\left(\frac{\omega_1 + \omega_2}{2}\right)t$$

The resultant wave is oscillating with average angular frequency $\left(\frac{\omega_1 + \omega_2}{2}\right)$

The frequency of the resultant wave is $2\omega_b = \omega_1 - \omega_2$

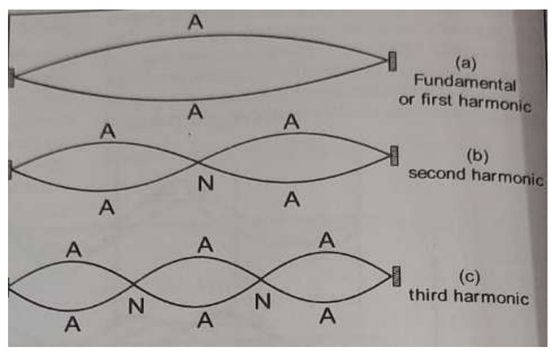
Beat frequency is given by

$$\upsilon_{beat} = \upsilon_1 - \upsilon_2$$

<u>VI. LONG ANSWERS QUESTIONS</u>

1. Explain the formation of stationary waves in Stretched strings and derive the equations for the frequencies of different modes of vibrations?

Ans: When a string is fixed at both the ends, and plucked, a transverse wave is generated, which in turn will travel and get reflected from the other end. This will go on until there is a steady wave pattern set up on the string. Such wave patterns are called standing waves or stationary waves.



Consider a wave travelling along the positive direction of x-axis

$$Y_1 = a\sin(kx - \omega t)$$

The reflected wave is along the negative direction of x-axis

$$Y_2 = a\sin(kx + \omega t)$$

The resultant wave on the string is

$Y = y_1 + y_2$ (Principal of superposition)

$Y = a[\sin(kx - \omega t) + \sin(kx + \omega t)]$

$Y = (2a\sin kx)\cos\omega t$

This equation represents a stationary wave nodes and antinodes are formed in stationary waves

Positions of nodes $x = n\frac{\lambda}{2}$, n=0, 1, 2, 3 _ _ _ _ _

Possible antinodes $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$, n=0, 1, 2_ _ _ _

Possible wave length of stationary waves

$$\lambda = \frac{2L}{n}, n = 1, 2, 3....$$

Corresponding frequencies

$$\upsilon = n\left(\frac{\vartheta}{2L}\right), \text{ here } \vartheta = \sqrt{\frac{T}{\mu}}$$

The lowest possible frequency is called fundamental or first harmonic for n=1

$$\upsilon_1 = \frac{\vartheta}{2L}$$

For n = 2

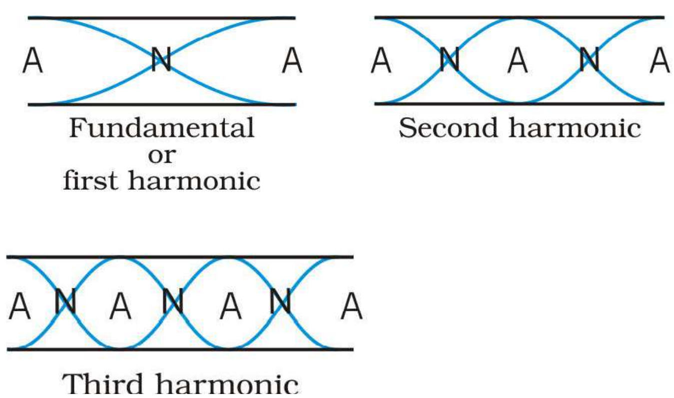
$$\upsilon_2 = 2\left(\frac{\vartheta}{2L}\right) \quad \text{Second harmonic}$$

For n = 3

$$\upsilon_3 = 3\left(\frac{\vartheta}{2L}\right) \quad \text{Third harmonic etc.}$$

2. Explain the formation of stationary waves in an air Column enclosed in open pipe. Derive the equations for to the frequencies of the harmonics produced.

Ans:



When the air in open pipe is vibrated, antinodes are formed at the open end and nodes are formed in the middle. Pressure changes are largest at the nodes while the displacement is minimum. At the open end the antinode, it is just the other way.

The possible wave lengths in an open pipe

$$\lambda = \frac{2L}{n}, \text{ n=1, 2, 3 \_ \_ \_ \_ \_}$$

The normal nodes the natural frequencies of the system are

$$\upsilon_n = n\left(\frac{\vartheta}{2L}\right)$$

The lowest possible frequency is called fundamental frequency

$\upsilon_1 = \left(\frac{\vartheta}{2L}\right)$ for n=1

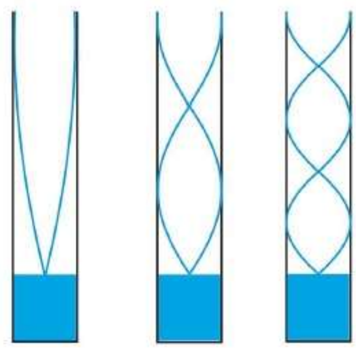
For n = 2 $\upsilon_2 = 2\left(\frac{\vartheta}{2L}\right)$ is second harmonic

For n = 3 $\upsilon_3 = 3\left(\frac{\vartheta}{2L}\right)$ is third harmonic etc

$\upsilon_1 : \upsilon_2 : \upsilon_3 = 1 : 2 : 3$

3. Explain the formation of stationary waves in closed pipe. Explain the various modes of vibrations and obtain relation for their frequencies.

Ans:



Fundamental or first harmonic third harmonic fifth harmonic

When the air in a closed pipe is vibrated, node is formed at the closed end and antinode is formed at the open end.

At the node the pressure changes are the largest, while the displacement is minimum. At the open end the antinode, it is just the other way.

The possible wave length in s closed pipe

$$\lambda_n = \frac{2L}{\left(n + \frac{1}{2}\right)}$$ for n=0, 1, 2, 3 _ _ _ _ _ _

The normal nodes or the natural frequency of the system are

$$\upsilon_n = \left(n + \frac{1}{2}\right)\frac{\upsilon}{2L}$$, n=0, 1, 2, 3 _ _ _ _ _ _

The fundamental frequency correspondents to n=0 is given by

$$\upsilon_1 = \left(\frac{\vartheta}{4L}\right)$$

Higher frequency are odd harmonics, odd multiples of fundamental frequency

$\upsilon_3 = 3\left(\frac{\vartheta}{4L}\right)$, Third harmonic

$\upsilon_5 = 5\left(\frac{\upsilon}{4L}\right)$, Fifth harmonic etc.

$\upsilon_1 : \upsilon_2 : \upsilon_3 = 1 : 3 : 5$

4. What are beats? Obtain an expression for beat, frequency where and how beats are made use of?

Ans: Beats: When two sound waves of slightly different in frequency interfere to produce waxing and waning of sound, called beats.

beats/sec = $\upsilon_1 - \upsilon_2$

Let us assume two longitudinal waves

S_1= cos ω_1 t and S_2= cos ω_2 t

When interfere, the resultant displacement is

S = S_1+S_2

S=a[cos ω_1 t + cos ω_2 t]

$$S=2a\cos\left(\frac{\omega_1+\omega_2}{2}\right)t\ \cos\left(\frac{\omega_1+\omega_2}{2}\right)t$$

The resultant waves is oscillating with average angular frequency $\left(\frac{\omega_1+\omega_2}{2}\right)$

The frequency of the resultant varies $2\omega_b = \omega_1 - \omega_2$

beat frequency is given by

$$\upsilon_{beat} = \upsilon_1 - \upsilon_2$$

Uses of beats:-

1) They are used in tuning musical instruments
2) They are used to find the unknown frequency of tuning fork.

VII. PROBLEMS

1. A stretched wire of length 0.6m is observed to vibrate, with a frequency of 30Hz in a fundamental mode if a string has a linear mass of 0.05 kg/m. Find (a) The velocity of propagation of transverse waves in the string (b) The tension in the string.

Ans: L = 0.6m

υ = 30Hz

μ = 0.05kg / m

a) Fundamental frequency of vibrating string $\upsilon = \frac{\vartheta}{2L}$

$\vartheta = \upsilon 2L$

$\vartheta = 30X2X0.6$

$\vartheta = 36m/\sec$

b) $\vartheta = \sqrt{\frac{T}{\mu}}$

T= $\vartheta^2\mu$ = $(36)^2$ x 0.05= 64.8N

2) A steel cable of diameter 3cm is kept under a tension of 10KN. The density of steel is 7.8g / cm^2. With what speed would transverse wave propagate along the cable.

Ans : T= 10KN =10^4N

D= 3cm $\Rightarrow r = \frac{D}{2} = \frac{3}{2}cm = \frac{3}{2}X10^{-2}m$

A = $\pi r^2 = \frac{22}{7}\left(\frac{3}{2}X10^{-2}\right)^2$

A $= \frac{22}{7}X\frac{9}{4}X10^{-4}m^2$

$$\vartheta = \sqrt{\frac{T}{\mu}}$$

$$\vartheta = \sqrt{\frac{T}{A\rho}}$$

$$\vartheta = \sqrt{\frac{10^4}{\frac{22}{7}X\frac{9}{4}X10^{-4}X7.8X10^{-3}}}$$

$\rho = 7.8X10^3 kg/m^3$

$\vartheta = 42.6m/\sec$

3. Two transverse waves given by y_1= 0.07 $\sin\pi$ (12x - 500t) and y_2 = 0.07sin π (12x+500t) travelling along a stretched string form nodes and antinodes what is the displacement at the (a) nodes (b) Antinodes? What is the wavelength of standing wave?

Ans : (a) At nodes, displacement

y = A_1- A_2 = 0.07 - 0.07 = 0 m

(b) At antinodes, displacement

y = A_1+ A_2= 0.07 + 0.07 = 0.14 m

(c) Wave length

$$\lambda = \frac{2\pi}{K} = \frac{2\pi}{12\pi} = 0.16m$$

4. A string has a length of 0.4m and the mass of 0.16g. If the tension in the string is 70N. What are the three lowest frequencies it produces. When plucked.

Ans: l = 0.4m, m = 0.16X10^{-3}kg, T = 70N

$$\mu = \frac{m}{l} = \frac{0.16X10^{-2}}{0.4} = 0.4X10^{-3}kg/m$$

$$n_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2X0.4}\sqrt{\frac{70}{0.4X10^{-3}}} = 523 \text{ Hz}$$

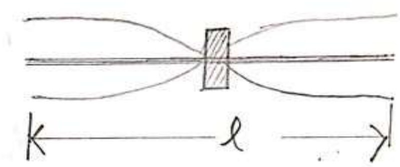
$n_2 = 2n_1 = 2X523 = 1046Hz$

$n_3 = 3n_1 = 3X523 = 1569Hz$

5. A metal bar when clamped at its centre resonates the fundamental frequency with longitudinal waves of frequency 4KHZ. If the clamp is moved to one end. What will be its fundamental resonance frequency?

Ans : A metal bar when clamped at the middle

In the fundamental mode

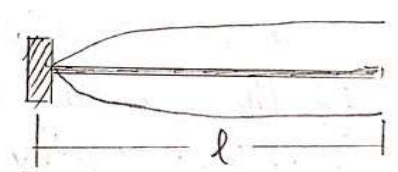


$$l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$$

Fundamental frequency $\upsilon = \frac{\vartheta}{\lambda} = \frac{\vartheta}{2l}$

$$4 = \frac{\vartheta}{2l} \ldots\ldots\ldots 1$$

When the clamp is moved to one end



$$l = \frac{\lambda^1}{4} \Rightarrow \lambda^1 = 4l$$

Fundamental frequency

$$\upsilon^1 = \frac{\vartheta}{\lambda^1} = \frac{\vartheta}{4l} \ldots\ldots\ldots\ldots 2$$

From 1 and 2

$$\upsilon^1 = \frac{1}{2}\left(\frac{\vartheta}{2l}\right)$$

$$\upsilon^1 = \frac{1}{2}(4KHZ)$$

$$\upsilon^1 = 2KHZ$$

6. A closed organ pipe 70 cm long is sounded. If the velocity of the sound is 331 m/sec. what is the fundamental frequency of vibration of the air Column?

Ans: Fundamental frequency $\upsilon = \frac{\vartheta}{4L}$

$$\upsilon = \frac{331}{4\left(70x10^{-2}\right)}$$

$$\upsilon = 118.2Hz$$

7. A vertical tube is made to stand in water so that water level can be adjusted. Sound waives of frequency 320 Hz are sent into the top of the tube If the standing waves are produced at two Successive water levels of 20cm and 13 cm. what is the speed of the sound waves in the air in the tube:

Ans: Given $\upsilon = 320Hz$

$l_1 = 20cm$

$l_2 = 73cm$

Speed of sound in air

$\vartheta = 2n(l_2 - l_1)$

$\vartheta = 2 \times 320(73 - 20)$

$\vartheta = 2 \times 320 \times 53 = 33920cm/\sec$

$\vartheta = 339$ m/sec

8. Two organ open pipes of lengths 65cm and 70cm respectively, are sounded simultaneously. How many beats per sec will be produced between the fundamental frequencies of the two pipes?
(Velocity of sound = 330 m/sec)

Ans: beats/sec $= \upsilon_1 - \upsilon_2$

beats/sec $= \frac{\vartheta}{\lambda_1} - \frac{\vartheta}{\lambda_2}$

$= \frac{\vartheta}{2L_1} - \frac{\vartheta}{2L_2}$

beats/sec$= \frac{\vartheta}{2}\left[\frac{1}{L_1} - \frac{1}{L_2}\right]$

beats/sec$= \frac{330}{2}\left[\frac{1}{65} - \frac{1}{70}\right] X 10^2$

beats/sec$= \frac{330 X 10^2}{2 X 65} - \frac{330 X 10^2}{2 X 70}$

$= 253.8 - 235.7$

$= 18 H_z$

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